SYNCHRONOUS PARTICLE AND NON-ADIABATIC CAPTURE*

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Introduction

In the theory of particle longitudinal motion, a classical definition of synchronous particle (synchronous energy, phase, and orbit) assumes that there is a one-to-one correspondence between the guiding magnetic field and the frequency of the accelerating electrical field. In practice, that correspondence may not be sustained because of errors in the magnetic field, in the frequency, or because sometimes one does not want to keep that relationship for some reason. In this paper, a definition of synchronous particle is introduced when the magnetic field and the frequency are independent functions of time. The result is that the size and shape of the bucket (separatrix) depends not only on the field rate of change but also on the frequency rate of change. This means, for example, that one can have a stationary bucket even with a rising field. Having the frequency, in addition to the field and voltage, as parameters controlling the shape and the size of the bucket, it is shown how to decrease particle losses during injection and capture.

Adiabatic Variables and Equilibrium Equations

In the theory of particle longitudinal motion, there are four values which we call adiabatic variables: B is magnetic field, f is supplied radio frequency, p is particle momentum, and R is the radius of synchronous orbit for that particle.

Suppose the state of the accelerator can be described uniquely by two variables B and f, while the state of the particle is described by p and R. Some of the particles, namely the synchronous one, can be at a state of equilibrium with the accelerator. Such an equilibrium can be expressed by the unique functions

$$p = p(B,f), \quad R = R(B,f), \tag{1}$$

which can be inverted to give unique functions

$$B = B(p,R), \quad f = f(p,R). \tag{2}$$

If relations such as (1) and (2) exist, we will call them adiabatic relations or equilibrium equations.[1] For example, the well-known equations

$$p_o = e\rho_o B_o, \tag{3}$$

$$f_o = (ch/2\pi R_o)\left[(E_r/ec\rho_o B_o)^2 + 1\right]^{-1/2} \tag{4}$$

are equilibrium equations for central (design, nominal, on-momentum) orbit of radius R_o. Here B_o and f_o are nominal field and frequency, p_o is particle nominal momentum (on-momentum particle), e and E_r are particle charge and rest energy, h and ρ_o are machine harmonic number and curvature radius.

The frequency-field relation (4) is the necessary and sufficient condition for the existence of the synchronous particle on R_o-orbit. Such a particle is synchronized with supplied frequency f_o by the equilibrium equation

$$h\omega_o = 2\pi f_o, \tag{5}$$

where ω_o is the particle revolution frequency.

By taking the total differential for each of equilibrium equations (1) and (2), we will come to the so-called differential relations[1,2] such as

$$dR/R = (\gamma^2 df/f - dB/B)/(\gamma_{tr}^2 - \gamma^2), \tag{6}$$

$$dp/p = dE/\beta^2 E = (\alpha\, dB/B - df/f)/\eta \tag{7}$$

where $\gamma_{tr} = E_{tr}/E_r$ is transition energy, $\alpha = \gamma_{tr}^{-2}$, $\eta = \alpha - \gamma^{-2}$, $\gamma = E/E_r$, $\beta^2 = 1 - \gamma^{-2}$. These two differential relations allow us to determine the state of the particle (p,R) or (E,R) knowing the state of the accelerator (B,f).

Synchronous Particle Out of Central Orbit

Usually the frequency-field relation (4) is kept during the whole accelerating cycle. However, sometimes there are errors in the magnetic field or in the frequency which make (4) invalid. Moreover, sometimes during injection and capture it is worthwhile to introduce deviations from (4) by frequency manipulation.

If we supply the field $B = B_o + \Delta B$ and frequency $f = f_o + \Delta f$, then it will change the synchronous radius by ΔR. Replacing the differential relation (6) by the difference equivalent, we will have a relative error in radius with respect to nominal R_o

$$\Delta R/R_o = (\gamma_o^2\, \Delta f/f_o - \Delta B/B_o)/(\gamma_{tr}^2 - \gamma_o^2) \tag{8}$$

where γ_o is the synchronous energy (in units of the rest energy) of the particle provided it stays on the non-distorted central orbit:

$$\gamma_o^2 = (ec\rho_o B_o/E_r)^2 + 1 = \left[1-(2\pi R_o f_o/ch)^2\right]^{-1}. \tag{9}$$

Thus, a new synchronous orbit will have a new radius

$$R_s = R_o + \Delta R. \tag{10}$$

Now a new synchronous energy can be found with the use of particle velocity $v = \beta c = \omega R_s$ and synchronization $h\omega = 2\pi f$ as

$$E_s = \gamma_s E_r = (1-\beta^2)^{-1/2} E_r = \left[1-(2\pi R_s f/ch)^2\right]^{-1/2} E_r. \tag{11}$$

*Work performed under the auspices of the U.S. Department of Energy.

