THE LONG1D SIMULATION CODE

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Abstract

LONG1D[1] is a tracking programme for the study of longitudinal dynamics in proton synchrotrons, in the presence of space charge. The model only considers motion along the beam axis. The beam is represented by an ensemble of macro-particles, tracked in parallel. LONG1D has a variety of ensembles available, as appropriate to coasting and bunched beams. It allows for the injection of additional particles, to facilitate phase-space painting studies. LONG1D can simulate the effect of radial and phase damping loops, and provides estimates of transverse incoherent betatron tune-shifts. This paper presents the physics algorithms used and some simulation results, and comparison with experimental observations.

Definitions

Synchronous particle

The synchronous particle is matched exactly to the magnetic bending field; and travels on the equilibrium orbit, with speed $v_s = \beta_s c$. The rest mass is m_0, and total energy $E_s = \gamma_s m_0 c^2$. The charge is qe. The dipole magnets (radius of curvature r_s) define the momentum: $\beta_s\gamma_s = (qer_s)/(m_0c) \times B_s(t)$. Let the peak accelerating volts be $\hat{V}$ and the effective orbit radius R_s. The orbital angular frequency is $\omega_s(t) = (\beta_s c)/R_s$. The *synchronous phase* (ϕ_s) is defined by: $\sin(\phi_s) \equiv (2\pi r_s R_s)/\hat{V}(t) \times (dB_s/dt)$.

RF-phase

The individual particle *RF-phase* is defined to be: the phase of the cavity electric field at the moment the beam particle crosses the accelerating gap. The equation is :

$$\dot{\Phi}_{b,rf} = -\frac{h\omega_s\eta_s}{\beta_s^2 E_s}(E_b - E_s) + (\dot{\phi}_s + \dot{\phi}_{rf}) \ . \tag{1}$$

The first term (in $E_b - E_s$) to the right is due to the lattice dispersion, with $\eta_s = |1/\gamma_t^2 - 1/\gamma_s^2|$. The middle term accounts for changes in ϕ_s. The final term (in ϕ_{rf}) is the additional phase advance that accrues from cavity RF-errors or the deliberate frequency variations induced by damping loops. Ideally $\dot{\phi}_{rf}$ is identically zero.

Constant emittance

The system is Liouvillian: the emittance (ε) is conserved – provided conjugate canonical coordinates are used. Denote energy E, RF-phase ϕ, and angular revolution frequency ω. Conservation in energy time coordinates implies conservation in $(E/\omega, \phi)$ space. Let

$$\text{Hence} \quad \frac{\varepsilon_2(E,\phi)}{\omega_2} = \frac{\varepsilon_1(E,\phi)}{\omega_1} \quad \text{or} \quad \varepsilon(t+T) = \frac{\omega(t+T)}{\omega(t)}\varepsilon(t) \ .$$

The area transforming properties of a mapping are given by the Jacobean determinant, J. Consequently, if we work in E, ϕ coordinates

$$J = \frac{\omega(t+T)}{\omega(t)} = \frac{\beta_2}{\beta_1} \quad \text{where} \quad \beta = \frac{v_s}{c} \ .$$

This property must be present in the equations of motion. When $\phi_s \equiv 0$, then $J \equiv 1$.

Single Particle Motion

The exact equations of motion are *difference equations* – solved by recursion to give a sequence. Successive members are labelled by the integer subscript n. It is conceptually advantageous to calculate the states of the ensemble at equal turn intervals. It simplifies matters if we simulate a single bunch domain, $-\pi \le \Phi \le +\pi$. We assume the other $h-1$ bunches to be identical. The equations of motion are set up for operation below transition energy. Let the fractional turn increment, between cavities, be $dm = 1/N_{\rm cav}$.

Firstly, the time is advanced according to $t_{n+1} = t_n + (2\pi R_s/v_s^n) \times dm$, the bending field $B_s(t_{n+1})$ calculated, and $\beta_s, \gamma_s, \eta_s$, etc updated. Each of the particles has some energy $\epsilon = E - E_s$ relative to the synchronous particle. For every macro-particle the following algorithm applies.

At an accelerating station, the energy changes discretely:

$$\epsilon_{n+1} = \frac{\beta_{n+1}^s}{\beta_n^s} \times \epsilon_n + qe\hat{V}\,[\sin(\Phi_n) - \sin(\phi_n^s)] \times dm \ . \tag{2}$$

The particles coast between accelerating stations, and the RF-phase accumulates :

$$\Phi_{n+1} = \Phi_n - \frac{2\pi h\eta_s}{\beta_s^2 E_s}[\epsilon_{n+1}] \times dm + \left(\phi_{n+1}^s - \phi_n^s\right) \ . \tag{3}$$

This is the integral of $\dot{\Phi}_{b,rf}$ from one cavity to the next. The last term of (3) represents the change of ϕ_s. For the moment, the RF-error (ϕ_{rf}) is assumed to be zero. These equations constitute a first order symplectic mapping and are exact. Their behaviour is different from the differential equations used to represent them.[2] The difference equations result in a distortion and rotation of the phase-space ellipse, and cause the synchrotron frequency Ω_s to be modulated at (roughly) $2\Omega_s$.

