

COUPLING IMPEDANCES OF A SET OF RESONATORS

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Abstract

The longitudinal and transverse coupling impedances are shown for an infinite pipe loaded with a finite number of cylindrical resonators or radial lines. The impedances are calculated with the code ICYRP based on a field matching technique which was given elsewhere. Special emphasis is put on the high-frequency behaviour of the impedances. The longitudinal impedance behaviour fits in very well with an asymptotic formula given elsewhere. In the case of a chain of resonators the resonances found for an infinite periodic structure are already reproduced by as few as 10 to 20 resonators.

Introduction

The calculation of the impedances of a chain of cylindrical resonators is an important problem from both the practical and theoretical points of view. In practice it can represent such different objects as an RF cavity, a small gap at a vacuum chamber flange or a bellows. On the other side it is one of the few problems which can be treated theoretically either with asymptotic or semi-analytical methods. It is therefore well suited for studying different aspects of the impedance such as the dependence on the size of the object, the high-frequency behaviour and the interference between several objects.

A very nice example of an asymptotic method is the high-frequency solution derived in [1]. Semi-analytical methods are mostly mode-matching techniques with two possible matching surfaces: the cross-section apertures between resonator and pipe or the cylindrical surface between the pipe region and the outer resonator region. The matching on the cross-section has first been chosen for the case of rectangular structures [2] and later for cylindrical devices [3, 4, 5]. The matching on the cylindrical surface was done for the case of pipes with finite length [6] or for the case of an infinite periodic structure [7]. In both cases a discrete set of eigenmodes exists in the pipe as well as in the resonator region.

For infinite pipes or a finite number of resonators the problem is more complicated in the sense that a continuous spectrum of waveguide modes exists in the pipe region. This approach was used to calculate the longitudinal [8] and transverse [9] impedances in the case of a single resonator or radial line and was later extended to a finite number of objects [10]. Recently the same technique was applied in order to derive an analytical expression for the high-frequency limit of the longitudinal impedance [11].

In the paper we will present some new numerical results obtained with the code ICYRP [10] which is based on this mode-matching technique. Hereby special emphasis is put on the high-frequency behaviour of the impedances and on the case of several resonators as compared to a single resonator or an infinite periodic chain.

Description of the problem

The problem dealt with is a point charge travelling parallel to the axis of an infinite pipe loaded with a finite number of cylindrical resonators or radial lines (Fig. 1). In order for such a structure to be treated with a mode-matching technique it is separated into subregions. In the pipe region I the

fields consist of the source fields, taking into account the point charge and a continuous spectrum of waveguide modes describing the fields scattered by the inhomogeneities. In the resonator regions II_i, a discrete set of waves is present travelling radially outward and inward (in the case of radial lines only outward travelling waves exist). The fields of the different sub-regions are matched on the common interfaces $\rho = a$, $il < z < il + 2g$. The result is an infinite set of inhomogeneous linear equations for the field expansion coefficients. The set of equations is solved numerically after truncation by a code called ICYRP (Impedances of Cylindrical Resonators with Pipes).

In some special cases such as short gap lengths or low frequencies it is sufficient to consider only the first mode in which case approximate formulae can be derived. The mathematical treatment is given in [10] and will not be repeated here.

Impedances of Single Objects

For all results given in the following the designations longitudinal and transverse refer to fields with azimuthal mode numbers $M = 0$ and $M = 1$ respectively. The dimensions chosen correspond to an average 3 GHz SLAC accelerating cell.

As a first example the impedances of a radial line (Figs. 2 and 3) are shown. In this case the impedances are essentially caused by the radiation into the line and they do not show resonant behaviour as for cavities. The real part of the longitudinal impedance increases rapidly from zero frequency onward, reaches a maximum around $\lambda/2 \approx 2g$, and decays. At each cut-off frequency $ka_c = n\pi a/2g$, $n = 0, 1, 2, \dots$ of the radial line, a new mode starts propagating and the impedance increases correspondingly. At the cutoff frequencies of the pipe $ka_c = j_{0n}$, $n = 1, 2, \dots$ the impedance is exclusively due to radiation into the line. The transverse impedance shows similar behaviour, except that it starts with zero slope at zero frequency.