Perhaps the most important relation is that for the rate of change of the synchronous energy. Dividing (7) by the time differential dt, one gets

$$\dot{E}_s = (\alpha\dot{B}/B - \dot{f}/f)\beta_s^2 E_s/\eta_s. \qquad (12)$$

It is easy to see that synchronous energy will be constant not only when f = const, B = const, but more generally when

$$\dot{f}/f = \alpha\,\dot{B}/B. \qquad (13)$$

For example, at the beginning of injection at the Brookhaven AGS, we have B = 250 Gauss, $\dot{B}$ = 4.9 Gauss/ms, f = 2520 kHz, α = 0.014, which gives us from (13) $\dot{f}$ = 0.68 kHz/ms to provide E_s = const.

If a particle stays on the central orbit R_0, then the particle's synchronous phase is determined by the energy gain in the electrical system

$$2\pi\dot{E}_s = \omega_s V\sin\phi_s, \qquad h\omega_s = 2\pi f_0 \qquad (14)$$

and by the corresponding "field gain" in the magnetic system

$$\dot{E} = \dot{p}\,dE/dp = v\dot{p} = \omega_s R_0\rho_0 B_0, \qquad (15)$$

which results in

$$\sin\phi_s = 2\pi R_0\rho_0\dot{B}_0/V. \qquad (16)$$

However, if the synchronous orbit is not the central one, then the energy gain (14) should be combined not with field-gain alone, but with the frequency-field gain (12), which results in

$$\sin\phi_s = h^2(\alpha\dot{B}/B - \dot{f}/f)/aVf, \qquad a = h\eta_s/\beta^2 E_s. \qquad (17)$$

Obviously, (17) will be identical to (16) if the frequency f is ideally adjusted to the field B through the frequency-field relation.

Computer Simulations

Since we have established the basic formulae for a synchronous particle (synchronous orbit R_s, energy E_s, and phase ϕ_s), we have a reference frame to describe a motion of all the other asynchronous particles.

Let $E = E_s + \Delta E$ and $-\pi \leq \phi \leq \pi$ be an energy and phase of an arbitrary particle. The dynamic equations for particle motion are

$$\frac{d}{dt}\left(\frac{\Delta E}{\omega_s}\right) = \frac{V}{2\pi}\left(\sin\phi - \sin\phi_s\right), \qquad (18)$$

$$d\phi/dt = a\omega_s\Delta E, \qquad 2\pi f = h\omega_s, \qquad (19)$$

where f is applied frequency, V is cavity voltage, and ω_s is the revolution frequency of the synchronous particle, which is not necessary staying on the central orbit.

A computer program was developed to solve dynamic equations (18) and (19) by the usual means of difference approximations. In the computer model simulating longitudinal motion in the AGS, we introduced aperture limits. They are ΔR = 2.5 cm distance in and out from the central orbit of radius R_0 = 128.5 m. During the tracking, each particle is subject to counting, provided the particle orbit radius R satisfies $R_0 - \Delta R < R < R_0 + \Delta R$. If this condition is not met, then the particle is lost (hits the aperture) and is removed from further calculations. Figure 1 presents the results of computer simulations of injection and capture at the Brookhaven AGS.[2]

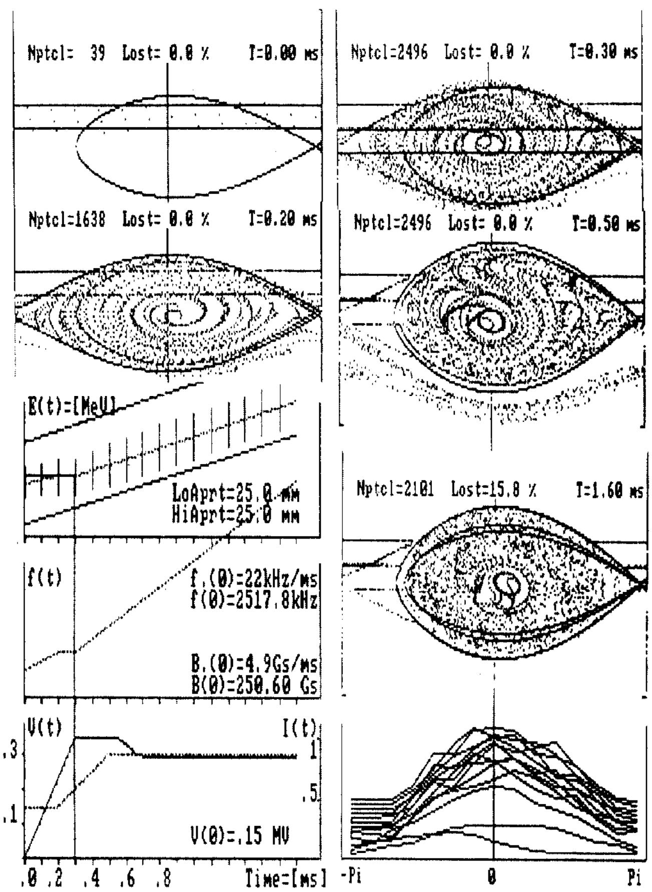

Fig. 1. Computer modeling representing AGS control program for injection and capture.

There are three graph sections in Figure 1. The upper graph has three curves. The middle curve $E_s(t)$ represents an evolution of synchronous energy with the time. After each 0.1 ms, the curve E_s is crossed by vertical lines representing the height of the bucket at that time. Two other curves below and above E_s are the aperture limits in energy units.

