ON THE INHERERENT PROPERTIES OF THE CYCLOTRON EQUILIBRIUM ORBITS

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Abstract: Critical cyclotron resonances could be successfully past in case that suficiently high energy gain per turn allows the particle jump over the resonance region or in the case that orbit is ideally centered containing no energy in the radial oscillation mode. Shift of the orbit center at the gap crossings which induces an oscillatory motion around EO giving rise to the energy content of radial oscilatory motion is an inherent property of an accelerated equilibrium orbit (AEO). At machine coupling resonances this energy content can be transfered into an axial mode eventually causing untolerable beam blow up. The positions of the AEO centers are predicted in an analytical way and compared with the results of numerical calculations. The elimination of the energy content of radial oscilation mode using beam centering techniques is considered and stability criteria are developed.

1. Introduction

Beam off-centering appears as a result of erroneous injection procedure or may be generated by imperfections in the cyclotron magnetic and electric fields. An inherent source of beam off-centering is the process of particle acceleration, due to the motion of the actual orbit centers at gap crossings |1|.

The aim of this work is to examine the stability criteria of the accelerated equilibrium orbits and to explore the effects of orbit centering using an AEO as an entry point.

2.1. Motion of the accelerated equilibrium orbit center

In the cyclotron magnetic field where the particle is accelerated by an RF system having N symmetrically distributed dees each D=180/N degrees wide, the charged particle of energy E and angular frequency ω equal to the one of the subharmonics h of the cyclotron RF frequency ω_{rf} at each of 2N gap crossings will increase its energy by an amount:

$$dE_{ij} = qV \sin hD/2 \cos \phi_{hf\ ij}$$

where

$$\phi_{ij} = \omega_{rf}\ t - h\ (\theta_{ij} - k_{ij})$$

is the high frequency phase of the particle measured relative to the phase of the dee voltage, having zero value when the particle phase coincides with the dee voltage, while

θ_{ij} - angular position of ith gap

k_{ij} - local phase slip at ith gap position.

Due the particle energy gain at the gap crossing, the orbit radius r will increas to the r+dr. The result is a change of the position of the orbit center:

$$dl = -\ dr = -R\ dE\ /\ 2\ E\ \nu_r^2$$

while the particle continue to move along the orbit radius of curvature r+k dr (k=1,2,...2N). The particle which was in phase with the RF field before the gap crossing will experience a phase slip after gap crossing. The phase slip reflects the diference in the effective length of the particle sector trajectories before and after gap crossings. The relevant features of the particle trajectory in a homogenous magnetic field for N=3 dees are shown in fig.1.

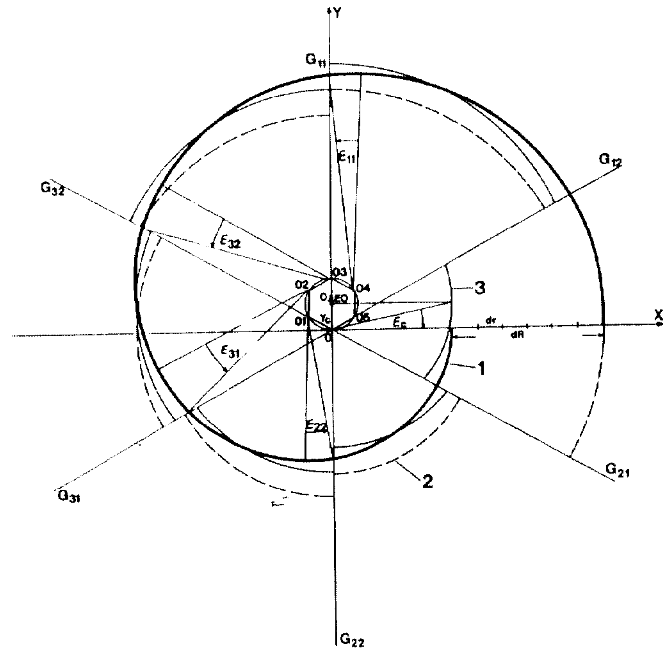

Fig.1. The actual particle orbit accelerated by 3-dee (60 deg wide) symmetrical accelerating structure in the homogenous magnetic field. The particle starts from SEO. The gap crossing lines are G_{ij} lines. The thick solid line represents the actual particle orbit while the thin solid line and dashed lines represent the efective and actual SEOs of the coresponding sectors, respectively.