In a resonator (Figs. 4 and 5) the situation is quite different. Radiation occurs only into the pipes with the cut-off frequencies $ka_c = j_{01}$ and $ka_c = j'_{11}$ for the longitudinal and transverse cases respectively. Below cut-off the real part of the impedances consists of δ -function resonances. The imaginary part behaves like a reactance. Above cut-off the resonances have a finite bandwidth owing to the radiation into the pipes. Some modes, especially those below cut-off, agree well with the modes of a closed resonator. Others are heavily degraded and/or shifted in frequency owing to the beam pipes. In general, the longitudinal resonances are shifted upwards in frequency since the pipes correspond to a reduction of the capacitance, whereas the transverse resonances are shifted downwards in frequency because the fields are essentially magnetic in the pipe region, thus increasing the inductance.

High-frequency Behaviour

In order to study numerically high-frequency behaviour it is easier to consider a radial line. At first, we can excite either even or odd modes when selecting the cut-off values $ka_c = n\pi a/2g$. Then, the system of linear equations reduces to half the order and we can go up to twice the frequency. This

has been done in [10] for the longitudinal impedance and a clear decay with $\omega^{-1/2}$ was found between $ka = 10$ and $ka = 100$. Secondly, the behaviour is smoother than for a resonator and allows a direct visualization of the decay with frequency.

But the proper way to study the ω dependence is the integration over bins, although it is quite computer-time consuming. This has been done here. Again the real part of the longitudinal impedance shows $\omega^{-1/2}$ behaviour in the case of a radial line. As expected the same behaviour was found for a resonator (Fig. 6). The imaginary part has the same behaviour and about the same magnitude, only 10% smaller. In the graph are also shown values obtained from an asymptotic formula derived in [13,11]. They agree very well with our numerical data, and we can write the asymptotic form of the longitudinal impedance as

$$Z_L \sim (1-j) \frac{Z_0}{2\pi} \sqrt{\frac{2g}{a}} \frac{1}{\pi ka}, \quad Z_0 \text{ free space impedance}$$

The high-frequency decay of the transverse impedance, shown in Fig. 7, is $\omega^{-3/2}$ as expected.

Chain of Resonators

Often it is of interest how a distinct resonance shifts or how the total impedance changes when going from a single resonator to many resonators. This is of particular interest since it would allow us to decide from what number on an infinite periodic structure is a good approximation.

Figure 8, taken from [10], shows the real part of the longitudinal impedance of a very small resonator as is used for bellows. The single undulation has a broad-band resonance located at $ka = 4.7$. This corresponds to a $\lambda/4$ resonance of the radial line. With an increasing number of undulations the resonance becomes broader owing to coupling between undulations, and is shifted in frequency. For more than 20 undulations, a real isolated resonance is formed at $ka = 3.45$. This resonance becomes smaller and smaller in bandwidth, although the losses per undulation stay constant. Finally, the resonance becomes δ -function-like for the case of an infinite periodic structure. The resonance frequency $ka = 3.45$ is that of the space harmonic, which has a phase velocity equal to the velocity of light.

For a large resonator the situation is very much the same apart from the fact that the resonances are spaced much more closely. Fig. 9 shows the case of a set of 20 resonators as used in Fig. 4. The resonances of a single resonator are narrowed in bandwidth and shifted towards the values of an infinite periodic structure. The process of narrowing the bandwidth, i.e. increasing the Q-value, is even more impressively viewed by the imaginary part.

