THE PARTICLES SCATTERING ON THE HARD PART OF SYNCHROTRON RADIATION IN THE ACCELERATORS AND STORAGE RINGS: THE INFLUENCE ON THE BEAM DYNAMICS

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Abstract

Circular relativistic beam has a field with a nonaveraged stochastic component of the spatially localized packages of the hard part of synchrotron radiation (so-called γ -regions). The scattering of beam particles in γ -regions is considered. In storage rings with colliding beams the scattering in γ -regions of particles having velocities of opposite directions is more essential. For LEP the losses per cycle are $\sim$ 10% of those by synchrotron radiation. Besides the beam swelling the scattering also causes a widening of the spectrum of betatron oscillation frequencies.

Introduction

The electrodynamic interaction forces between curvilinearly accelerated relativistic electrons and influence of these forces on the electron bunch shape in the synchrotron and millitron have been originally investigated in 1947-1948 by I.E.Tamm [1]. Later on, an unjustified rectilinear approximation of particles trajectories was used (see, for example [2]), therefore I shall remind the results of [1]. The general expressions for twobody electrodynamic forces were obtained by approximate solution of the retardation expression in the Lienard-Vichert potentials. There was introduced a region of "small distances", where trajectories curvature is negligible and interaction forces are Coulomb-like. The azimuthal component of resulting force was calculated and the stability of the bunch with respect to the electrons azimuthal oscillations was examined.

The particles trajectories curvature have been taken into account when calculating the collective beam fields in [3-7] too. Besides electrodynamic forces the field pattern of one particle is analysed here. It is shown that a γ -dependent field region (γ -region, γ is the Lorentz-factor of the particle) is separeted out of Coulomb region which has small transverse sizes and stretches out along radius near particle's orbit. This γ -region is the space localization of the hard part of synchrotron radiation.

In this work the value of particles scattering in γ -regions is estimated. The influence of such scatterings on beam dynamics is discussed.

Field of one charge

The field of charge moving on circle with radius R has complicated structure. The field is Coulomb-like in the region with sizes R . Let us introduce the cylindrical system (R+x , Φ , z), where azimuth Φ is counted along the motion of particle, x and z are **transverse** deviations. Outside the Coulomb region a γ -region can be separated out along the line $3\Phi=-(2x/R)^{3/2}$, z=0 and has small transverse size $\sim R\gamma^{-3}$ in orbital plane and vertical size $(xR)^{1/2}\gamma^{-1}$ (here the field is $\sim 2\sqrt{2}e\,\gamma^4 R^{-3/2}x^{-1/2}$).

The total field of the beam is defined by particles density μ . In the total γ -regions intersecting case ($\mu \gg (R\,\delta_x)^{-3/2}\gamma^4$, δ_x - is the bunch's radial size) the beam may be looked upon as continuous one. This continuity condition however is not fulfilled for electron accelerators and storage rings with energy $\geqslant$ 1 GeV. Here collective beam field consists of smooth component and peaks of hard part of synchrotron radiation, which don't intersect in average. The peaks structure is defined by particles microscopic distribution of the beam, therefore in [7] this component is called stochastic.

The field smooth component, in particular, shifts the betatron oscillations frequency ν_x by the value $\Delta\nu_x \sim \Lambda\mu r_e R\,\delta_z/\nu_x\gamma$, where δ_z is the vertical size of the beam, $r_e=e^2/mc^2$, $\Lambda \sim 10$ [5,6] . It is necessery to note the work [8] , where beam self field was considered in continuous charged ring model. In such model the stochastic component is absent.

Shift of momentum

In electron beam the particles scattering in γ -region takes place statistically independently of each other, therefore it is possible to separate two particles interaction. In [7] the shift of the momentum (within the finite retarded time interval (t_1', t_2')) of the charge e_0 resting in the point $\vec{r}_0$ is calculated, caused by Lienard-Vichert field of the charge e moving by arbitrary but given trajectory. It is given by the expression:

$$\Delta\vec{p}_0 = \frac{e e_0}{c^2}\left\{\int_{t_1'}^{t_2'} dt' \cdot \frac{\vec{n}}{(t-t')^2} + \left.\frac{\vec{n}-\vec{\beta}}{(t-t')(1-\vec{n}\vec{\beta})}\right|_{t_1'}^{t_2'}\right\}, \quad (1)$$

where $t = t' + |\vec{r}_0 - \vec{r}(t')|/c$, $\vec{n} = (\vec{r}_0 - \vec{r}(t'))/|\vec{r}_0 - \vec{r}(t')|$, $\vec{\beta} c = d\vec{r}(t')/dt'$.

This expression by Lorentz transformation can be generalized for the case, when charge e_0 is moving rectilinearly: rectilinear approximation for scattering charge trajectory only is evidently acceptable because we are interested in charge e_0 intersecting with narrow γ -regions.