After a particle rotation of 360 degrees the radius of orbit increases by:

$$dR = 2N\ dr$$

while the positions of the orbit center have described the N-polygonal regular geometrical form of circumference dR.Obviously the central ray of the accelerated orbit does not follow the line of a centered SEO, implying that the center of the AEO does not coincide with the center of the cyclotron magnetic field. The center of the polygon described by the motion of the orbit centers during the considered period of particle acceleration can be identified as the center of an AEO.

In the coordinate system defined in fig.1, the position vector of the AEO center is given by:

$$x_c = 0$$

$$y_c = dR/6$$

Thus the AEO motion can be described in terms of center and circle motion |2|. The AEO central position phase will then correspond to the central position phase of the equivalent static equilibrium orbit which has to be superimposed on the motion of the orbit center in order to describe the actual particle motion. The central position phases of the static equilibrium orbit and the equivalent static equilibrium orbit then define the reference phase axis.

From fig. 1 it appears that the inherent phase slip of an AEO is given by

$$h \varepsilon_c = - h \arcsin (y_c/R)$$

The definition of the AEO center as the polygon center, implies that the position of the AEO center depends on the configuration of the accelerating dee structure. The orbit center moves paralel to the gap line. Thus, in the case of spiral gaps, the polygon representing the motion of the orbit centers will rotate around the axis which passes through the origin of the coordinate system, by an angle α which equals the angle between the vector representing the electric force and the tangent to the circle of radius r at the given gap position. This means that in the case of spiral gaps the position vector of AEO center reflects the direction of the electric force in the gap. The coordinates of the orbit center in the case of the symmetrical dee configuration with N = 3 are given by:

$$x_c = (dR/2N) \sin\alpha$$

$$y_c = (dR/2N) \cos\alpha$$

2.2. Phase space behaviour of AEO

Knowledge of the orbit center position in the course of one revolution in x,y coordinate space in fig.1, where $y=x'= dx/d\theta$, makes it possible to map orbit center motion from real space into (x,px) phase space. Using coordinate transformations:

$$px = (p/r)x'$$

or in cyclotron units

$$px/m_o\omega_o = \gamma \nu_r x'$$

where γ is the relativistic factor, the phase space coordinates of the AEO (polygon) center becomes:

$$x_c = x_c$$

$$px_c = \gamma \nu_r y_c$$

while α has to be replaced by α' in such way that

$$tg\alpha' = (tg\alpha) / \gamma\nu_r$$

If the particle entry point coincides with the AEO, the particle phase point will, after one revolution, coincide with one of the point describing the actual AEO.

2.3. Phase slip induced by gap crossing

The particle which starts its motion at equilibrium orbit (see fig. 1) and crosses the first gap in the phase with peak dee voltage will have a phase slip ε_{22} at the position of the gap G_{22}, due to the shift of the orbit center at point 0_1, phase slip ε_{22} at position G_{13}, due to the shift of the orbit center at point 0_2 etc. These considerations define the local HF phase at a particular gap crossing:

$$\Phi_{hf\ ij} = \omega_{rf}\ t - (\theta_{ij} - \varepsilon_{ij})$$

In the case of an AEO motion, the nonuniformities in the energy gain due to the shifts of the orbit centers at the gap crossings tend to be compensated so that the average value of the HF phase is given by:

$$\Phi_{hf} = \langle\omega_{rf}\ t_i - h\ (\theta_i - \varepsilon_i)\rangle + h\ \varepsilon_c$$

where ε_c defines the central position phase as

$$\Phi_{cp} = h\ \varepsilon_c$$

These considerations lead to the conclusion that if the particle follows an AEO i.e. the best centered orbit during the process of the particle acceleration the actual phase slip should equal the phase slip of SEO i.e.

$$\Phi_{hf} = \langle\omega_{rf}\ t_i - h\ \theta\rangle = f(E)$$

since $\langle\varepsilon_{ij}\rangle$ equals $-\varepsilon c$. This value of Φ_{hf} represents the phase of the maximum energy gain per turn. The SEO phase slip can be calculated from the magnetic field error function F(E) |4|.