References

- [1] J.D. Lawson, Rutherford Lab. internal report, RHEL/M 144 (1968).
- [2] H.G. Hereward, Internal report CERN/ISR-DI/75-47 (1975).
- [3] H. Hahn and S. Zatz, IEEE Trans. Nucl. Sci. NS-26, 3626 (1979).
- [4] L. Vos, Internal report CERN-SPS/86-21(MS) (1986).
- [5] S.A. Kheifets, K.L.F. Bane and H. Bizek, Internal report SLAC-PUB-4097 (1987).
- [6] E. Keil, Internal report CERN/ISR-TH/68-49 (1968).
- [7] E. Keil, Proc. 7th Internat. Conf. on High Energy Accelerators, Yerevan, 1969, Vol. II, p. 551.
- [8] H. Henke, Internal report CERN/LEP-RF/85-41 (1985).
- [9] H. Henke, IEEE Trans. Nucl. Sci. NS-32, 2335 (1985).
- [10] H. Henke, Internal report CERN-LEP-RF/87-62 (1987).
- [11] S.A. Kheifets and S.A. Kheifets, Internal report, CERN-PR-87-030 (1987).
- [12] E. Keil, Nucl. Instrum. Methods 100, 419 (1972).
- [13] G. Dôme, IEEE Trans. Nucl. Sci. NS-32, 2531 (1985).

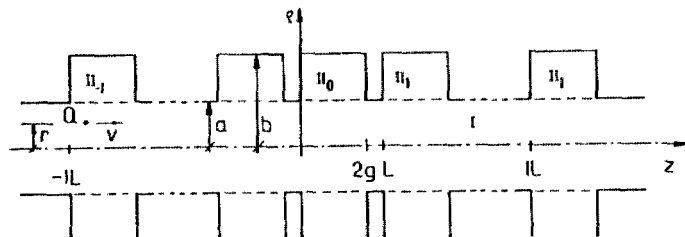


Fig. 1 Point charge travelling parallel to the axis of an infinite pipe loaded with cylindrical resonators (upper part) or radial lines (lower part).

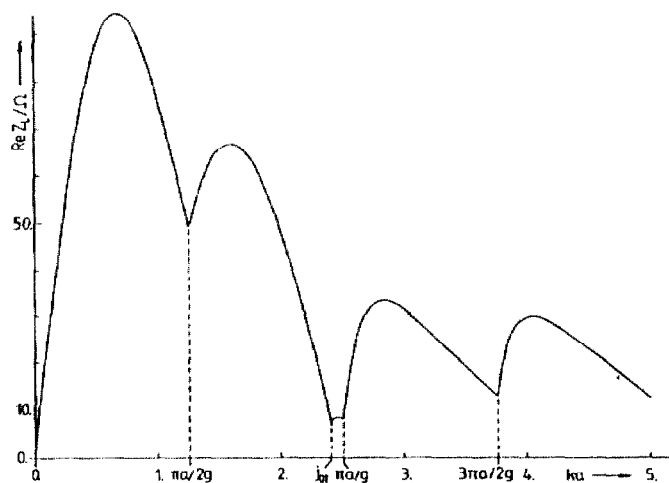


Fig. 2 Real part of the longitudinal impedance of a radial line (geometry of Fig. 1 with $I = 1$, $a = 11.63$ mm, $2g = 29.15$ mm, $\gamma = 10^6$).

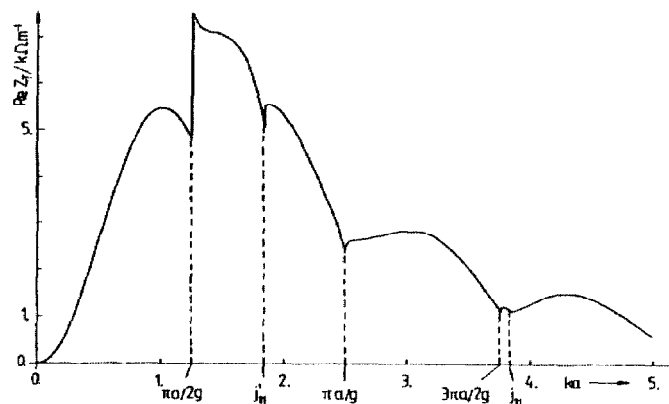


Fig. 3 Real part of the transverse impedance of a radial line (parameters of Fig. 2).