Motion with space-charge

There are various forces which act on the beam other than those produced by the guiding electric and magnetic fields. They originate from the beam charge, and result in collective effects. At present, only the *space-charge force* is modelled. The physics is outlined by Hofmann.[3] The effects will be simulated by step-wise integration. Generally, the number of steps ($N_{\rm scf}$) does not equal the number of cavities ($N_{\rm cav}$). The mean beam energy is $E_b = \gamma_b m_0 c^2$. Let $\lambda(\phi)$ be the number of particles per unit rf-phase. The space-charge force acts continuously ; and during the (space-charge) step $dn = 1/N_{\rm scf}$ produces an incremental energy change :

$$-\left(\frac{q^2e^2}{4\pi\epsilon_0}\right)\frac{g_0}{\gamma_b^2}\frac{2\pi h^2}{R_s}\frac{d\lambda}{d\Phi} \times dn \ . \tag{4}$$

Considerations

The bunch is represented by N_p macro-particles, which are binned into N_b sub-intervals of $[-\pi, +\pi]$ to give a histogram representation of the bunch shape λ^*. The techniques of Lagrange assignment and interpolation are used to hasten computations. Details are given by Koscielniak.[4] λ^* is smoothed by fitting of a Fourier series, and the harmonic components recombined to give the derivative $d\lambda/d\Phi$. Smoothing is accomplished by truncating the harmonic series to some value $N_h \le N_b/2$. The choice of N_b, $N_{\rm scf}$ and N_h are related issues. Consideration of a coasting beam suggests that $N_p \ge N_b^2$. In this case, statistical fluctuations go as $\sqrt{N_b/N_p}$; unless this is of order 1% smoothing is required. Thus, the length scale for variations in λ is set by the harmonic cut-off N_h. If we believe variations of $\lambda^*(\Phi)$ to be real on a length scale $L = \pi/N_h$, then particles must move much less than L during each time-step. In fact, it is adequate to make the phase step-length 1/4 of the wavelength of variations in λ. The maximum rate of phase-advance for particles within an RF-bucket is $2\Omega_s$, as occurs on the separatrix at $\Phi = \phi_s$. Combining these observations, gives the *minimum* number of space-charge integration steps per turn :

$$N_{\rm scf} \simeq 16 \times N_h \times \frac{\Omega_s}{\omega_s} = 16.N_h.Q_s \ . \tag{5}$$

Conservative motion

Motion under space-charge is derivable from a Hamiltonian. This has two consequences. (1) The system conserves the sum of kinetic and electromagnetic energy. Thus whenever the particle KEs are incremented (due to space-charge), $\lambda(\Phi)$ and space-charge energy must also be updated. If this practise is not adopted, non-physical clumpings of the particles results. (2) The system is Liouvillian and the integration technique must be symplectic. Such canonical integrators have been considered by Ruth.[5] Because space-charge acts continuously, a symmetric 2[nd] order symplectic mapping is appropriate.

Canonical integrator

The algorithm is easily understood when presented formally. Let the initial and final states be (q_1, p_1) and (q_2, p_2) respectively. and the time-step be Δt.

$$\text{Let} \quad H = \frac{p^2}{2} + V(q,t) \;\; ; \;\; \dot{q} = \frac{\partial H}{\partial p} \;\; ; \;\; \dot{p} = -\frac{\partial H}{\partial q}$$

The algorithm is:

$$\begin{aligned} q_2 &= q_1 + p_1.\Delta t/2 \\ p_2 &= p_1 - \frac{\partial}{\partial q} V(q_2, \Delta t/2) \times \Delta t \\ q_2 &= q_2 + p_2.\Delta t/2 \end{aligned}$$

The force is evaluated at the midpoint of the time step. Often several space-charge integration steps are required during the time for the synchronous particle to travel between adjacent cavities, that is $N_{\rm scf}/N_{\rm cav} > 1$.

