LONGITUDINAL INSTABILITY OF BUNCHED BEAM INTERACTING WITH HIGH-FREQUENCY IMPEDANCE

V.I.Balbekov

Institute for High Energy Physics, Serpukhov, 142284

Abstract. The longitudinal bunched-beam instability is considered when a vacuum chamber has a high-frequency impedance. It is shown that the microwate criterion is, in certain cases, the sufficient stability condition. For a resonance impedance, the necessary and sufficient stability condition is obtained in the form similar to that for a coasting beam. The role of statical effects which may cause the increase of momentum spread sufficient for beam stabilization is discussed.

I. INTRODUCTION

The microwave model is often used for the description of the longitudinal bunched-beam instability. The model gives the following stability criterion [1]:

$$\left|\frac{Z_k}{k}\right| < \frac{\beta^2 E B |\eta|}{eJ\Lambda}\left(\frac{\Delta p}{p}\right)^2. \tag{1}$$

Here Z_k is the longitudinal impedance for the k-th harmonic of the revolution frequency, J is the average beam current, B is the bunch factor, $E=mc^2\gamma$ and β are the energy and normalized velocity of particles, $\eta=\alpha-\gamma^{-2}$, α is the momentum compaction factor, $\pm\frac{\Delta p}{p}$ is the maximum momentum spread, and the factor $\Lambda\sim 1$ depends on the bunch distribution function. The model explains satisfactorily some experimental data [1-2] but has no convincing theoretical ground.

Equation (1) was initially obtained by analogy to the known stability criterion for a coasting beam, with the local increase of the beam current taken into account by factor B. This analogy is in fact far from being rightful because the conditions for perturbations to propagate in coasting and bunched beams differ essentially. The perturbation produced by current fluctuations in a coasting beam is propagating from particle to particle against the beam motion until the revolution closes and so on. That is how back-coupling is realized and the "self-action" of particles occur, which is the indispensable condition for the usual (regenerative) instability to arise. In a bunched beam, perturbation can propagate from the bunch head to its tail without passing to the next bunch because of a rapid field damping (for a wide-band impedance). Back-coupling cannot be closed synchrotron oscillations either since this mechanism is effective only for a low-frequency impedance when multipole oscillations of bunches arise [3]. For simultaneous excitation of several multipoles, which is typical for a high-frequency impedance, this channel is actually locked.

There may be a different interpretation of inequality (1). The interaction of a bunch with an inductive impedance ($Z_k\sim ik$) was investigated in papers [4-5]. It may be seen that the self-consistent momentum spread satisfying inequality (1), where $\Lambda=1/\pi$, is established after the transition, with this distribution being stable. This suggests that restriction (1) can, in some cases, have "static" origin and at the same time be the sufficient stability condition.

It was also shown in paper [6] that inequality (1) can be the sufficient condition for stability or, more precisely, for the absence of a strong instability. However, the problem regarding the necessity of this condition remains open for discussin.

The role of static effects was studied in paper [7] in a different way for a freely drifting bunch. The condition for its stability is shown to be the same as for a coasting beam (restriction (1), without factor B). However, violation of restriction (1) (with factor B included) leads to a sharp amplification of the beam reaction to external perturbations which is interpreted as a static effect.

The present work discusses the scope of these problems with reference to the standard acceleration mode.

2. THE SUFFICIENT CONDITION FOR STABILITY

Let us take the system of equations for Fourier-harmonics of the electric field of the beam without the equilibrium field:

$$E_k(\Omega)=Z_k(\Omega)\sum_{k'}\Pi_{kk'}(\Omega)E_{k'}(\Omega), \tag{2}$$

where $\Pi_{kk'}(\Omega)$ is the beam conductivity matrix [8]. The solvability condition of this system gives the spectrum of the characteristic beam frequencies $\{\Omega\}$. The presence of the frequencies with a positive imaginary part means instability of the beam.

Let us suppose that the impedance is considerable only for the harmonics

$$|k\pm k_o|\lesssim\Delta k,\quad \Delta k\ll k_o,\quad k_o\Delta\theta\gg\frac{\Omega_s}{\Delta\Omega_s}. \tag{3}$$

where $2\Delta\theta$ is the bunch azimuthal length, Ω_s, $\Delta\Omega_s$ are the synchrotron frequency and its spread. The excitation frequency estimated in paper [3] (see also Section 3) is

$$|\Omega|\sim k_o\Omega_s\Delta\theta\gg\frac{\Omega_s^2}{\Delta\Omega_s}. \tag{4}$$

