# STRATEGIES FOR LONGITUDINAL PAINTING IN THE EHF BOOSTER AT INJECTION

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# Abstract

Proposals for high-current hadron facilities all rely on "painting" during charge exchange injection as a nearly lossless technique to transform a continuous stream of linac microbunches into a number of bunches matched to the characteristics of a circular machine. In order to find the best strategy to fill the RF buckets of the European Hadron Facility Booster, meeting the design goal of a bunching factor better than 0.33, the LONG1D computer simulation code, including space charge, was modified to cope with the EHF scenario and upgraded to higher spatial resolution. Strategies consist in phase and energy ramping the locus of the injected microbunches in the bucket and the methods compared in this study can be divided into two groups: (i) painting with two linac bunches (corresponding to the EHF linac specifications); the best results (bunching factor  $\simeq 0.38$ ) have been achieved with a stationary injection position in phase-space ("no-paint" scheme); (ii) more ambitious strategies aiming at obtaining a square-shaped linear density, which require painting with a single bunch. For 200 turns injected into the EHF Booster, these "spiral filling" schemes, however, do not yield substantially better bunching factors, the best results being comparable to (i) when tracking with higher spatial resolution. Another distinctive difference between the two approches is the sensitivity of group (ii) to tracking parameters, mainly to the spatial resolution involved in the computation of the space-charge force; this sensitivity is absent in the simple painting strategies (i). A criterion for the resolution required is given and all relevant painting strategies have been checked with this resolution. The more favourable results of the simple strategies led to the recommendation of one of those for the EHF.

#### Introduction

"Painting" is a term now generally employed to describe the task of populating a 2-dimensional phase plane with many small spots so as to fill a much larger surface. The spots are (small) bunches injected from a linac, and the surface to be filled is a (larger) bunch in a circular machine with an accelerating RF system. This way of building a bunch structure is appropriate for the charge-exchange injection of a long (a few hundred turns) linac pulse, not possible with classical multiturn injection.

## **Filling Strategies**

In our study we distinguish between "simple" strategies and others aiming at flat-topped distributions.

By "simple" strategies we mean linear motion (in time) of the locus of the injected bunch. The case where the linac bunch is injected at fixed locus in the phase plane and the painting is left to synchrotron motion is particurly instructive; Such a bunch will paint an annular domain yielding a characteristic double-hump projection. In the EHF Booster<sup>[1]</sup> five such annuli are painted, and one can guess the bunch shape to be expected from a slow linear motion of the locus as a superposition of five different double-humped projections. Typically the injected linac bunches are short and tall with respect to the shape of the macrobunch to be created and microbunches with the highest possible energy spread yield the flattest, smoothest and thus most suitable projections. For these "smooth" distributions, space-charge hardly alters the results for zero intensity, i.e. the purely kinematic approach is valid.

On the other hand, the square bunch has the lowest peak density for a given bunch length. In order to see how to obtain it, it is useful to consider the Abel transform relating phase-space density and its projection:

$$p(x) = \frac{1}{\pi} \int_x^R \frac{drn(r)}{\sqrt{r^2 - x^2}}$$

which we have written here for rotationally symmetric density distributions P(r);  $n(r) = 2\pi r P(r)$  is the radial density. Such a symmetry is achieved for an injection point moving on a radius slowly with respect to the synchrotron motion. The radial increment  $\Delta r$  is then simply

$$\Delta r = \frac{1}{n(r)N_{inj}}$$

where  $N_{inj}$  is the number of spots painted, i.e. the number of turns injected. Since n(r) has to be zero at r = 0 for all well-behaving distributions, special attention has to be paid to the painting of the very first spots in the vicinity of the origin. As  $\Delta r$  formally grows to infinity, they are calculated individually according to n(r), such as to yield a local phase-space density as uniform as possible. As the linac bunches have a shape not at all matched to the bucket, it makes a difference if the radius in phase-space is along the abscissa (painting by "phase ramping" between linac and booster RF) or the energy coordinate ("energy ramping"). With phase ramping, the short and tall microbunches, produced by the EHF 1.2 GeV linac, paint steep slopes of the line density, resulting in five thin spires well distinguishable in the projection (Fig. 1b) and in a bunching factor below its theoretical limit. On top of that, these steep slopes produce strong local spacecharge forces destroying the quasi-stationary distribution one wants to produce. This is clearly revealed when spatial resolution in computation of the space-charge forces is pushed to its limits. Energy ramping (in the EHF case) produces phase space distributions where the micro bunches are placed side by side and the complications due to steep gradients do not occur, but the gain in bunching factor is marginal with respect to simple strategies.

