THE EFFECT OF PERTURBATION ON THE LONGITUDINAL MOTION OF PARTICLES IN A STEPPED PHASE VELOCITY LINAC

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Abstract

We investigate the influence of the deviations of the electrical and geometrical parameters from their design values on the longitudinal motion of particles in the side-coupled linac. The behaviour of the longitudinal acceptance (displacement in energy, decreasing and formation of breaks) for different types of perturbations is determined. We suggest a method of mutual compensation of various errors in the accelerating modules. It is found that the definite spatial variation of the accelerating field within each module results in substantial increasing longitudinal acceptance.

Introduction

The particle loss in the intense linear accelerators of meson factories with average current 1 mA are restricted strongly by requirement of radioactive purity. Therefore, after increasing of radio frequency (by factor of 4÷5) and according decreasing of longitudinal acceptance in the high-energy part of linac (W 100 MeV), we shall have serious problems with the particles near the separatrix. It becomes necessary to design damping system for the coherent oscillations, longitudinal and transverse particle filters and to work out special procedures for tuning the accelerator [1,2]. One of the main reasons of particle loss is connected with deviation of parameters of the accelerating structure from their design values.

The high-energy part of the linac of Moscow Meson Factory (MMF) constitutes 27 cavities. Each cavity consists of four tanks interconnected by coupling bridges. The tank incorporates from 19 to 28 accelerating cells of the same length. The errors of the accelerating channel include deviations of the sizes of the accelerating structure elements from the design values, space distortions of the field along the cavity, as well as the inaccuracy of setting the amplitude and phases of the accelerating RF field. The instability of the amplitude and phase of the RF field leads to increasing of the effective beam sizes, which can be obtained by numerical simulation. Below, for studying the effect of perturbations on the beam motion, we use an approach based on the notion of quasi-synchronous particle. This method enables one to determine the degree of effect of perturbation depending on its wavelength for any cavity of the accelerator.

Perturbations of the phase motion of particles in the tank

The accelerating tanks consist of identical cells, i.e. have a constant phase velocity β_p . In this structure there is no usual synchronous particle, i.e. no particle which moves along the whole length of the accelerator with a velocity equal to the velocity of the wave and, consequently, performs no phase oscillations. On the other hand, if the tank is considered as one accelerating cell, it is possible to introduce the notion of a quasi-synchronous particle, which is analogous to the definition of a synchronous particle in the accelerator with a smooth change in the accelerating cells. A particle is called quasi-synchronous which in each macroperiod "tank + drift space" has a velocity increase equal to the jump of the phase velocity from tank to tank[3]. Using the definition of quasi-synchronous motion, namely the equality of the input phases in each tank and constant velocity increase in the tank, it is possible to find to what changes in the parameters of the quasi-synchronous particle some or other deviations of the tank parameters lead. In this case the particles perform phase oscillations in the cavity with respect to a "new" quasi-synchronous particle. It follows from the analysis of the solutions of the equations of motion that the change in the velocity of the quasi-synchronous particle (or in the effective phase velocity), associated with deviations of the geometrical sizes of the cavity from the design values, takes the form [3]

$$\frac{\delta\beta_s}{\beta} = \frac{\Delta L}{L}\,, \tag{1}$$

where $L = L_t + L_d$, L_t is the length of tank L_d is the length of drift space before and after the tank. The relation (1) yields the important conclusion about the possibility of simple compensation in practice for the errors of tank lengths by changing the lengths of the drift spaces between the tanks result to $\Delta L = 0$.

Using the conditions of quasi-synchronous motion formulated above, we find that in the case of slope of the field in the tank with amplitude $\mathcal{E} = \frac{\Delta E}{E}$

$$\frac{\delta\beta_s}{\beta} = \frac{\mathcal{E}}{3}\cdot\frac{L_t}{L}\cdot\frac{\beta_o-\beta_p}{\beta_p}\,, \tag{2}$$

β_o is the input design velocity of the tank. Fig. 1 presents for these perturbations the value of $\frac{\delta\beta_s}{\beta}$ as a function of the cavity number N (curves I, II).

