GALOPR, A BEAM TRANSPORT PROGRAM, WITH SPACE-CHARGE AND BUNCHING

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Abstract

GALOPR is a first order beam transport code including three dimensional space-charge forces and the beam bunching process. It deals with usual optical devices (bending magnets, lenses, solenoids, drifts, bunchers) and can take into account any special optical device represented by its transfer matrix with space-charge the ("Müller inflector" was recently introduced as one of these devices). The beam can be continuous or undergoing a bunching or debunching process. The beam line parameters can be optimized in order to fit at will the 6 x 6 transfer matrix and the 6 x 6 covariance matrix for a maximum beam intensity. The results are presented with useful data tables and graphical displays.

1) Introduction

The computer code GALOPR (GANIL beam Lines OPtics including Radiofrequency bunchers) has been developed at GANIL from the code PREINJ (Ref. 1, 2) which was written at CERN to study the focusing and bunching characteristics of the Low Energy Beam Transport system (LEBT) for the actual proton linac injector, including space charge forces.

Firstly, a version without buncher but allowing to choose the ion mass number and charge state was used at GANIL to calculate the transfer lines between the 3 cyclotrons ; the space charge forces were taken into account in the dipoles.

Since 1985, we have been developing the GALOPR version. Its universality makes it possible to calculate with linear approximation, the optics of a beam line either existing or being designed, for continuous or bunched beams, or beams in the process of being bunched, including space charge forces. Any buncher or rebuncher and any kind of element with an analytically known transfer matrix (drifts, quadrupoles, dipoles, solenoids) or a numerical transfer matrix are taken into account (Ref. 3).

The beam is considered as an hyperellipsoid in the 6 dimensional phase space. All forces, including space charge are linear with respect to the reference particle which in the present version is energy-fixed .

We have used GALOPR to study the axial injection beam line into the two injector cyclotrons for the GANIL modifications to come (O.A.E and O.A.I) including a "Müller inflector" in the 20 kV injection line and a "Pabot-Belmont inflector" in the 100 kV injection line.

2) Beam parameters

Each particle at a point S of the beam line is located in the 6-dimensional phase space by its coordinates : x, x' = dx/ds, y, y' = dy/ds , z and z' = dz/ds, where s is the curvilinear coordinate. The fifth coordinate z is proportional to the difference between the time of arrival of a particle at point S and the corresponding time t_o for the reference particle,

$$z = v_r\,(t - t_o) \tag{1}$$

where v_r is the velocity of the reference particle.

The beam is entirely defined by its intensity and the second order moments of the particle distribution in the 6-D phase space ; we will sometimes call these moments "rms" values". They can be written, for a centered distribution :

$$\overline{uv} = \frac{\oint_{V_0} u\,v\,\rho(V)\,dV}{\oint_{V_0} \rho(V)\,dV} \quad \text{with } u,v \in \{x,x',y,y',z,z'\} \tag{2}$$

where dV and V are the volume element and the volume in the 6-D space.

If the 6 coordinates of each of the N particles of the distribution are known, the "rms" values become merely:

$$\overline{uv} = \frac{1}{N}\sum_{1}^{N} u_i\ v_i \tag{3}$$

and define the covariance matrix Σ of the beam.

The type of distribution is assumed to be hyperellipsoidal, i.e the surfaces of equal density are homothetic to the hyperellipsoid defined by Σ .

If $\hat{u}$ represents the envelope of the hyperellipsoid for the coordinate u, then the ratio $k = \hat{u}/\sqrt{\overline{u^2}} = \hat{u}/\tilde{u}$ is characteristic of the distribution. For an uniform distribution (ρ = cst) in a n-D space, $k = \sqrt{n + 2}$. A continuous beam, is represented by a infinitely long cylinder (n = 2, k = 2), and a bunched beam by an ellipsoid (n = 3, $k = \sqrt{5}$), both being uniformly charged.

The rms emittance in each of the 3 phase planes (x, x'), (y, y') and (z, z') is given by :

$$\tilde{E}_u^2 = \overline{u^2}\ \overline{u'^2} - \overline{uu'}^2 \tag{4}$$

and is connected to the marginal emittance by :

$$E_u = k^2\ \tilde{E}_u \tag{5}$$

As demonstrated in Ref. 4, the evolution of the rms values depends mainly on the linear component of the space charge force, in the case of linear external forces. Therefore, all types of distributions having the same rms values can be treated by such linear programs ; the tuning of the physical parameters of the line is then the same for any distribution.

