THE LONGITUDINAL COUPLING IMPEDANCE OF A CYCLIC ACCELERATOR VACUUM CHAMBER WITH SMALL CROSS-SECTION VARIATIONS

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Abstract. The method for calculation of the coupling impedance of perturbations sequence, which are small variations of radius, in an accelerator cylindrical vacuum chamber is developed. The general expressions for the impedance and characteristic features of resonances are obtained in the case when perturbations are disposed periodically. The dependence of the impedance on random violations of periodicity is investigated in the framework of a statistical approach. The longitudinal impedance of conical transitions and fast demountable joints in the UNK 1st stage chamber is calculated.

1. Introduction

The conditions of beam longitudinal stability are typically formulated in the form of bounds on tolerable value of the coupling impedance (e.g. [1]). Then the problem is to calculate the impedance for the given geometry of the vacuum chamber. The present work develops the method of calculating the longitudinal impedance of a chamber with small cross-section variations to be termed below ε-expansion. The idea of ε-expansion was put forward in paper [2] and was used in [3,4] to calculate the impedance. In contrast to paper [3], our version of the method takes into account the finite conductivity of the chamber walls and considers an arbitrary beam energy (Sect.2). In addition, we succeeded in generalizing ε-expansion for quasiperiodic structure (Sect.3) with the help of statistical approach. In Sect.4 the longitudinal impedance of some UNK-1 chamber elements is calculated. More details can be found in [5,6].

2. ε-Expansion Method: a Periodic Case

Consider an axial-symmetric vacuum chamber whose boundary is $r=b(z)=bw(z)$, where b is the mean chamber radius and $w(z)=1+\varepsilon s(z)$ has the period $D=2\pi R/N$. Here R is the mashine radius, N is an integer. Let $\vartheta=2\pi z/D$ and we will normalize $s(\vartheta)=\Sigma_{p=1}^{\infty}(C_p\cos(p\vartheta)+S_p\sin(p\vartheta))$ by the condition $Var[s(\vartheta)]=2$. Let a transverse - homogeneous beam of a radius a moves with velocity βc along the chamber axis. Consider the n-th mode of the longitudinal perturbation of beam current density

$$j_z = \rho_n\beta c\,\exp(ikz - i\omega t)\,, \qquad (1)$$

where $k=n/R$, $\omega=\beta ck$. The fields produced by current (1) have the form (the factor $i\rho_n/(\varepsilon_0 k)\exp(-i\omega t)$ is omitted)

$$E_z = \sum_{m=-\infty}^{\infty} e^{ik_m z}\left[A_m f(\chi_m r)-\delta_{mn}\left\{\begin{matrix}1\\ g(\chi r)\end{matrix}\right\}\right]\,, \qquad (2)$$

and similarly for E_r, H_φ. In (2) $k_m=m/R$; $\chi_m^2=k_m^2-(\omega/c)^2$; $\chi\equiv\chi_n=k/\gamma$; $\gamma=1/\sqrt{(1-\beta^2)}$; $f(u)=I_0(u)$, $g(u)=\chi a[K_1(\chi a)I_0(u)+I_1(\chi a)K_0(u)]$; the upper line in $\{\ldots\}$ corresponds to $0\le r\le a$, the lower one to $r\ge a$.

To find A_m, let us impose on fields (2) the boundary condition (3)

$$\left[E_z+b'(z)E_r+(1-i)\frac{\delta\omega}{2c}\sqrt{1+(b'(z))^2}\,Z_0H_\varphi\right]_{r=b(z)}=0,$$

where $Z_0=120\pi$ Ohm; δ is the skin depth at frequency ω. After substituting (2) into boundary condition (3) decompose the obtained equation over the system of functions $\{\exp(ik_m z)\}$ which is complete in $\{z: 0<z<2\pi R\}$. As a result, we obtain the infinite system of linear equations for the field coefficients $F_q\equiv A_{n+qN}$ ($A_m=0$, if $m\neq n+qN$; $q=0,\pm1,\ldots$):

$$\sum_{q=-\infty}^{\infty} M_{jq}F_q = R_j\quad,\ j = 0,\pm1,\ldots, \qquad (4)$$

where

$$M_{jq}=\Big\langle e^{i(q-j)\vartheta}\Big(f(x_q w) - \frac{f'(x_q w)}{x_q}\big[iw'G(\xi+qG)+(1+i)\eta\sqrt{1+G^2w'^2}\,\big]\Big)\Big\rangle\,, \qquad (5)$$

$$R_j=\Big\langle e^{-ij\vartheta}\Big(g(x_0 w) - \frac{g'(x_0 w)}{x_0}*\big[iw'G\xi+(1+i)\eta\sqrt{1+G^2w'^2}\,\big]\Big)\Big\rangle\,.$$