Phase and radial damping loops

Some of the design features of damping loops including delays are presented by Koscielniak.[6] The starting point is the phase equation (1). The last term (ϕ_{rf}) is the extra phase-advance due to damping loops; we assume that it can be set in a controllable manner. The ideal phase loop control signal is $\Phi_b^s = \Phi_{b,rf} - \phi_s$. The radial loop control signal is $(E_b - E_s)$. The frequency error is:

$$-\partial\phi_{rf}/\partial n = F_0(E_b - E_s) \; + \; G_0(\Phi_{b,rf} - \phi_s) \; . \qquad (6)$$

These choices give standard phase and radial loop control, leading to the equation:

$$\frac{\partial^2 \Phi_b^s}{\partial n^2} + G_0 \frac{\partial \Phi_b^s}{\partial n} + \Big(\frac{2\pi h \eta_s}{\beta_s^2 E_s} + F_0\Big) q e \hat{V} \cos(\phi_s) \times \Phi_b^s = 0 \; . \qquad (7)$$

Delays

We have assumed the control signals are applied at the instant they are measured. Suppose the energy error ($E_{b,s} = E_b - E_s$) and phase error ($\Phi_b^s = \Phi_{b,rf} - \phi_s$) are measured between every cavity crossing, and a series of the ordered pairs stored in a first-in first-out memory. Using the $(n-k)^{\rm th}$ pair to control the RF at the $n^{\rm th}$ cavity crossing, amounts to introducing a time delay of $k \times \tau_s/N_{\rm cav}$.

Algorithm for motion

Let the superscript i denote individual particle values. Using the phase advance defined in (6), the algorithm including delay is:

(a) Find average energy ($E_n^{b,s} = <\epsilon^i>_n$) and mean phase deviation ($\Phi_n^{b,s} = <\Phi^i>_n - \phi_n^s$) and store them.

(b) Integrate individual particle phases as for a normal traversal between cavities.

(c) Find the additional change ϕ_{rf} due to the damping-loop signals:

$$-\Delta\phi_{rf} = G_0 \Phi_{n-k}^{b,s} \times dm + F_0 E_{n-k}^{b,s} \times dm$$

(d) For all the macro-particles perform the mapping : $\Phi^i \Rightarrow \Phi^i + \Delta\phi_{rf}$.

Phase-Space Painting

By painting it is meant that during the injection, the centre of the micro-bunch emittance is moved over the phase-plane in a controlled manner. LONG1D has the option to accumulate macro-particles and to simulate longitudinal painting. Strategies for phase-space painting are discussed by Koscielniak.[7]

Verification in ISIS

The bunch shape is directly related to the disposition of particles in phase-space. Verification of LONG1D centred upon comparing a continuous sequence of experimentally measured bunch-shapes with those predicted by simulation. Shapes were recorded in the ISIS synchrotron (at Rutherford Laboratory) using a LeCroy transient digitiser (sampling at 200 MHz) and acheiving a resolution of 3° of rf-phase. 5.3×10^{12} protons are injected at 70 MeV into a ring of 26 metre radius, and this corresponds to significant space-charge. The synchrotron cycles at 50 Hz, and ejects beam at 800 MeV. The synchrotron frequency is low, and it is permissable to use one space-charge step per cavity crossing.

The measured and computed bunch shapes are found to be in one to one correspondence over the first millisecond of acceleration. This implies that the difference of synchrotron frequencies between experiment and computation is $\Delta\Omega_s/\Omega_s = 10^{-2}$. Voltage, phase and timing errors combine making it unreasonable to expect exact correspondence at later times. Figures 1(a)-(b) show representative examples from the bunch-shape sequence. Experimental results are to the left, and computed to the right.

KAON Factory

LONG1D has been used to study the TRIUMF KAON Factory[8] suite of rings. The A and B rings each hold 1.3×10^{13} protons, and the C, D and E rings 6.5×10^{13}. The respective machine radii are 34 m and 170 m. These machines have high synchrotron frequencies, and so require several space-charge integration steps per cavity crossing. A preliminary exercise[9] was to inject a stationary ensemble.[10] This was transported from 450 MeV to 30 GeV with 0.1% loss. The rf-voltage requirements were 650 kV per turn in the Booster and 2.5 MV in the Driver ring.

More intriguing has been a study of accumulation in the A ring. Beam is delivered from a cyclotron to a storage ring by $H^{\ominus}$ injection. Filling is limited to 20 ms. During this time, the first injected beam strikes the stripping foil many times, degrading the transverse emittances. The number of foil traversals is reduced if the beam is injected off-energy, so as to produce an annulus in phase-space. The computer simulations suggest this distribution to be unstable. Qualitatively, the annulus evolves cyclically into a crescent and then back to an annulus. The explanation is relatively simple. A particle phase-space distribution with exact circular symmetry produces an anti-symmetric space-charge voltage and is stable. However, due to injection there are unequal numbers of particles on either side of the annulus. This imbalance produces an asymmetric space-charge voltage. Thus Ω_s is lower where the line density is highest. As a result there is a net transfer of particles from low to high density. This enhances the difference of synchrotron tune and leads to further transfer. Eventually the annulus is replaced by a crescent. The dependence of Ω_s on amplitude allows the crescent to stretch out until an annulus is formed. Figures 2(a)-(b) show snap-shots from this behaviour. The effect can be eliminated by varying the injection energy so as to fill the anulus, at the expense of increased foil traversals.