The above-mentioned (cylinder and ellipsoid) have space charge forces varying linearly with the coordinates and will be chosen in the following treatment as models.

3) Space charge force calculation

The components of the radial electric field inside an infinitely long cylinder with elliptical section are given by :

$$E_x = \frac{b}{a+b}\,x \qquad E_y = \frac{a}{a+b}\,y \qquad E_z = 0 \tag{6}$$

and inside an ellipsoid by :

$$E_x = \frac{\rho}{\epsilon_0} J_a\ x \qquad E_y = \frac{\rho}{\epsilon_0} J_b\ y \qquad E_z = \frac{\rho}{\epsilon_0} J_c\ z \tag{7}$$

where a, b and c are the axes, if these volumes are in their principal axes, and where :

ϵ_o = dielectric constant of vacuum

ρ = space charge density

$$J_a = \frac{1}{2}\,a\,b\,c \int_0^{\infty} \frac{d\lambda}{(a^2+\lambda)\sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}} \tag{8}$$

and analogous for J_b and J_c are dimensionless elliptic integrals numerically computed by the Gauss' method.

Choosing $s = v_r t$ as the independant variable, the equations of motion can be written for both models :

$$\frac{d^2x}{ds^2} = \frac{e}{2\,\epsilon_0\,(W/\epsilon)} E_x = K_x\, x \qquad (9)$$

and same for y and z
with W = energy per nucleon of the reference particle.
ε = charge to mass ratio.

Introducing the rms values, we then get :

$$K_x^c = \frac{\rho}{2\,\epsilon_0\,(W/\epsilon)} \frac{\tilde{y}}{\tilde{x}+\tilde{y}} \qquad \text{for the cylinder (same for y)} \qquad (10)$$

$$K_x^e = \frac{\rho}{2\,\epsilon_0\,(W/\epsilon)} J_{\tilde{x}} \qquad \text{for the ellipsoid (same for y, z)} \qquad (11)$$

4) Transfer matrices

The beam line is composed of thick and thin elements ; the space charge forces act only in the thick ones, which are subdivised into steps, the length of which is chosen in such a way that it has only a small effect on the transverse dimensions.

We use 4 types of transfer matrices :

4.1. Thin elements

The transfer matrix for a pole-face rotation at one end of a dipole, and for a rotation of the transverse coordinate system around the curvilinear coordinate are given in Ref. 5.

Any element (or combinaison of elements) previously calculated and given in a numerical form (for instance, the Pabot - Belmont inflector) can be introduced in the program as a 6 X 6 matrix.

4.2. Thick elements with decoupled phase planes

If the reference frame of the ellipsoid and of the motion are the same, then the space charge forces (K_u) can be added to the external ones (Q_u) in each phase plane, leading to the Hill-type equation.

$$\frac{d^2u}{ds^2} + (Q_u - K_u)\,u = 0 \qquad (12)$$

The following table gives the constants Q_u corresponding to the external forces of the different elements :

	Q_x	Q_y	Q_z
Drift Space	0	0	0
Quadrupole	G/BR	$-Q_x$	0
Solenoid(*)	$\frac{1}{2} B_s/BR$	Q_x	0
Rebuncher	$\frac{\pi V_R T}{2\pi \beta\lambda (W/\varepsilon) L_G}$	Q_x	$-Q_x$

BR = magnetic rigidity
$\beta\lambda$ = distance between 2 consecutive bunches adjacent
G = quadrupole gradient (G > 0 is focusing in the x direction)
B_s = maximum field in the solenoid
$V_R T$ = maximum voltage times transit time factor on the axis
L_G = gap length for a single-gap element
L_S = solenoid length

(*) A rotation by an angle $Q_x L_S$ must be added afterwards to get the transfer matrix in the initial reference frame.

In each plane, the 2 x 2 transfer matrix contains sine and cosine terms if $Q_u - K_u > 0$ and cosh and sinh terms if $Q_u - K_u < 0$.