Similarly one can also consider the influence of the other field perturbations in the tank. With decrease in their wavelength Λ the deviation of the quasi-synchronous velocity is described by Λ^2 . In order to answer the question about the influence of perturbations with a wavelength greater than the macroperiod length, it is necessary to consider the particle motion in the cavity. Determination of the value $\frac{\delta\beta_s}{\beta}$ for each cavity is of great importance for its tuning and characterizes the degree of "nonideality" of the cavity [2].

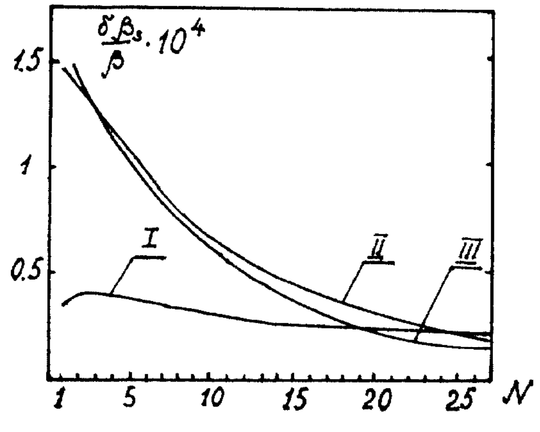

Fig. 1 Dependence of the value of $\frac{\delta\beta_s}{\beta}$ on the cavity number N with a change in the macroperiod length by ΔL=100 mkm (curve I), field slope amplitude in the tank ε = 0.1 (curve II), field slope amplitude in the cavity ε = 0.01 (curve III)

Small oscillations of particles in the cavity

For the deviations from the design values of the phase $\Delta\varphi = \varphi - \varphi_s$ and the velocity $\Delta\beta = \beta - \beta_s$, in the conservative approximation we obtain equations which describe small oscillations of the particles in the cavity with perturbation of field $E_0[1+\varepsilon\mathcal{F}(z)]$

$$\frac{d^2\Delta\varphi}{dz^2} + K_0^2\,\Theta(z)[\Delta\varphi(1+\varepsilon\mathcal{F})+\varepsilon\mathcal{F}\,tg\,\varphi_s], \quad (3)$$

$$\frac{d\Delta\varphi}{dz} = -\frac{\omega\Delta\beta}{c\beta\beta_s}, \quad (4)$$

where $K_0^2 = eE_0\,|\sin\varphi_s|\,\omega/(m_p c^3\beta_p\gamma_p^3)$,

function $\Theta(z) = \begin{cases} 1, & z \in L_t \\ 0, & z \in L_d \end{cases}, \quad \gamma_p = (1-\beta_p^2)^{-1/2}$.

Any perturbation of the field in the cavity can be represented as the superposition of harmonic perturbations. For linear oscillations the solution is the sum of solutions found for the harmonics of the Fourier expansion. For $\mathcal{F}_n = \cos\frac{\pi n z}{4L}$, where integre n = 1,2..., the deviation of the synchronous phase is

$$\delta\varphi_s = -\frac{\varepsilon\,tg\,\varphi_s}{(\frac{\pi n}{\mu})^2-1}\cdot\frac{[1+(-1)^n]}{2} \quad (5)$$

and the change in the quasi-synchronous velocity is determined by the relation

$$\frac{\delta\beta_s}{\beta} = -\frac{\varepsilon\,tg\,\varphi_s}{(\frac{\pi n}{\mu})^2-1}\,\frac{K\beta_s c}{\omega}\,\frac{\sin\mu}{1-\cos\mu}\,\frac{[1-(-1)^n]}{2}, \quad (6)$$

