ITACA: A NEW COMPUTER CODE FOR THE INTEGRATION OF TRANSIENT PARTICLE AND FIELD EQUATIONS IN AXI-SYMMETRICAL CAVITIES

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INTRODUCTION

We present a method for the numerical simulation of the interaction between high charged bunches and axi-symmetric fields inside a cylindrical structure (RF cavity or wave guide). A computer code named ITACA (Integration of Transients in Axi-symmetrical Cavities for Accelerators) has been written on the basis of the method presented here.

ITACA is an electromagnetic particle-in cell code able to study both the radial and axial motion of a bunch of particles moving through a cavity, and the propagation of the wake-field excited by the bunch itself inside the cavity. The code can inject the particles of the bunch into the resonant field of the accelerating mode present inside the cavity, making possible to study the dynamics of the bunch in presence of all the relevant forces acting on it: the accelerating field, the space charge forces due to the self-field of the bunch and the wake-field excited by the interaction between the self-field of the bunch and the boundary condition imposed by the cavity surface.

The computer code ITACA can handle, presently, axisymmetrical fields and bunches, allowing to study monopole wake fields and their effects on the particles. The code has been designed in order to represent with accuracy the behaviour of the fields and the dynamics of the particles: the first tests indicate that this goal has been reached.

INTEGRATION OF TRANSIENTS AXI-SYMMETRICAL FIELDS

It is well known that axisymmetrical fields in a cylindrical cavity can be expressed as functions of a scalar potential, which is given by $\Phi_h=r\cdot H_\phi$ for fields which can be expanded in a sum of TM_{0np} modes (i.e. TM-like fields), and is given by $\Phi_e=r\cdot E_\phi$ for fields which can be expanded in a sum of TE_{0np} modes (i.e. TE-like fields).

Writing down the wave equations for axi-symmetrical E and H fields in a system of cylindrical coordinates, it can be seen at a glance that, in case of a cylindrical driving current, having only radial and axial components, the two set of fields (the TM-like, specified by H_ϕ, E_r, E_z and the TE-like, specified by E_ϕ, H_r, H_z) are independent. Moreover, the driving current excites only the TM-like fields. Since a particle moving in a TM_{0np} field experiences only a radial and an axial force (but no azimuthal one), it produces, inside a cavity, a driving current which can couple only to TM-like fields. Then, the interaction between a bunch of particles and the field inside the cavity can be fully described solving the wave equation for the $\Phi=r\cdot H_\phi$ potential, as a function of r,z,t, in presence of a driving current $J(r,z,t) = (J_r(r,z,t),J_z(r,z,t))$:

1) $$\mathcal{L}\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = r\frac{\partial J_z}{\partial r} - r\frac{\partial J_r}{\partial z}$$

$$\mathcal{L} = \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}$$

together with the boundary condition $\frac{\partial\Phi}{\partial n} = 0$ on the cavity surface.

This typical hyperbolic equation gives, starting from some initial condition of the field distribution at t=0, the time evolution of the fields, provided that the driving current is a known function.

To solve this equation we adopt the standard technique of the FDM, discretizing the fields over a regular rectangular mesh covering all the cavity section in the r-z plane. The numerical stability of this well established technique is assured if the time integration step (τ=ct) is below a threshold given by [2]:

$$\tau < 1/\sqrt{\frac{1}{\epsilon^2} + \frac{1}{h^2}}$$

where ε and h are the mesh-steps in r,z respectively.

A special treatment for curved boundary has been developed. At the location of all the mesh points close to the boundary, the field has been expanded in a Taylor series up to the II order and the boundary condition has been imposed on the intersections between the mesh lines and the boundary. This procedure allows to handle all the special points near to the boundary with the same equation as for the normal ones, making possible to describe the field over a regular mesh with the same accuracy of an irregular one. The total amount of memory needed is reduced: in fact the gain in memory given by the regular mesh overcomes the lost due to the special boundary points handling. We think moreover that a regular mesh becomes recommended for short high charged bunches, because the field contains high order spatial harmonics (of wavelength comparable to the mesh step) which are excited by the bunch and propagate through the cavity. These harmonics, eventually excited in some point of an irregular mesh, couldn't propagate through other regions of the mesh with larger discretization steps.