4.3. Thick elements with coupled phase planes

We consider the most complicated case of a dipole with the bunch tilted with respect to the axes. We first calculate the eigenvalues of the covariance matrix in the real space and the eigenvectors matrix (V) which transforms the (x, y, z) frame into the (X, Y, Z) frame of the ellipsoid; the "rms" values $\overline{X^2}, \overline{Y^2}, \overline{Z^2}$ of the ellipsoid are the above eigenvalues.

According to the previous paragraph, the space charge forces K_X, K_Y and K_Z can be calculated.

The new 6-D covariance matrix may be obtained in two ways.

1st. method

The 6-D covariance matrix is calculated in the (X, Y, Z) frame ; then, a new one is obtained by the action of a thin lens with a strength equivalent to the effect of the space charge force over one step.

This new matrix is projected on the original reference frame, and is finally transfered through the next element step.

2nd. method

The forces $K_X X$, $K_Y Y$ and $K_Z Z$ are projected on the original axes, giving :

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [V]^{-1} \begin{bmatrix} K_X & & \\ & K_Y & \\ & & K_Z \end{bmatrix} [V] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [K_{uu}] \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad (13)$$

These constants K_{uu} must be added to the external forces implied in the classical system :

$$\frac{d^2x}{ds^2} - \frac{n}{\rho^2} x = \frac{1}{\rho}\frac{dz}{ds}$$

$$\frac{d^2y}{ds^2} + \frac{n}{\rho^2} y = 0$$

$$\frac{d^2z}{ds^2} = -\frac{1}{\rho}\frac{dx}{ds} \qquad (14)$$

with n = field index
R = radius of curvature of the reference frame.

And the resulting system is solved by classically going to a Runge-Kutta method applied to a six first-order differential equation system.

The final transfer matrix is composed of the 6 vectors, solutions of the system. It is however necessary to perform a matrix transformation at both ends in order to take into account the difference between entrance and output reference frames.

This second method is more time consuming but it is more physical to add all the forces before solving the equations.

4.4. Special elements

The Müller inflector can be given as an example since it can be represented by an analytical matrix from the origin to any points inside it. Care of the space charge effect is taken by adding a perturbation matrix (Ref. 6). In GALOPR, we are only using the total transfer matrix (Ref. 6) for a 90° deflection angle ; since an inflector is necessarily attached to the central region of a compact cyclotron, two additional factors must be taken into account : the magnetic field variation along the trajectory and a rotation to match the reference frame of the accelerated motion.

5. Transition from a continuous to a bunched beam.

All the elements studied above have a linear action on the beam. When a continuous beam, with a negligible energy spread traverses a buncher, the energy modulation resulting from the sinusoidal voltage generates a longitudinal emittance. In our model, it is of importance to include in this emittance only the particles that will be captured further while taking into account the effect of all the particles (including those which will be lost) for the space charge force calculations ; we proceed in 3 steps :

5.1. Generation of the longitudinal emittance at the buncher

Before the buncher, the emittance is a single line along the phase axis (zero energy spread). The relative momentum spread is given by :

$$z' = \frac{\delta p}{p} = - \frac{e\,V_B T}{2\,(W/\epsilon)} \sin\left(2\pi \frac{z}{\beta\lambda}\right) \qquad (15)$$

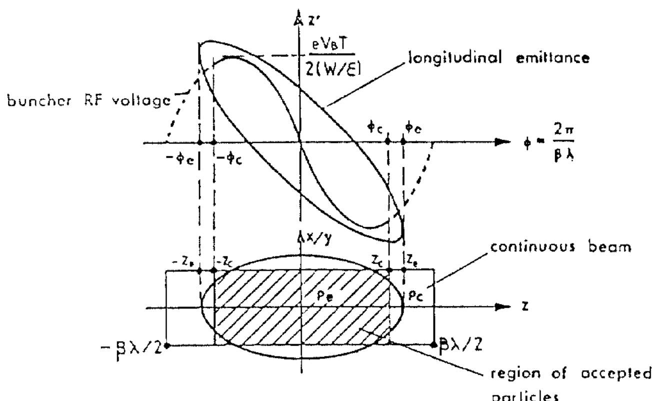

Figure 1

The assumption is that, at the buncher, the trapped particles lie inside the hatched region of figure 1 ; the separation between the trapped and rejected particles is called the "cut-off" phase Φ_c .