where $K = \sqrt{\frac{L_t}{L}}\,K_0$, $\mu = 4KL$ is phase advance in the cavity, ω is radio frequency. At n=1,3 $\delta\varphi_s$ =0 and $\frac{\delta\beta_s}{\beta}$ =0 at n=2,4. With increasing number n the effect of perturbation on the quasi-synchronous motion decreases ($\sim n^2$). The maximum effect on the beam is caused by perturbations with n=1 (in many cavities $\mu \sim \pi$). At n=1 we have the so-called field slope in the cavity. Fig. 1 (curve III) presents the dependence of the value of $\frac{\delta\beta_s}{\beta}$ on the cavity number in MMF at the relative amplitude of the field slope in the cavity $\varepsilon = \frac{\Delta E}{E}$ =0.01.

Similar calculations can also be made for $\mathcal{F}_n = \sin\frac{\pi n z}{4L}$. In this case

$$\frac{\delta\beta_s}{\beta} = -\frac{\varepsilon\,tg\,\varphi_s}{(\frac{\pi n}{\mu})^2-1}\,\frac{\pi n}{\mu}\,\frac{K\beta_s c}{\omega}\cdot\frac{[1+(-1)^n]}{2} \quad (7)$$

For perturbations, which do not change the average field level in the cavity, n=2.4, and $\delta\varphi_s$ =0.

The effect of change in the quasi-synchronous parameters on the longitudinal acceptance (results of numerical simulation)

The change in the parameters of a quasi-synchronous particle describes the coherent behaviour of the acceptance. The results obtained make it possible, for example, to calculate the change in the level of the synchronous energy at the accelerator output in the presence of perturbations which lead to the deviation $\frac{\delta\beta_s}{\beta} \neq 0$ of the same sign in different cavities. This fact was verified during the numerical simulation of the acceptance of the second part of the MMF for the changed lengths of the accelerator macroperiods [3]. Similar results have also been obtained for the deviations of the electrical parameters of the system: in the case where the field slopes have the same sign in the tanks or cavities of the accelerator on the phase plane (φ, β) the capture region is lifted or lowered coherently (depending upon the sign of ε) relative to the design synchronous energy value.

At the same time the alternation of sign of the deviation $\frac{\delta\beta_s}{\beta}$ from cavity to cavity in the case of phase advance of small longitudinal oscillations $\mu \sim \pi$ leads to the resonance build-up of coherent phase oscillations, the capture region being decreased. Depending upon the initial perturbation phase (e.g. the sign of slope in the first cavity) "breaks" appear on one or the other side of the acceptance (see Fig. 2).

The simultaneous effect of these perturbations results in restoring the capture region. For comparison Fig. 3 gives the design and restored capture regions at the input of the second part of the MMF. The perturbation amplitudes (ε = 0.02 and ΔL = 600 mkm) are selected so that in the first cavities the resulting deviation is $\frac{\delta\beta_s}{\beta}$ =0 (see Fig. 1).

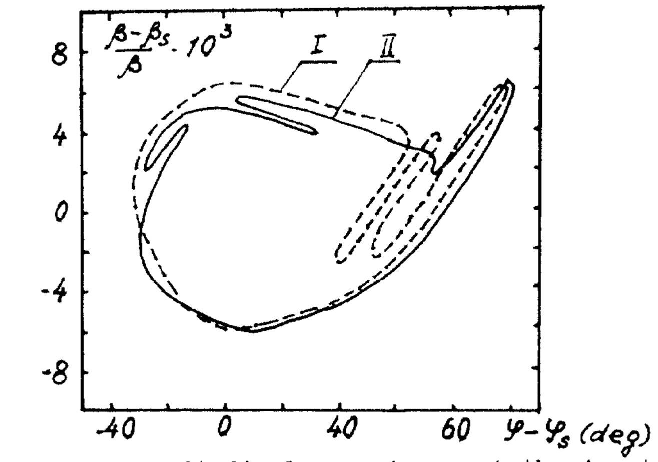

Fig. 2 Longitudinal acceptance at the input of the high-energy part of the MMF with the field slopes in the cavity with the amplitude ε =0.02 and changing of signs according to the law $\frac{\Delta E}{E} = \varepsilon (-1)^N$ (curve I) and with the deviations of the lengths of the drift spaces between the tanks ΔL_d=600 $(-1)^{N+1}$ mkm (curve II), where N is the cavity number