### The Tracking Code

Amongst the few existing codes doing longitudinal tracking with space charge, we have selected LONG1D<sup>[2]</sup>. Its attractive feature is the use of FFT as a tool for efficient smoothing of the fluctuations of the line density due to the unavoidbly poor statistics. In order to match the EHF requirements, the code needed several important modifications: (i) Simulation of bucket filling over many injection turns. (ii) A correct definition of synchronous phase for injection at constant RF frequency, on the bottom of a guiding field  $B(t) = B_0 - B_1 \cos(ft)$ , given by

$$\sin\phi_s = \frac{2\pi R\rho}{\eta\gamma_s^2 V_o} \frac{dB}{dt}$$

Then the closed-orbit radius changes during injection and, in general, the beam will be off-centre when injection ends. Owing to the jump of the synchronous phase, dipolar instabilities arise when switching abruptly to the constant radius acceleration regime, where the synchronous phase is calculated by

$$\sin\phi_s = \frac{2\pi R\beta E}{c\epsilon V_o} \left[\frac{1}{B}\frac{dB}{dt} + \frac{\gamma_t^2}{R}\frac{dR}{dt}\right]$$

(iii) Simulation of a radial control loop, to assure a smooth transition, by imposing

$$R(t) = A \frac{(t-t_0)}{\tau} \exp[-(t-t_0)/\tau] + C \exp[-(t-t_0)/\tau] + D$$

 $(t_o \text{ end of injection})$  with appropriately chosen constants. (iv) Computation of the bunching factor as the ratio of the average over the highest populated bin.

### **Resolution Limits of Simulation**

There is an obvious tendency to reduce the number of bins in the presentation of the line density (and in the computation of the bunching factor). In fact, if the binning is finer than actual structural details, fluctuation due to poor statistics may yield a pessimistic result.

On the other hand, pushing the number of Fourier harmonics Hand the number of bins  $N_{bins} \ge 2H$  to their reasonable limits changes the results dramatically in those cases where the line density shows pronounced peak structure (cf. results for spiral filling, Figs. 1a-c). Fig. 1a, produced with  $N_{bins} = 36$  and H truncated by the original automatism, suggests a bunching factor of 0.507. High resolution  $(N_{bins} = 240)$  in Fig. 1b, but H still limited as above, makes this figure drop to  $B_f = 0.373$ ! By pushing H higher, results change more and more and the spiral structure is completely destroyed with H = 120 (Fig. 1c). These results raise the question as to what the "reasonable limits" to the spatial resolution (expressed in terms of Hor  $N_{bins}$ ) may be.

Two effects compete: (i) Very short wavelength modulation of line density will not produce equal modulation of the fields at the walls of the pipe and the depth of the potential well will not follow this variation. Formally, the  $g_o$ -factor  $g_o = 1 + 2\ln(b/a)$  rolls off at short wavelength <sup>[3]</sup>:

$$g_{o} = \frac{4}{x^{2}} - \frac{4}{x} \left[ K_{1}(x) + I_{1}(x) \frac{K_{o}(xb/a)}{I_{o}(xb/a)} \right]$$

with a and b being beam and pipe radius respectively,  $\lambda$  the wavelength of the line density modulation, and  $\mathbf{r} = 2\pi a/\gamma \lambda$ . The wavelength  $\lambda$  (and  $\mathbf{r}$ ) are related to  $N_{bins} = 2H$ :