In (5) $G=2\pi b/D$; $\xi=\omega b/\beta c$; $\eta=\delta/2b(\omega b/c)^2$; $x_0=\chi b=\xi/\gamma$; $x_p^2\equiv(\chi_{n+pN}b)^2=x_0^2+2pG\xi+(pG)^2$; $\langle\ldots\rangle$ means averaging over $\vartheta\in[0,2\pi]$ (e.g. $\langle w\rangle=1$). The system which is a particular case of (4)-(5) for $\gamma\to\infty$ и $\delta\to\infty$ was considered in paper [7].

By definition (e.g. [8]) the longitudinal impedance is

$$Z_n = -\frac{1}{\rho_n\beta c\pi a^2}\int_0^{2\pi R} dz\; e^{-ikz}\bar{E}_z\,, \qquad (6)$$

where $\bar{E}_z$ is the beam cross section-averaged value of the field z-component amplitude. From (2) and (6) we obtain

$$\frac{Z_n}{n} = -\frac{2iZ_0}{\beta(ka)^2}\left[\frac{2I_1(\chi a)}{\chi a}F_0 - 1\right]. \qquad (6')$$

System (4) truncated by the conditions $|j|,|q|\le Q_{max}$ can be solved numerically. However, here we confine ourselves to the case when $h\equiv Var[b(z)]\ll b$, i.e. $\varepsilon=h/2b\ll 1$. Then, the truncated system can be solved analytically, by perturbation method. If $\varepsilon|x_q|\ll 1$ for all $|q|<Q_{max}$, then matrices M_{jq} and R_j may be expanded into a series over the powers of ε. Now we seek for F_q in the form of $F_q^{(0)}+\varepsilon F_q^{(1)}+\varepsilon^2F_q^{(2)}+\ldots$ Solving reccurently the systems of equations at $\varepsilon^0,\varepsilon^1,\varepsilon^2$ etc., we find $F_q^{(i)}$ in analytical form. As a result, $Z_n=\Sigma_{i=1}^{\infty}Z^{(i)}$, where $Z^{(i)}=O(\varepsilon^i)$. Here $Z^{(0)}$ is the impedance of a smooth chamber with radius b, $Z^{(1)}\equiv 0$ (this was noted in [2-4]), and

$$\frac{Z^{(2)}}{n} = -iZ_0\frac{\varepsilon^2}{4\beta}\left[\frac{2I_1(\frac{a}{b}x_0)}{\frac{a}{b}x_0}\right]^2\left[I_0(x_0)+(1+i)\eta\frac{I_1(x_0)}{x_0}\right]^{-2}$$

$$*\sum_{p=1}^{G_{max}}(C_p^2+S_p^2)[(\frac{\xi}{\gamma^2}+pG)\frac{(\frac{\xi}{\gamma^2}+pG)+(1+i)\frac{\eta}{\xi}(2-F(x_p))}{F(x_p)-(1+i)\eta}$$

$$+ (p\to-p)+\gamma^{-2}+(1+i)\frac{\delta}{2b}\beta^2(2+p^2G^2)] , \quad (7)$$

$$\text{where } F(x)=\begin{cases} xI_0/I_1(x), & x^2>0 \\ |x|J_0(|x|)/J_1(|x|), & x^2<0. \end{cases}$$

For $\gamma\to\infty$ and $\sigma\to\infty$ the limit of (7) coincides with the result given in [3]. The energy dependence of $Z^{(2)}/n$ at low frequencies ($\xi<1$) and/or at high energies (for example, for UNK: $\gamma>75$) appears to be weak. However, during acceleration in U-70, with $\beta\gamma$ varying from 3 to 75, the resonance force may increase by several times.