Since, at this point, the beam is continuous, the trapped intensity is proportional to ϕ_c :

$i_{trap} = i_o\ \phi_c/\pi$ where i_0 is the current upstream.

$\eta = \phi_c/\pi$ is the "bunching efficiency". In order to inject this bunch into the following sections of the line, the hatched portion of the cylinder is modified into an ellipsoid with the same volume and the same transverse rms values $\tilde{x}$ and $\tilde{y}$. The rms values are calculated assuming an uniform distribution between $-\phi_e$ and ϕ_e in the ellipsoid or between $-\phi_c$ and ϕ_c in the cylinder (the $\overline{zz'}$ and $\overline{z'^2}$ expressions are detailed in Ref 1, 2).

When several bunchers are present, the modulation is no longer a simple sine and it is much easier to make a numerical treatment to calculate the modulation and therefore to calculate the rms values, which is the case in our computer code where a double drift harmonic buncher is considered.

5.2. Introduction of the space charge

We then have to fulfill the following requirement: there must be continuity of the space charge forces during the transition from a 4-D to a 6-D phase space.

The chosen model is a combination of the actions of a infinite cylinder with the density ρ_c of the rejected particles and of an ellipsoid with the density $\rho_e - \rho_c$, where ρ_e is the density of the trapped particles. These densities are given by :

$$\rho_e = \frac{i_0\ T_{RF}}{V}\,\eta \qquad\qquad \rho_c = \frac{i_0\ T_{RF}}{V_0 - V}\,(1-\eta) \qquad (16)$$

with $V_0 = 4\pi\beta\lambda\ \tilde{x}\,\tilde{y}$ = volume of the cylinder having the length $\beta\lambda$

and $V = 8\pi\ \tilde{x}\,\tilde{y}\ z_e$ = volume of the hatched cylinder

and can be replaced in formulae (10) and (11), giving :

$$K_x = K_x^c + K_x^e = \frac{1}{2\,\epsilon_0\ (W/\epsilon)}\left(\rho_c\,\frac{\tilde{y}}{\tilde{x}+\tilde{y}} + (\rho_e - \rho_c)\,J_{\tilde{x}}\right)$$

$$K_z = K_z^e = \frac{\rho_e - \rho_c}{2\,\epsilon_0\ (W/\epsilon)}\,J_{\tilde{z}} \qquad (17)$$

5.3. Longitudinal matching

Up to this point, the choice of ϕ_c and $V_B T$ was arbitrary. If no space charge effect exists and if an upright ellipse is required at the end of the line, the phase acceptance ϕ_a is given by the intersection of the ellipse with the phase axis and depends only on ϕ_c and not on $V_B T$.

In the presence of space charge forces, one must use a minimization method acting on ϕ_c and $V_B T$ in order to obtain the desired ϕ_a value.

6. General optimization

The previous sections show that it is possible to transport a beam with space charge forces along a beam line of given structure. The goal of the program is also to determine the tuning parameters of the elements in order for the beam to be matched, i.e. for its 6-D emittance to fit the required acceptance.

In the transverse planes, the main parameters are the quadrupole gradients (and rarely the drift lengths); they are varied in order to :

- fulfill given conditions on some elements either of the covariance matrix or of the transfer matrix at the matching point,
- limit the beam dimensions locally or along the whole line.

In the longitudinal plane, the matching is possible by varying ϕ_c, $V_B T$ of the first buncher and the voltage of a rebuncher.

Due to the various couplings between the transverse and longitudinal planes, all the parameters, conditions and limits are included in the same optimization which is carried out by a least-squares method.

7. Conclusion

The code GALOPR is now running onto the IBM 3090 (VM/CMS system) of the C2 IN2P3 (IN2P3 computer centre in Lyon) and into the 32 bits MODCOMP at GANIL : an off-line code allows to draw the beam envelopes onto a "BENSON" plotter.

Two further improvements to this code are planed by introducing :

- a thin lens taking into account the space charge effects in an element whose transfer matrix is given in a numerical form,
- elements in which the central particle is accelerated.

References :

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