The algorithm for the field integration has been tested starting the computation at t=0 from the field distribution of the fundamental accelerating π-mode of the LEP SC cavities: during three RF periods (ν=352 MHz) of integration, the field distribution follows the harmonic evolution in time with a great accuracy, reproducing at the end of the third period the starting condition with a maximum error of a few parts per thousand (in the case of a mesh with 20000 points, i.e. a mesh step of 6 mm). Other tests have been performed against the two typical analitycal cases of the pill-box and of the spherical resonators, giving similar results with meshes having a few thousands of points.

COUPLING THE PARTICLES TO THE FIELD

The driving current is produced by the bunch of particles which moves around the axis of the cavity. Since the driving current must be axi-symmetrical, the particles of the bunch are free to move only in the r-z plane, so that the current and charge densities associated to them are the same as the ones produced by rings of charge centered on-axis, free to move axially and to expand radially. In order to derive, from the distributions of both particles and their velocities, the current density distribution, we adopt the gaussian assignment algorithm [3], which treats the bunch as constituted by gaussian axi-symmetrical sub-bunches.

Choosing a value for the gaussian width close to the mesh step, it can be seen that the unphysical fluctuations in the charge density (and in the current density) distribution, due to the assignment algoritmh, are quite low (less than 1% for a uniform distribution)[3]. This property of the assignment algorithm is very important, because each fluctuation of the driving current distribution excites an unphysical spatial harmonic of the field and it can cause instability in the field integration [4].

THE CODE ITACA

The computer code ITACA solves simultaneously the equation 1) for the field propagation, and the equation of motion for the particles.

The electric field components are derived integrating versus the time one of the Maxwell equation, over a reduced mesh which covers the region around the axis where the bunch is expected to move.

$$\begin{cases} \dfrac{\partial E_r}{\partial \tau} = -\dfrac{\partial H_\phi}{\partial z} - J_r \\ \dfrac{\partial E_z}{\partial \tau} = \dfrac{1}{r}\dfrac{\partial (rH_\phi)}{\partial r} - J_z \end{cases}$$

Knowing the field H_ϕ at each point of the mesh (both at the present time and at the next integration time), this equation can be integrated with respect to the time with a standard R.K. procedure

During the integration of the whole set of equations, the program monitors the energies stored in the bunch and in the e.m. field. The computation of the total e.m. field energy requires an integration of the Poynting vector flux across the cylindrical surface separating the reduced mesh (where both the **E** and the **H** fields are known) and the outer part of the cavity.

The code is able to handle mesh with up to 200000 points for the H_ϕ field inside the cavity, requiring a core memory of about 3.5 Mbytes; the CPU time needed for the case presented in the next section (1000 particles in the bunch traced over 2.5 m, 20000 mesh points) is about 6 hours of VAX-8600. The CPU time scales like $T+(n+.08N)\sqrt{N}$, where n is the number of particles and N is the number of mesh points (the factor $\sqrt{N}$ is due to the numerical stability criterium, which states that the integration time step must scale like the mesh step, i.e. like the inverse of $\sqrt{N}$).

THE FIRST TESTS

As a reference case, we chose a bunch of 1μC charge and 10 MeV energy injected in the empty LEP SC cavity. The bunch is initially gaussian in the r-z plane and semi-gaussian in the two phase spaces (gaussian in r and uniform in r′, gaussian in z and uniform in Δγ), with a normalized emittance of $4\cdot10^{-4}$ m·rad and an initial radius (rms) of 20 mm. The bunch length at injection is 40 mm (rms). The wake-field excited in the cavity by the bunch passage, is shown in Fig.1 at some integration times. The excitation and propagation of the higher order modes is evident: the driving current, shown in Fig.2 at the same bunch position, is initially gaussian, but it exhibits an increasing splitting while the bunch approaches the last two cells, due to the strong effects of the induced wake-field. The splitted driving current starts to excite higher frequencies, which propagate more as in free space than in a cavity: these frequencies are at the limits of the mesh capability of propagating high order modes, but the numerical stability is not yet violated, as can be deduced from the fact that these modes propagate through the mesh and that the total energy (e.m. energy