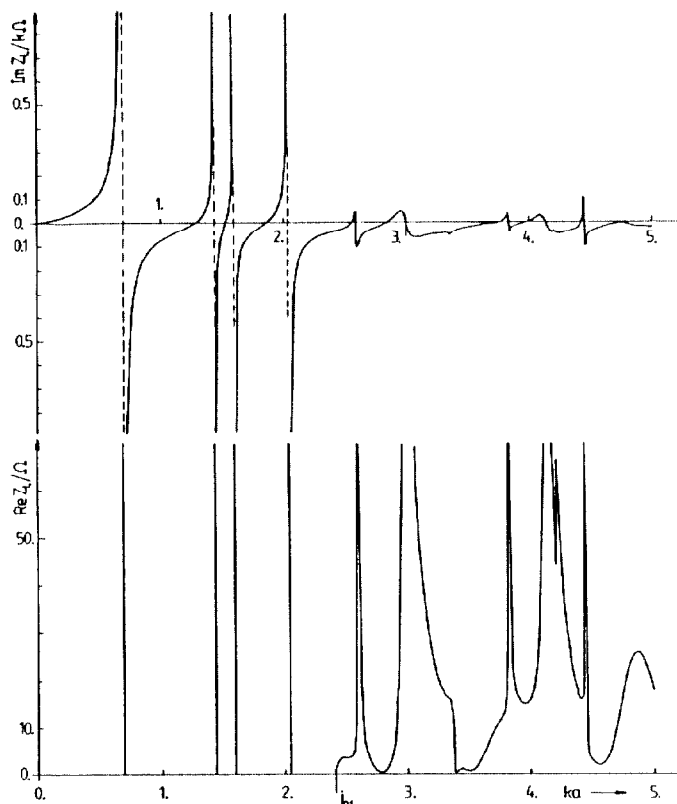


Fig. 4 Longitudinal impedance of a resonator (geometry of Fig. 1 with $I = 1$, $a = 11.63$ mm, $2g = 29.15$ mm, $b = 41.3$ mm, $\gamma = 10^4$).

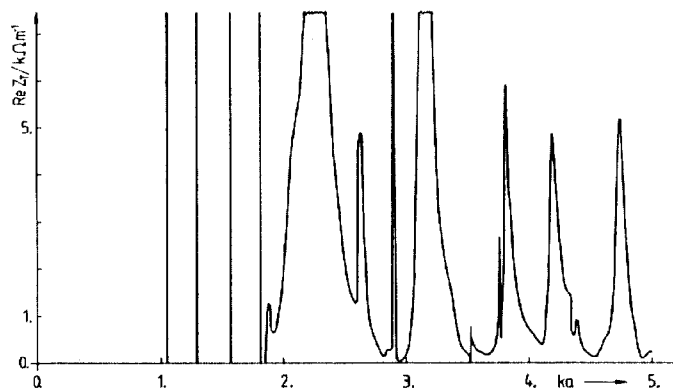


Fig. 5 Real part of the transverse impedance of a resonator (parameters of Fig. 4).

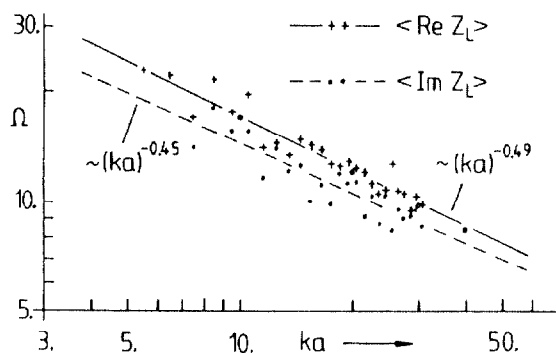


Fig. 6 High-frequency dependence of the longitudinal impedance of a resonator (averaged over intervals $\Delta ka = 1$, parameters of Fig. 4) $\circ\circ\circ$ data from asymptotic formula [11,13].