$$x = \frac{haN_{bins}/2}{R\gamma}$$

where h is the RF harmonic number. In the EHF Booster ( $b \simeq 3$ cm,  $a \simeq 1$  cm)  $q_a$  rolls off from its d.c. value of 3.2 to 3 for 100 bins and to 2.3 for 240 bins. We conclude that the effect is not dramatic for the number of bins envisaged. (ii) Another critical parameter is the cut-off frequency of the waveguide, i.e. when the lowest TE mode begins to propagate inside the pipe, and one may expect any perturbation of the line density of this or shorter wavelength to be "radiated away". This limit was also quoted by MacLachlan<sup>[4]</sup>, who developed a tracking code similar to LONG1D. The cut-off for the TE11 mode of a circular waveguide of radius b is  $\lambda_c = 3.413b$ . This corresponds to  $N_{bins} = 240$  for the EHF Booster and that is the figure that has been chosen for testing painting strategies with high-resolution tracking (normally the number of bins is 100). An immediate consequence is that the number of super-particles has to be increased in order to avoid excessive fluctuation due to the poorer statistics: as fluctuations are in general proportional to the square root of the number of particles per bin, the total number of super-particles has to be scaled with the square of  $N_{bins}$ ; in our test runs with  $N_{bins} = 240$ , the number of super-particles was chosen to be 60 000. This number limits the time the ensemble is trackable to the painting process proper. Tracking of early acceleration (3 ms) was performed with standard 100 bin resolution only.

### **Results of Tracking Study**

**Painting process:** In the EHF scenario<sup>[1]</sup>, a tandem of "2-out-of 8" of linac bunches 45 RF degrees apart are injected during 200 turns into the 50 MHz buckets of the Booster to paint a bunch of  $\pm 110^{\circ}$ . The simplest scheme ("no-paint") gives bunching factors slightly below 0.4, i.e. better than the (conservative) specification. It is also "wellbehaving" over the injection interval (360  $\mu$ s), under all possible tests up to highest spatial resolution. This, which is our choice, is depicted in Fig. 2a. The strategies aimed at a "square bunch" do not give significantly better results in the EHF environment (Figs. 1a-c). The fine structure of the spirals bears the risk that bunches painted this way develop a lumpy phase-space structure entailing peaks in the line density and possibly instabilities.

Early Acceleration: The first 3 ms of the total 30 ms acceleration cycle have been tracked for the chosen filling strategy (Fig. 2b). Three tracking modes have been compared : (i) Automatic truncation of the Fourier representation of the line density to a resolution related to r.m.s. bunch length: Fig. 2. (ii) Highest possible resolution possible for 100 bins: Fourier harmonics up to 50: Fig. 3. (iii) No space-charge. The somewhat surprising result is that (i) shows the development of some azimuthal asymmetry of dipolar character - an effect dubbed bunch "sloshing" This effect is completely absent without space-charge and hardly visible in (ii), but there are particles leaking out of the original bunch area; in fact bunch length at the base has grown. This growth deserves further attention and tracking over a longer interval. On the other hand, "bunch sloshing" does not necessarily entail loss: the ISIS synchroton at RAL shows this effect throughout the whole cycle at high intensity without suffering from loss<sup>[6]</sup>

#### References

[1] Proposal for a European Hadron Facility, Rep. EHF-87-18.

[2] S.R. Koscielniak, The LONG1D Simulation Code, contribution to this Conference.

[3] W. Hardt, Proc. 9th Int. Conf. on High Energy Accelerators, Stanford 1974 (US Atomic Energy Commission, Washington, 1974), pag. 434.

[4] J.A. MacLachlan, FN-446, FNAL, 1987.

[5] S.R. Koscielniak, private communication

[6] G. Rees and Ch. Planner, private communication.

## Figures

Phase space plots (top) are presentend in units of RF radians and MeV. Projected line density (bottom) in super-particles per bin.





Fig. 1: Attempts to paint a square bunch by phase ramping in the EHF Booster (with one linac bunch), for different display resolution and treatment of space-charge: spatial resolution in space-charge calculation is limited to the typical r.m.s. bunch length by automatic

truncation of higher Fourier harmonics in the code in (a): 36 bins, truncation of Fourier harmonics, and (b): 240 bins, truncation of Fourier harmonics. (c): 240 bins, 120 Fourier harmonics. Note the strong dependence of the results upon the processing parameters!



Fig. 2: Result of painting (a) and evolution during early acceleration, strobed at 3 ms (b), for the recommended constant injection locus strategy. Position and size of linac bunches are drawn in (a). Beam parameters are the nominal ones for the EHF Booster. 20.000 super-particles, 100 bins, automatic truncation of Fourier harmonics.



Fig. 3: Same as Fig. 2b, but spacecharge forces computed with 50 Fourier harmonics (maximum resolution possible with 100 bins).