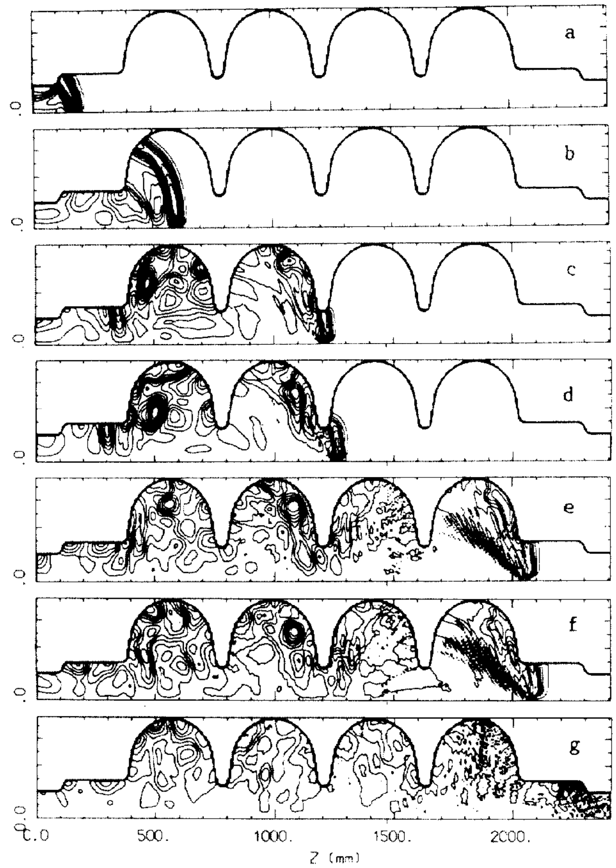

Fig.1 - Motion of a 10 MeV 1μC electron-bunch and wake fields excitation in an initially empty LEP SC cavity. $r\cdot H_\phi$ = const. lines are shown.

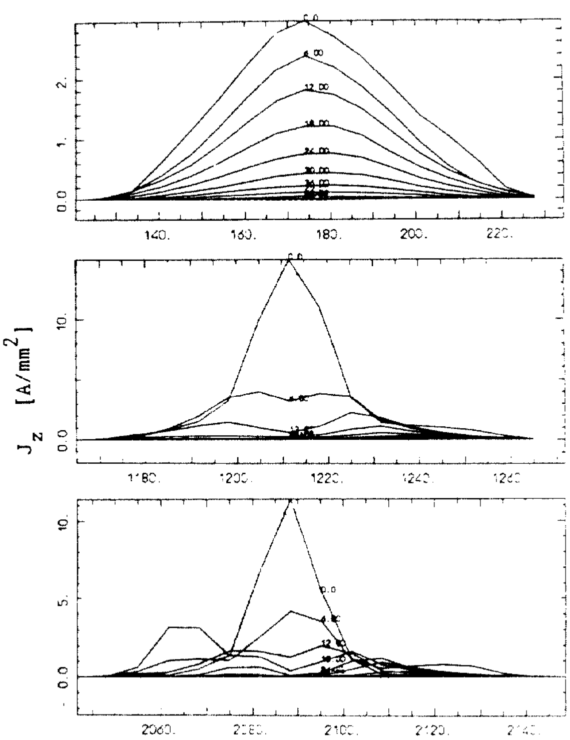

Fig.2 - Current density J_z as a function of z (at some indicated radii), produced by the bunch shown in Fig.1 at the positions a,c,f.

plus bunch energy) still stays constant until the bunch exits from the cavity. At this time the bunch has lost 1.02 joule of its initially 10 joule energy in the cavity: the computation of the e.m. field energy when the bunch has left the cavity gives .98 joules stored in the field, with an error of a few percent in the energy exchange.

The electric field on axis, shown in Fig.3, reflects evidently the wake associated to the bunch: a peak of the order of 10 MV/m propagates just behind the bunch, and a great fluctuation of the field inside the bunch induces the explosion of the particles off the axis while the bunch propagates in the cavity.