We have found previously the variation of the field, which leads, according to the relation (5), to a change in the phase of the quasi-synchronous particle $\delta\varphi_s$ with no change in the average amplitude of the field in the cavity. During the numerical simulation of the effect of perturbation $\mathcal{F}_n = \cos\frac{\pi n z}{4L}$ at n=2 the corresponding increase in the acceptance in comparison with the design case for $\varepsilon < 0$ and its decrease for $\varepsilon > 0$ were revealed (see Fig. 4).

As has also been checked numerically, the synchronous energy level has the design value if, as a result of the effect of various perturbations, $\frac{\delta\beta_s}{\beta} = 0$ in each of the cavities.

In tuning the accelerator in the LAMPF, initially attempts to obtain the capture region of the design value failed due to the

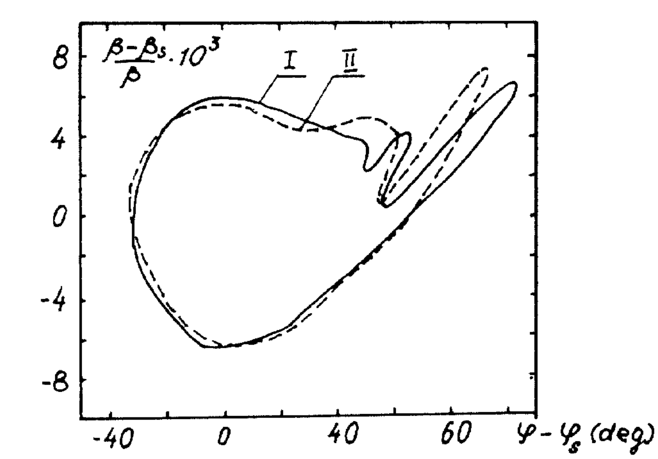

Fig. 3 Restored (curve I) and design (curve II) longitudinal acceptance at the input of high-energy part of the MMF

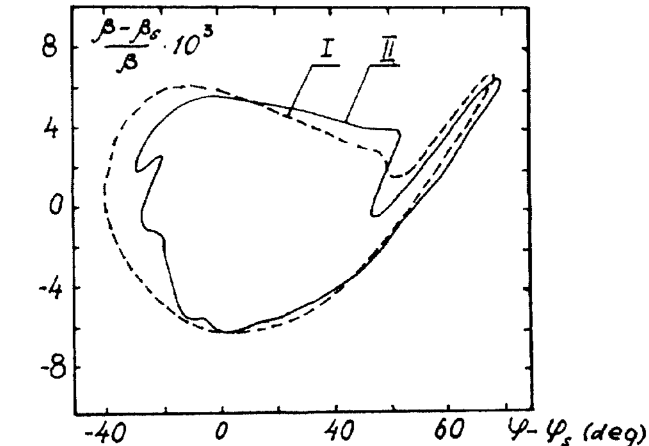

Fig. 4 Longitudinal acceptance at the input of the high-energy part of the MMF in the case of field perturbations in the cavity of the type $\varepsilon \cos\frac{2\pi z}{4L}$ at ε = -0.05 (curve I) and ε = 0.05 (curve II)

deviations of the longitudinal sizes of the accelerating tanks [4]. In that case an attempt was made to preserve it by creating slopes of the field amplitude in the cavities. However, the searches had random character and have not led to the desirable results [5]. This seems to be attributable to the fact that compensation for the perturbation is only possible at the point of its appearance. It was demonstrated [2] that the tuning of the accelerator with the aid of the ΔT-procedure makes it possible to determine the value of the deviation $\frac{\delta\beta_s}{\beta}$ in the cavity tuned and, consequently, to directly compensate for perturbations.

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