The first term in (1) is Coulomb-like and is independent of γ . The second term vanishes in the integral over whole trajectory, i.e. the radiation doesn't contribute in the momentum shift in first order on the field.

Integral of friction force

The particles scattering on the hard part of synchrotron radiation is to be studied by means of quantum methods. However, this is difficult since the synchrotron radiation dispersion consists of $\sim \gamma^3$ main harmonics. For preliminary evaluations we use the estimates of the friction forces due to the charge e_0 . In spite of the fact that the field of one charge isn't quasi-classical for large number of scattering perhaps the classical result will be restored [9] .

The component of the friction force $\vec{f}$ acting on the charge e_0 moving with velocity $\vec{B}c$, contributing into the integral along the trajectory $\vec{r}(t')$ is written as

$$\vec{f} = -\frac{2e^4}{3m^2c^4}\vec{B}\Gamma\left\{\left(\vec{E} + [\vec{B}\times\vec{H}]\right)^2 - \left(\vec{B}\vec{E}\right)^2\right\}, \quad (2)$$

where $\Gamma = (1-B^2)^{-1/2}$, $\vec{E}$ and $\vec{H}$ are electric and magnetic fields of the charge e . The crossing time of γ -region by charge e_0 is of order

$$\Delta t \sim \frac{4}{3}\frac{R}{c\gamma^3}\frac{1+(1-B)\cos\theta}{1-B\cos\theta}, \quad (3)$$

where θ is the angle between the directions of the motion of γ -region and the $\vec{B}c$.

Taking into account the fact that the electric field is directed along the γ -region and the magnetic field is perpendicular to the orbit plane, we find an expression for energy losses of particle during a single act of scattering:

$$\Delta\mathcal{E}_{s\gamma} \sim (64/9)(r_e^3/R^2x)\gamma^7(1-B\cos\theta)\cdot mc^2, \quad (4)$$

where x is the radial deviation between external scattered and internal scattering particles and β is set $\sim B$.

First, let us consider the scattering of particles moving in one beam. Since γ -region moves perpendicular to its direction, we find $\theta \sim \sqrt{2x/R} \gg \gamma^{-1}$ for distances x greater than interparticle distances. For beams with angular spread of order γ^{-1} we obtain therefore $\Delta\mathcal{E}_{s\gamma} \sim (64/9)(r_e^3/R^3)\gamma^7\cdot mc^2$. This is a small quantity even in comprasion with the energy change due to the Coulomb part of interaction $\Delta\mathcal{E}_c$.

If scattering takes place in γ -region of colliding beam particle, than $\Delta\mathcal{E}_{s\gamma} \sim (128/9)\cdot(r_e^3/R^2x)\gamma^7 mc^2$. For head-on collision the $\Delta\mathcal{E}_{s\gamma}/\Delta\mathcal{E}_c \sim (64/9)(r_e x/R^2)\gamma^6$ in distances as little as $x > 10^{-5}$ cm.

Influence on beam dynamics

The intersections of γ -regions occur practically instantly for any value of θ . It means that the effect of scatterings of particles in γ -regions is similar to the quantum swinging caused by self radiation.

The number of such acts per cycle is proportional to that of particles in the beam and to the radial shifts of particle from equilibrium trajectory. This causes the widening of the spectrum of frequencies of betatron oscillations.

In the storage rings with colliding electron beams the scattering in γ -regions of particles with velocities of opposite directions is more essential. The energy losses per cycle per particle are $\sim(32/8\pi)(r_e^2/R\bar{x})\gamma^3\bar{N}$ from overall losses by synchrotron radiation, where $\bar{N}$ is the number of scattering acts, $\bar{x}$ is the mean value of impact parameter. For LEP this ratio is $\sim 10\%$, if $\bar{N}$ is set to be of order of particles total number and $\bar{x}$ is ~ 1 cm. However, the influence of the such scatterings is less rough than in the effect of quantum swinging, because the number of radiated hard quanta per cycle $\ll \bar{N}$.

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References

1. Tamm I.E.-Coll.of Sci.Works,v.1,Moscow, Nauka,1975,pp.390-423.
2. Bruck H.,Accelerateurs Circulaires de Particules,Saclay,1966.
3. Arutunian S.G.-Preprint YPI-387(45)-79, Yerevan,1979.
4. Arutunian S.G.,Nagorsky G.A.-Preprint YPI-453(60)-80,Yerevan,1980.
5. Arutunian S.G.-Preprint YPI-477(20)-81, Yerevan,1981.
6. Arutunian S.G.-Proc.of XIII Int.Conf.on High En.Accel.(Aug.1986,Novosibirsk), Novosibirsk,1987,v.2,pp.182-185.
7. Arutunian S.G.-Proc.of X National Accel. Conf.(Oct.1986,Dubna),Dubna,1987,v.1, pp.456-458.
8. Talman R.-Phys.Rev.Lett.,1986,56,14,1429-1432.
9. Jackson J.D.,Classical Electrodynamics, Wiley,New York,1975.