If the bucket reaches either of those limits, then some of the bucket particles hit the aperture and will be lost. The vertical time line crossing all three graphs represents the duration of injection which starts at T = 0 and finishes at T = 0.3 ms. Within the aperture corridor, the solid horizontal line ending at the time line represents constant injection energy.

The middle graph shows the frequency program f(t). It starts with constant $\dot{f}$ = 22 kHz/ms and during 100 μs between T = 0.2 ms and T = 0.3 ms it stays with $\dot{f}$ = 0, then it switches to $\dot{f}$ = 33 kHz/ms.

The lower graph shows the voltage program, which is linear between T = 0.2 ms and T = 0.5 ms and which is constant elsewhere. We can also see an intensity curve I(t), which is the total number of particles at each instant or total charge vs time.

The five pictures in Figure 1 represent particle position in phase space ($\Delta E, \phi$). The first picture shows the bucket at the beginning of injection, injection ribbon, and the first 39 particles after the first turn (4.7 μs ≃ 0.00 ms). The third picture (T = 0.3 ms) shows two adjacent ribbons representing energy shift between the first and last portion of injected particles. At the bottom of the last picture we can see mountain ranges. The mountain range is a series of curves representing particle intensity distribution versus phase angle for each nth turn, in this case for each 20th turn.

Each of the five pictures shows bucket instantaneous contour. The last picture shows three buckets: at the beginning of injection (T = 0), at the end of injection (T = 0.3), and at a later time (T = 1.6 ms) when 16% of the particles have been lost.

Another controlling program proposed for loss reduction is presented in Figure 2. Because the magnetic field increases as B(t) = 249.3 + 4.9t ([B] = Gs, [t] = ms), the frequency rate, calculated from (17) should be $\dot{f}$ = 0.7 kHz/ms to produce a stationary bucket for 300 μs of injection. After 0.2 ms of injection, the voltage ramps up from 50 kV to 300 kV linearly with $\dot{V}$ = 650 kV/ms. At the end of injection, the frequency rate changes from $\dot{f}$ = 0.7 to $\dot{f}$ = 10 kHz/ms and stays at that level for 0.2 ms before switching to a final 33 kHz/ms. This intermediate period serves as a transition from stationary to fully accelerating mode, preventing particle spill below the bucket due to particle inertia.

Acknowledgments

I am very thankful to Drs. Leif Ahrens and Michael Goldman with whom I had fruitful discussions. I am glad to express my gratitude to Mrs. Marion Heimerle who has done an excellent job of preparing this manuscript.

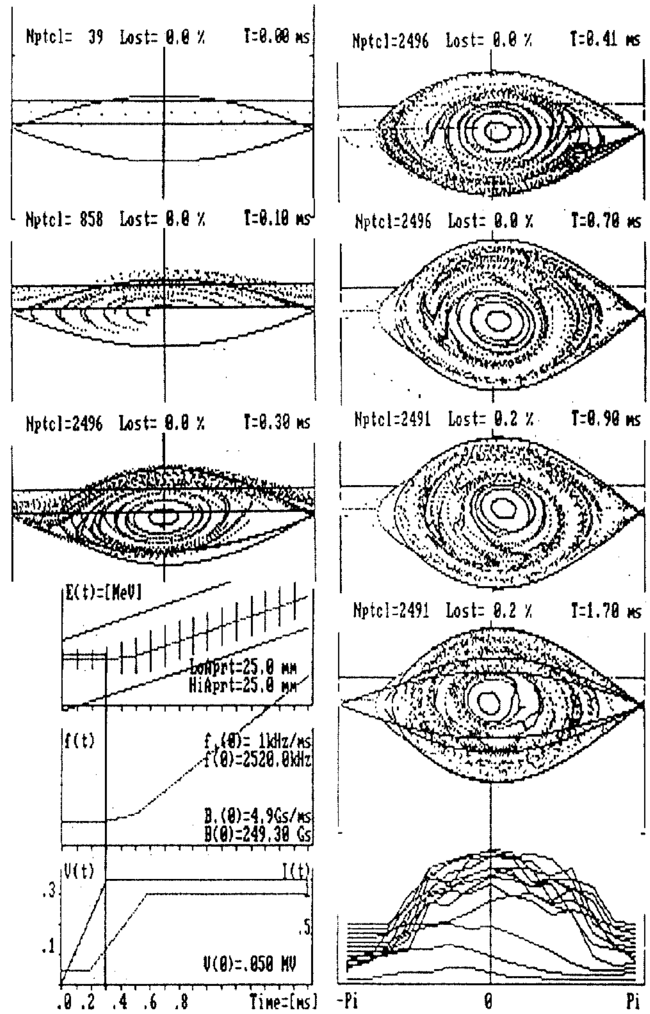

Fig. 2. Proposed controlling program for the AGS.

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2. E. Raka, L. Ahrens, W. Frey, E. Gill, J.W. Glenn, R. Sanders, W. Weng, Increased Intensity Performance of the Brookhaven AGS, IEEE Trans. on Nucl. Sci., Oct. 1985, Vol. NS-32, No. 5, Part II, 3110 -3112.