3. Numerical results and discusion

The concept derived above were applied in the analyses of beam responses to the properties of the magnetic field of the Milan superconducting cyclotron, described elsewere |4|. The analyses were performed using isochronized average magnetic field data with a superimpose modulation field of three-fold symmetry.

The only perturbation which can cause the shift of the orbit center in the case of the ideally symmetric AVF cyclotron magnetic field is the action of the electric field at the gap crossings. The average values of x, px coordinates at the positions of the middlines of three consecutive hills

$$x_c = (x_1 + x_2 + x_3) / 3$$

$$px_c = (px_1 + px_2 + px_3) / 3$$

should reflect this perturbation thus becoming a measure of the corresponding shift of the orbit center. If compared to the values calculated from the derived formulae

$$xc = (dR(E) / 6) \sin \alpha(E)$$

$$px = \gamma\nu_r\ (dR(E) / 6) \cos \alpha(E)$$

the agreement of the results can be taken as a measure of the validity of the introduction of the concept of an AEO center.

The dynamical phase plots representing the process of the acceleration of the ion representative Z/A=0.5 at B_o=2.2 T (at applied centering procedure) in the first 100 turns during which the significant effects of the shift of AEO center is expected are shown in figs. 2a, 2b and 2c. The formulae predictions are found to be in very good agreement with the results of the numerical calculations.

The phase slips computed using the EQUILIBRIUM ORBIT code for an SEO and SPIRAL GAP code for an AEO for a representative ion Z/A=0.5 at center field B_o=2.2 T are given in fig.3. As may be expected, the phase slip of centered AEO, nearly reproduces the phase slip of the SEO. As may be expected |3| , the phase slip of 5mm off-centered particle oscillate around the average phase slip with the frequency of the precession of the phase plot.

The number of ways that the particle can miss the SEO is given by the set of points $Nr=1/|\nu_r-1|$ which describe the phase space contour in the linear region around an SEO. Each of these points may be considered as the central ray of the off-centered beam.

The scheme used in the investigation of the influence of the off-centered ray initial conditions on the orbit properties of the ion representative Z/A=0.5 B_o=2.2 T is shown in fig.4a. The set of the points marked A,B,C,D on the Δr = 5mm phase space contour has been chosen as test particles.

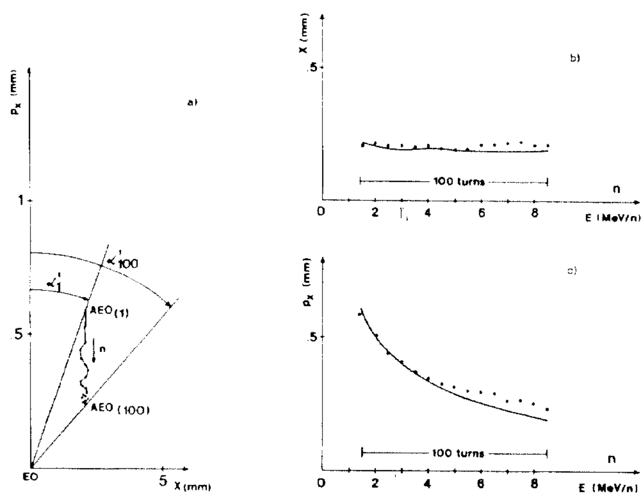

Fig.2. The change of the positions of x, px coordinate pairs during the accelerations of the ion representative Z/A=0.5 at B_0=2.2 T over the first 100 turns. The effective spiral angle and the changes of the position of AEO center are shown in fig. 2a (solid line indicates the formulae predictions, the dots are the resultults of the SPIRAL GAP calculations in three points). The energy dependence of the values of the coordinates is shown in figs. 2b and 2c.