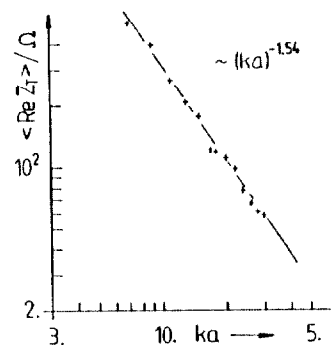


Fig. 7 High-frequency dependence of the transverse impedance of a resonator (averaged over intervals $\Delta ka = 1$, parameters of Fig. 4).

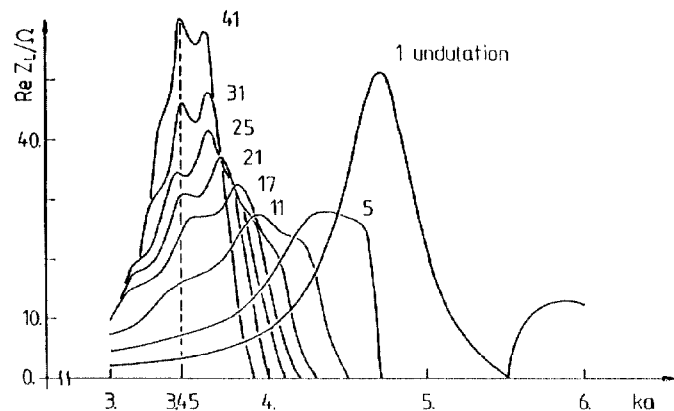


Fig. 8 Real part of the longitudinal impedance of a set of resonators (geometry of Fig. 1 with $a = 15$ mm, $2g = 2$ mm, $b = 19$ mm, $L = 4$ mm, $\gamma = 10^6$).

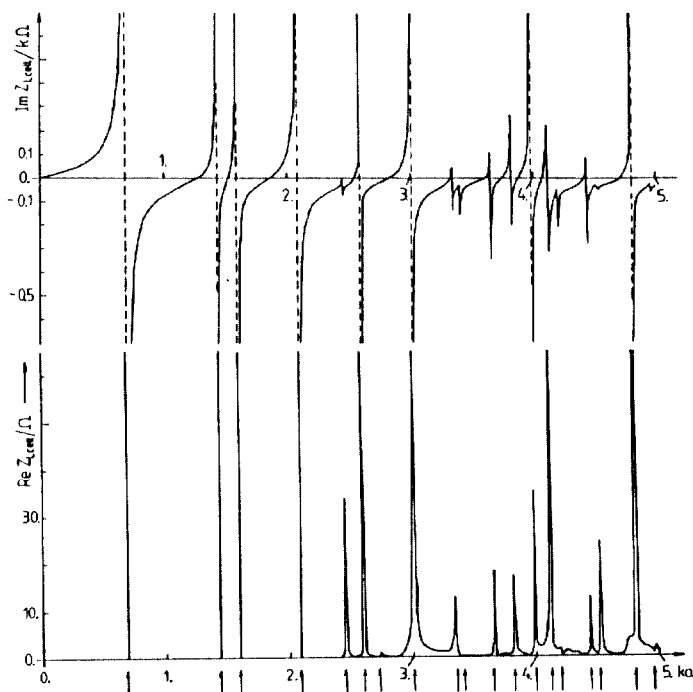


Fig. 9 Longitudinal impedance per resonator for the case of a chain of 20 resonators (geometry Fig. 1 with $I = 20$, $a = 11.63$ mm, $2g = 29.15$ mm, $L = 35$ mm, $b = 41.3$ mm, $\gamma = 10^4$). The arrows $\uparrow$ indicate space harmonics in an infinite periodic structure with phase velocity equal to velocity of light (calculated with KN7C [12]).