Following [3] one may obtain in these approximations the formula for the beam conductivity:

$$\Pi_{kk'}=i\frac{eJ}{\mathcal{L}}\int_{-\infty}^{\infty}\frac{F'_{k-k'}(u)du}{\Omega+k_o\eta\omega_s\frac{u}{p_s}}, \tag{5a}$$

$$F_k(u)=\frac{1}{2\pi}\int_{-\pi}^{\pi}F(\theta,\ u)e^{-ik\theta}d\theta, \tag{5b}$$

$$\int_{-\infty}^{\infty}F_o(u)du=1. \tag{5c}$$

Here $\mathcal{L}$ is the orbit length, $u=p-p_s$ is the momentum deviation from the synchronous one, ω_s is the angular velocity of a synchronous particle, $F(\theta,\ u)$ is the equilibrium distribution function normalized according to (5c).

Let us rewrite system (2) as

$$iZ_k\sum_{k'}G_{k-k'}E_{k'}=\lambda E_k, \tag{6}$$

where G_k are the Fourier harmonics of the function

$$G(\theta)=\frac{\Delta p^2}{2}\int_{-\infty}^{\infty}\frac{\partial F}{\partial u}(\theta,u)\frac{du}{u+\frac{\Omega p_s}{k_o\eta\omega_s}} \tag{7}$$

and λ is the eigenvalue of the system. The comparison of (2) and (5a) yields the dispersion equation

$$\lambda(\Omega) = \frac{\beta^2 E k_o \eta}{eJ}\left(\frac{\Delta p}{p}\right)^2 \tag{8}$$

Whereof there follows the sufficient condition for stability:

$$\max|\lambda(\Omega)| < \frac{\beta^2 E k_o |\eta|}{eJ}\left(\frac{\Delta p}{p}\right)^2 \tag{9}$$

for $\mathrm{Im}\,\Omega>0$. To estimate the maximum eigenvalue we use the known inequality:

$$|\lambda| \leq \max_{(k)}|Z_k| \sum_{k'} |G_{k'}| . \tag{10}$$

For the Gaussian distribution we have

$$F \propto \exp\left(-\frac{\theta^2}{\theta_o^2} - \frac{u^2}{u_o^2}\right), \quad G_k \propto \exp\left(-\frac{k^2\theta_o^2}{4}\right), \tag{11}$$

therefore

$$\sum_k |G_k| = \left|\sum_k G_k\right| = |G(0)| = \frac{\Lambda}{B}, \tag{12a}$$

$$\Lambda = \max_{(u_\Omega)}\left|\frac{\Delta p^2}{2\pi} \int_{-\infty}^{\infty} \frac{F_o'(u)du}{u+u_\Omega}\right|. \tag{12b}$$

The substitution of (12a) into (10) and then into (9) leads to inequality (1). A similar result has been obtained in paper [6], However, the assumption used there, $\mathrm{Im}\,\Omega \gg \Omega_s$, meant a strong instability. In fact, this condition is not obligatory, and formula (1) remains valid even for $\mathrm{Im}\,\Omega \to 0$, allowing one to interpret it as the sufficient condition for stability. In addition, it is easy to see that the smoothness of the distribution function is the most essential point for relation (12a) to be fulfilled. This allows one to believe that the obtained result is true for a sufficiently wide class of realistic distributions, for which, as estimated, $\Lambda \sim 1$.

3. RESONANCE-TYPE IMPEDANCE

Let us consider an impedance

$$Z_k = \frac{R_r}{1-i\dfrac{(k\omega_s+\Omega)^2-\omega_r^2}{2\omega_s\Delta k(k\omega_s+\Omega)}} \simeq \frac{R_r}{1-i\dfrac{k^2-\omega_r^2/\omega_s^2}{2k\Delta k}} \tag{13}$$

It is possible to obtain from (2), (5a) and (13) the differential equation for the function $E(\theta)=\sum E_k e^{ik\theta}$:

$$E''-2\Delta k E' + \frac{\omega_r^2}{\omega_s^2}E = -i\frac{2eJR_r\Delta k}{\mathcal{L}}\left[E\int_{-\infty}^{\infty}\frac{\frac{\partial F}{\partial u}(\theta,\ u)du}{\Omega+k_o\eta\omega_s\frac{u}{p_s}}\right]' \tag{14}$$

Its solution is $E=Ae^{ik_o\theta}+\bar{A}e^{-ik_o\theta}$, where $k_o \simeq \frac{\omega_r}{\omega_s}$, and the slowly varying amplitude satisfies the equation

$$A'(\theta)+g(\theta)A(\theta)=0, \tag{15a}$$

$$g(\theta)=i\frac{eJR_r\Delta k}{\mathcal{L}}\int_{-\infty}^{\infty}\frac{\frac{\partial F}{\partial u}(\theta,u)du}{\Omega+k_o\eta\omega_s\frac{u}{p_s}} - \Delta k - i\left(\frac{\omega_r}{\omega_s}-k_o\right). \tag{15b}$$