Resonance frequencies are determined by the equation $F(x_{-p})-\eta=0$. The resonance parameters are expressed in the simplest way for $\gamma>1$. So, resonance frequency $f_{p,r}$ corresponding for the specified p to the r-th radial mode is

$$f_{p,r}=\frac{\beta cp}{2D}\left[1+(j_{0r}/pG)^2\right] , \quad (8)$$

where $J_0(j_{0r})=0$, $r=1,2,\ldots$ The resonance width is

$$(2\Delta f)_{p,r}= f_{p,r}\frac{\delta}{2b}\left[1+(j_{0r}/pG)^2\right] \quad (9)$$

and the value of Re $Z^{(2)}/n$ in the peak is

$$\frac{R^{(2)}_{p,r}}{n} = Z_0\varepsilon^2\frac{2b}{\delta}(C_p^2+S_p^2)\left[1+(j_{0r}/pG)^2\right]^{-2} . \quad (10)$$

These extreme expressions, (8)-(10), are analogous to those obtained in [3].

3. Generalization for a Quasiperiodic Case

Let the inserts of the same shape be placed along the ring in such a way that the position of i-th ($i=1,\ldots,N$) insert $z_i=(i-1/2)D+x_i$ differs from its position $z_i^{(0)}=(i-1/2)D$ in rigorously periodic structure by a random variable x_i. We assume all x_i to be independent and have Gaussian distribution; naturally, the r.m.s. deviation $d\ll D$. Let us choose some set of (x_i), $i=1,\ldots,N$. Then the period of a structure is $D=2\pi R$ and all changes in (7) are reduced to replacements $G\to G/N$ and $C_p,S_p\to C_p^{(N)},S_p^{(N)}$. Averaging over the distribution of (x_i) yields

$$\overline{(C_p^{(N)})^2+(S_p^{(N)})^2}=(C^2_{p/N}+S^2_{p/N})\Phi_p(d) ; \quad (11)$$

$$\Phi_p(d)=\begin{cases} \frac{1}{N}\left(1-\exp\left\{-\left[\frac{2\pi dp}{D\ N}\right]^2\right\}\right), & p\neq jN \\ \exp\left\{-\left[\frac{2\pi d}{D}j\right]^2\right\}+\frac{1}{N}\left(1-\exp\left\{-\left[\frac{2\pi d}{D}j\right]^2\right\}\right), & p=jN, \quad j=1,2,\ldots \end{cases}$$

The coefficients C_j,S_j in the r.h.s. of (11) already corresponds to the D-periodic structure, the index j is replaced by p/N. Note that $\Phi_p(0)=\delta_{p,jN}$.

The following two important general results are easily obtained from (7) with account of (11):

(i) in the nonresonant region, $\omega<j_{01}c/b$, the impedance of a quasiperiodic structure coincides with that of a periodic one (it is natural because there are no propagating waves).

(ii) the factor determining suppresion due to periodicity violation for the periodic structure resonance (p,r) is

$$\varphi_p(d)=\exp\left\{-\left[\frac{2\pi d}{D}p\right]^2\right\}+\frac{\alpha_pN_p+1}{N}\left(1-\exp\left\{-\left[\frac{2\pi d}{D}p\right]^2\right\}\right). \quad (12)$$

In eq.(12) N_p is the number of "new" (occuring in quasiperiodic structure) resonances hitting the band $(2\Delta f)_p$ of the "old" resonance (p,r); α_p ($1/2<\alpha_p<1$) takes into account their overlapping. Denote the distance between "old" resonances through $\delta f_p=f_{p+1}-f_p$, then that between "new" ones is $\delta f=\delta f_p/N$. Obviously, $N_p=0$ if $(\Delta f)_p<\delta f$, otherwise $N_p=(2\Delta f)_p/\delta f$. As seen from (12), $\varphi_p(0)=1$; and when $d\gg D/2\pi p$ (resonance wave length $\lambda_p\ll 4\pi d$) the suppresion is maximum: $\varphi_p(d)\approx(\alpha_pN_p+1)/N=\alpha_p(2\Delta f)_p/\delta f_p+1$.

Note that for overlapping "old" resonaces ($(\Delta f)_p\geq\delta f_p$) $\alpha_p\to1$, $N_p\to N-1$. Therefore in the region where periodic structure resonances overlap (near the frequency cut-off, $\omega=j_{0r}c/b$ – see (8),(9)) the impedance does not decrease with periodicity violation.

4. Application of the Method: the Impedance of Some UNK Chamber Elements

Tests of ε-expansion by comparison with the results on numeric calculations of resonances (we used the MULTIMODE code [9] for short period model structures) shows a good agreement for $\varepsilon G=\pi h/D\ll1$ (see paper [6]). However, numeric method is inapplcable for long-period structures ($D\gg b$, as in UNK) because the time used to compute each resonance increases as D^2, with the number of resonances increasing as D. On the contrary, the ε-expansion method works well for long periods because the necessary condition for its applicability, $p\ll D/\pi h$, for two cases considered below has the form of $p\ll7500$ and $p\ll400$. Besides, near lower radial mode resonances we have $\varepsilon|x_{-p}|\ll1$ regardless of the value of p because in this case $|x_{-p}|\approx j_{0r}$.