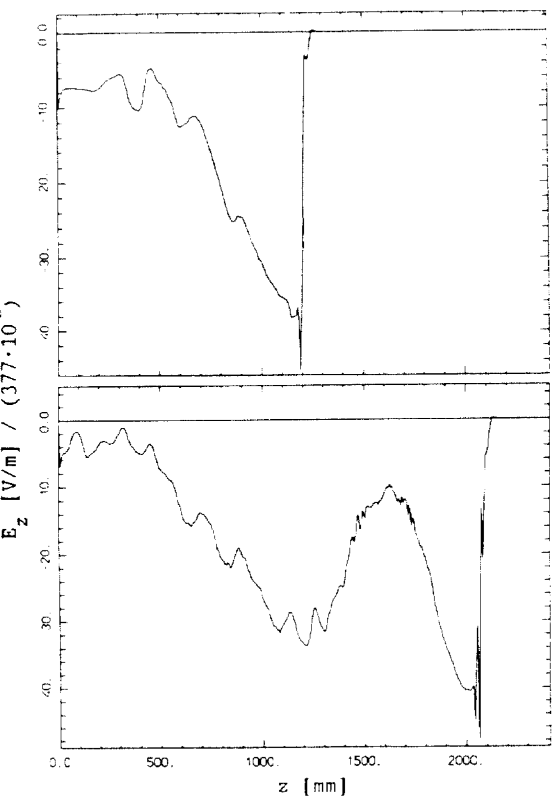

Fig.3 - Electric field on axis produced by the bunch shown in Fig.1 at the positions c and f.

We present also the acceleration of a 1 MeV 100 nC bunch injected in the cavity ($1.5\cdot10^{-4}$ of normalized emittance), when a 115 joule of e.m. energy is stored inside (which corresponds to an accelerating field of ~6.5 MV/m on the active length). In this case the accelerating field is much higher than the wake-field excited by the bunch, as can be seen from Fig.4, where the wake field is visible only at those phases of the accelerating field where the H_ϕ field is close to zero. Here it is important to note the reliability of the field integration algorithm, which after three RF periods reproduces the starting spatial distribution of the H_ϕ field with a great accuracy. The bunch is focused during the acceleration by the radial component of the electric field, but it emerges from the cavity with a large energy spread due both to the wake field excitation and to its large phase spread (about 40° RF). In this case the driving current associated to the bunch stays fairly gaussian, with some distorsions, all over the acceleration process. The bunch exits from the cavity at an average energy of 11.12 MeV: the total energy, given by the sum of the e.m. energy and the bunch energy, remains constant near its initial value of 115.1 Joule with a maximum oscillation of .05 joule.

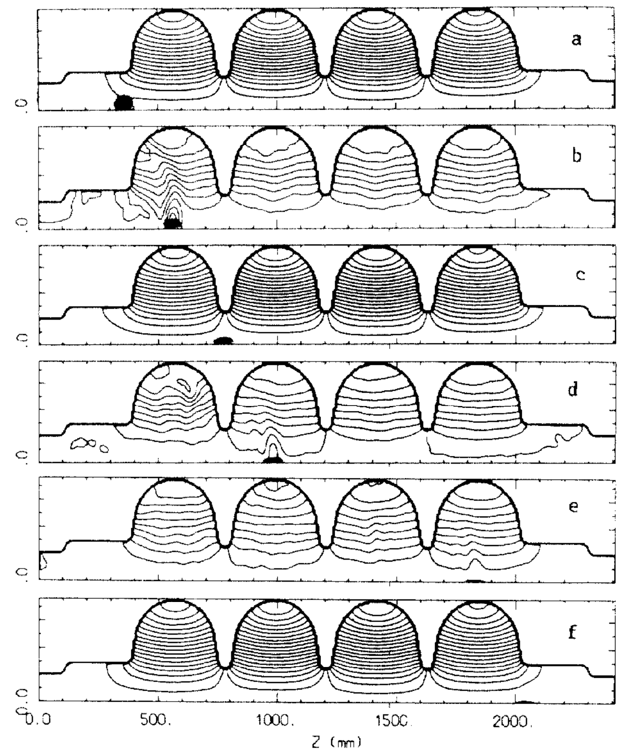

Fig.4 - Acceleration of a 1 MeV 100nC electron-bunch and wake fields excitation in a LEP SC cavity, initially stored with a resonating π-TM_{010} mode. $r\cdot H_\phi$ = const. lines are shown.

The final energy gained by the bunch is 1.02 J, against an energy lost by the e.m. field of 1.07 J. The final normalized emittance of the bunch is $4.6\cdot10^{-3}$ m·rad.

CONCLUSION

This preliminary short paper presents a few results obtained in the days just before this conference, so that the time to properly analyze the outputs and to run different cases was lacking. Nevertheless, in our opinion, the two given examples should be sufficient to show the program performances, even if they are not useful for a real design.

In the near future a more complete presentation of ITACA will be published, together with a description of the overall package (which includes a resonating modes finder and a graphic postprocessor).

We assume that, once fully tested, ITACA will be ready for external users by the end of this year, in a version compatible for vector computation.

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