Equation (15a) has the periodic solution only under the condition that

$$\int_0^{2\pi} g(\theta)d\theta = -2\pi i n, \tag{16}$$

which, with (13) and (15b) taken unto account, can be presented as

$$1 = i\frac{eJZ_k}{\mathcal{L}}\int_{-\infty}^{\infty}\frac{F_o'(u)du}{\Omega+k_o\eta\omega_s\frac{u}{p_s}}, \tag{17}$$

where $k=k_o+n$ is an integer. This expression coincides almost exactly with the dispersion equation for a coasting beam having the distribution function $F_o(u)$, the difference being an inessential replacement $k \to k_o$ in the integral. The stability condition is reduced to inequality (1) without the bunch-factor B, thus depending on the average beam current only.

4. THE ROLE OF EXTERNAL PERTURBATIONS

A more consistent approach should include the initial conditions for the beam current and field and also such external factors as variations of the accelerating voltage amplitude, RF noise, etc. Though they are many, still they can be taken into account by introducing an additional current $J_{ext}(\theta)e^{-i\Omega t}$. As a result, system (2) and equation (4) become nonuniform. In particular, equation (15a) becomes

$$A'+gA = -i\frac{R_r\Delta k}{\mathcal{L}k_o}J_{ext}'e^{-ik_o\theta} \tag{18}$$

and has the following periodic solution:

$$A(\theta) \simeq \frac{R_r\Delta k}{\mathcal{L}}\int_{\theta}^{2\pi+\theta}\frac{\exp\left[-ik_o\theta'+\int_{\theta}^{\theta'}g(\theta'')d\theta''\right]}{\exp\int_0^{2\pi}g(\theta'')d\theta''-1}J_{ext}(\theta')d\theta'. \tag{19}$$

This expression has a pole in the complex plane Ω whose position is determined from the dispersion equation (17). If this pole is located in the upper half-plane, usual stability occurs. Besides, the amplitude growth may be related to a large value of the exponential multiplier in the numerator of formula (19). One may get convinced that for a narrow-band impedance, $\Delta k \to 0$, this does not lead to additional limitations on the intensity except for those following from (17). For a wide-band impedance, $\Delta k \Delta\theta > 1$, a rough condition of such a growth can be written as

$$\max \mathrm{Re}\, g(\theta) > 0. \tag{20}$$

Whereof with account of (15b) the inequality reciprocal to (1) is obtained with

$$\Lambda = \max\left|\frac{B\Delta p^2}{2}\frac{\partial F}{\partial u}(\theta,\ u)\right| \tag{21}$$

It makes no difficulty to see that for realistic distributions $\Lambda \sim 1$.

Of course, these results make sense only if the current J_{ext} is sufficiently large. Here lies the distinction from the traditional approach attributing the appearance of J_{ext} to small fluctuations of the distribution function. The appearance of J_{ext} during typical acceleration is, apparently, related mainly to the beam deviation from the self-consistent state due to variation of particle energy or of any other parameters of the machine. If so, the effect considered above should be interpreted as an evolution of the equilibrium distribution with the external conditions varied. As a result, the momentum spread satisfying inequality (1) is established in the beam, which eliminates a possibility of "usual" instability. However, with a rapid variation of the machine parameters, the evolution of the distribution function becomes nonadiabatic and must be accompanied by a noticeable modulation of the bunch current, i.e., by a burst of high-frequency radiation. The most pronounced effect is to be expected in the region of the transition energy. This is confirmed by experimental data [1-2].

REFERENCES

1 . D.Boussard. - CERN Report LABII/RF/INT/75-2, 1975.
2 . V.I.Balbekov et al. - Preprint IHEP 85-129, Serpukhov, 1985.
3 . V.I.Balbekov, S.V.Ivanov. - Atomnaya Energia, 1985, v. 59, p. 42.
4 . A.N.Lebedev, E.A.Zhilkov. - NIM, 1966, v.45, p. 238.
5 . A.N.Lebedev. - Atomnaya Energia, 1968, v. 25, p. 100.
6 . I.M.Wang, C.Pellegrini. - Proc. XI Intern. Conf. on High Energy Accel., Geneva, 1980, p. 554.
7 . V.I.Balbekov. - Preprint IHEP 86-73, Serpukhov, 1986.
8 . V.I.Balbekov, S.V.Ivanov. - Proc. of XIII Intern. Conf. on High Energy Accel., Novosibirsk, 1987, v. 2, p. 124.