1) Conical Transitions (CT) of UNK-I. Each period of the normal magnetic lattice ($N=160$) is $D\approx92$m long. Inside a period the chamber consists of two equal-length cylinders (in calculations the elliptic cross sections were replaced by round ones with radii $b_1=2.8$cm, $b_2=3.2$cm) connected with 30-cm CT. In this case, $b=3$cm, $\varepsilon=(b_2-b_1)/2b=1/15$ and we accept $\sigma=1.43\cdot10^6\,(\mathrm{Ohm\cdot m})^{-1}$. The calculation performed according to formula (7) yields at low frequencies Im $Z^{(2)}/n\approx-0.3$mOhm which is much less than Im $Z^{(0)}/n$. Near the minimum resonance frequency $f_1^{min}\approx(2\pi)^{-1}j_{01}c/b\approx$ 3.825GHz

($p \simeq p_1^* = j_{01}/G \simeq 1173$) about 20 resonances overlap, which yields Re $Z^{(2)}/n \simeq 0.4$mOhm. From (7) and (8)-(10) we obtain:

for $f = 10^9 \div 10^{12}$ Hz

$$\text{Re } Z^{(2)}/n < 1\text{mOhm}; \; -1\text{mOhm} < \text{Im } Z^{(2)}/n < 0. \quad (13)$$

If the transitions were short, $l \lesssim 1$cm, this estimate would have been $|Z^{(2)}/n| < 0.13$ Ohm.

Since the periodicity of the CT is violated (due to the presence of special sections, etc.) the value of split resonances in UNK-I decreases by about $(\delta f)_p/(2\Delta f)_p \simeq 2 \div 5$ times (see Sect.3). This allows one to decrease twice the estimate of Re $Z^{(2)}/n$ in (13). For $l \lesssim 1$cm, the estimate is left unchanged because max Re $Z^{(2)}/n$ is attained near the cut-off frequency f_1^{min} (overlapping resonances).

2) Fast-Demountable Joints (FDJ). When the vacuum chamber sections are assembled, small cavities are left on its walls. Let us consider the following model in our calculations: 3 mm long, 5 mm deep inserts with the distance between them D=6m. We study inserts of two types: (A) – cosine-shaped and (B) – triangular. Since b=3cm parameter $\varepsilon = 1/12$. At low frequencies, Im $Z^{(2)}/n = -56.4$mOhm and -45.1mOhm for (A)- and (B)-shaped inserts, respectively, which are noticeably larger than for CT. The resonance overlapping yields the maximum value of Re Z/n for the 3d radial mode: 85mOhm near $f_3^{min} \simeq 13.763$GHz. For the 1st radial mode Re $Z^{(2)}/n \simeq 30$mOhm in overlapping point and for r=2 ($f_2^{min} \simeq 8.782$GHz) it is about 55mOhm. The estimate of a FDJ impedance in the resonance region for rigid periodicity is

$$\text{Re } Z^{(2)}/n < 0.2 \text{ Ohm}; \; |\text{Im } Z^{(2)}/n| \lesssim 0.1 \text{ Ohm}. \quad (14)$$

In the real lay-out of FDJ's periodicity is violated. Due to suppression of split resonances R_p/n decreases by $(\delta f)_p/(2\Delta f)_p \simeq 10 \div 20$ times. As a result, max Re $Z^{(2)}/n$ is reached near the cut-off frequency f_3^{min} and instead of (14) we have $|Z^{(2)}/n| < 0.15$ Ohm.

Thus, for $f > f_{cut} \simeq 3.8$ GHz the impedance of FDJ's is larger than that of a smooth chamber. And still, the impedance induced by CT's and FDJ's is essentially smaller than the upper limit imposed by the longitudinal beam stability requirement for UNK: $|Z/n| \lesssim 5$ Ohm (see paper [1]).

5. Conclusion

The ε-expansion method makes it possible to calculate the impedance of a set of vacuum chamber elements with small cross section variations both in the low and high frequency ranges. This method is very convenient for long-periodic structure for which numeric methods are difficult to apply. Statistical account of periodicity violation (see Sect.3) allows one to estimate the impedance of real accelerator structures.

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