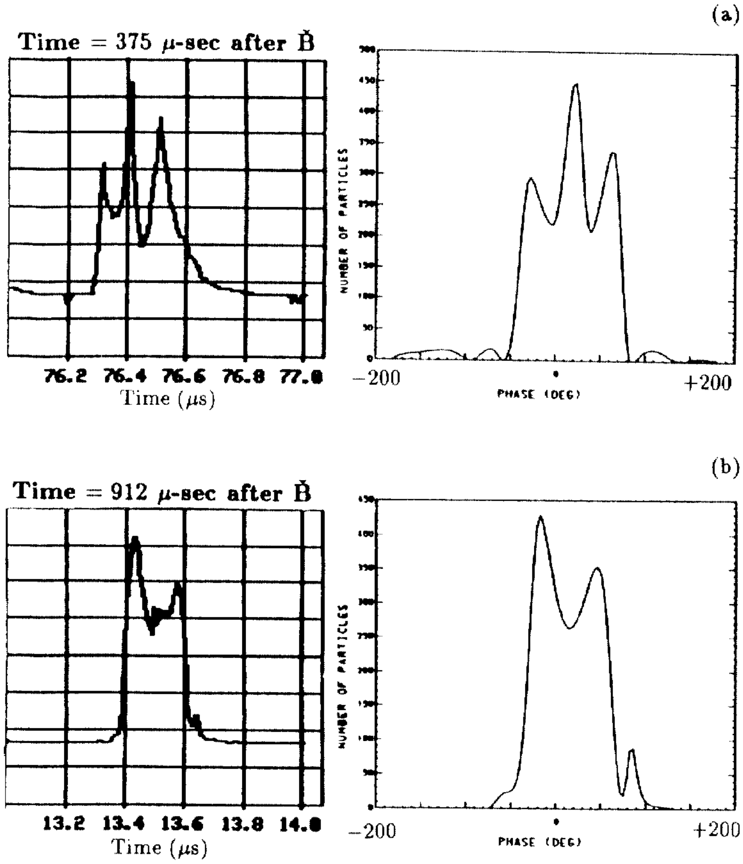

Fig. 1. Comparison of (a) experimental and (b) computed bunch shapes in ISIS.

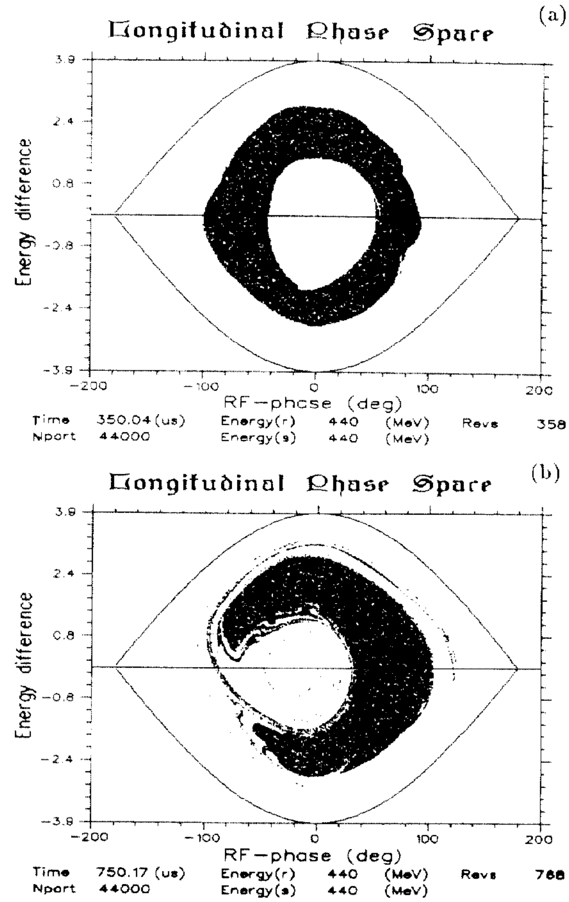

Fig. 2(a),(b). Evolution of annular ensemble into crescent distribution in TRIUMF-A.

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