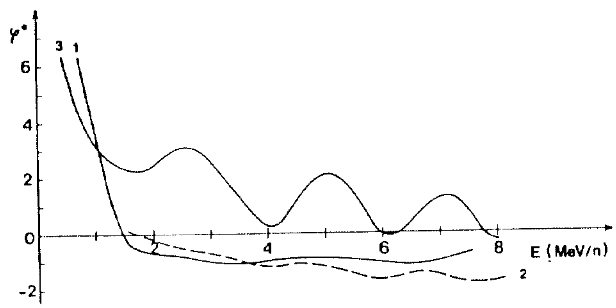

Fig.3. Phase slips, as calculated by EQUILIBRIUM ORBIT code for SEOs (1), and the SPIRAL GAP code for AEOs (2) for ion representative Z/A=0.5 at B_0=2.2 T. The phase slip of 5mm off-centered particle (3) oscillates with the frequency of the precession cycle.

It the entry point coincides with one of these points this particle will oscillate around the AEO i.e. its phase point will precess. The off-centered orbit coresponds to a radial oscillation with amplitude

$$A = ((x1-xc)^2 + (px1-pxc)^2)^{1/2}$$

combined with an angular oscillation amplitude |3|

$$\Delta\theta = A/R1$$

which introduces an oscillatory component in the phase history of the local HF phase. The angular frequency of such a precession is given by

$$\omega_p = 2\pi(\nu_r-1)$$

The average value of the local HF phase is strongly dependent on the initial phase space coordinates of the off-centered particle as is shown in the fig. 4b.

Assuming again that the average values of the x,px coordinates at the positions of the middline of the three hills should reflect this perturbation we find that each of the starting phase points A,B,C,D define diferent (x_c, p_{xc}) coordinates pair as is shown in fig. 4b. The result of this analysis clearly indicate that the high frequency phase changes and the central position phase changes are orthogonal coordinate pair.

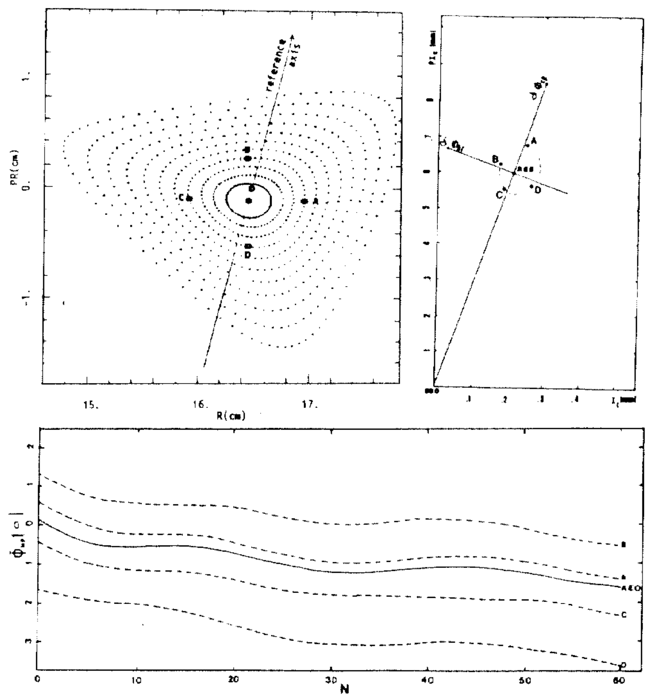

Fig.4. Static phase plot of ion represetative Z/A=0.5 at Bo=2.2 T and energy E/A=1.5 MeV. The chosen set of test particles are indicated by A,B,C,D. (a). The phase space histories of the average HF phases for test particles (b). The (x_c, p_{xc}) coordinates responces of test particles. The ϕ_{cp} changes and ϕ_{HF} changes axes are also indicated in the plot (c)

Summary

The energy content of the particle oscillation mode may appear in the consequence to an error in the evolution of the orbit matching procedure.

The positions of the AEO center may be predicted applyng the approximative formulae. These positions can be determined accurately by applyng a corresponding numerical procedure.

A centered AEO approximately reproduces the same phase slip, as is obtained in SEO calculations.

It appears also that the central position phase is the conjugate coordinate to the HF phase.

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