ELECTROMAGNETIC WAVE GENERATION WITH HIGH TRANSFORMATION RATIO BY INTENSE CHARGED PARTICLE BUNCHES

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Charged particle acceleration in wake fields excited by intense relativistic bunches refers to a class of new methods allowing to have acceleration gradient above [1-3]. Clearly, such a scheme may be useful only in case it will allow to accelerate a particle up to energy higher than that of the driving bunch of particle. For thisreason , a necessity arises as to excite electromagnetic waves with high transformation ratio defined as a ratio of maximally possible gain of the test charge to initial energy of particles in the driving bunch.

The purpose of this work is to study the structure of excited electromagnetic fields versus the longitudinal shape of the driving bunch in order to obtain high transformation ratio.

Let a point-like relativistic bunch of charged particles with a velocity V_z close to the velocity of light c at a time moment t=0 enter some structure (a cavity, diaphragmatic waveguide, plasma) where it induces electromagnetic waves with electric field longitudinal component E_z . If we put Z=0 at the arrival point ,then the energy acquired (or lost) by the test charge q moving along the same trajectory with a time delay τ will be

$$V_o(\tau) = q\int_0^\infty E_z\left(z, t=\tau+\frac{z}{c}\right)dz \qquad (1)$$

The Green function formalism allows to express the loss function $V(\tau)$ for arbitrary longitudinal distribution of driving bunch $P(\tau)$ through the loss function of point-like bunch

$$V(\tau) = \int_0^\tau \rho(\tau')\,V_o(\tau-\tau')\,d\tau' \qquad (2)$$

If we assume that the energy exchange process is lasting up to resting (going out of relativism) of the driving bunch particle undergoing maximal deceleration V^-_{min} , then the transformation ratio will be $k = -V^+_{max}/V^-_{min}$, where V^+_{max} is a maximally possible energy gain of the test charge.

An expression for the loss function $V(\tau)$ can be found using for searching for the excited electromagnetic fields the eigenmode expansion method for the case of usual accelerating structure or by solving the kinetic equation on linear approximation for the case of cold isotropic plasma [4]. In both cases a general expression $V_o(\tau)$ for the ultrarelativistic point-like driving bunch takes a relatively simple form:

$$V_o(\tau) = -\eta(\tau)\sum U_n \cos\omega_n\tau\,, \qquad (3)$$

where $\eta(\tau)$=0, 1/2, 1 at $\tau<0$ (in front of the bunch), $\tau=0$, $\tau>0$ (behind the bunch), respectively; ω_n is frequency of the n-th excited mode; U_n are expansion coefficients determining a contribution of the n-th mode to the wake-wave field. In case of plasma , a single mode on plasma frequency ω_p is excited, and

$$E_z(\tau) = -2\eta(\tau)\frac{Q\,\omega_p^2}{c^2}K_o\left(\frac{\omega_p}{c}\tau\right)\cos\omega_p\tau\,, \qquad (4)$$

where Q is total charge of the driving bunch, τ is the deviation from the trajectory, $K_o(x)$ is the Bessel function of the 11 kind [5]. Note , that in front of the bunch electromagnetic fields are absent due to the causality principle. In our further consideration of single-mode approximation we shall not specially mention the plasma case.

From (3),(4) one can readily see that the basic excited mode yields transformation equal 2. This fact is known as the basic property of excited wake fields [6]. Let us show that with account of all excited modes the transformation ratio in the case of point-like driving bunch does not exceed 2 . Indeed, if we introduce a parameter $\alpha = |Q/q|$, then the energy lost by driving bunch will be $V_1 = \alpha|V_o(0)|$, while the energy acquired by the test charge will be $V_2 = V^+_{max} - |V_o(0)|/\alpha$. Then according to the energy conservation we have

$$V^+_{max} - \frac{1}{\alpha}\left|V_o(0)\right| \leq \alpha\left|V_o(0)\right| \qquad (5)$$

whence $k \leq (1+\alpha^2)/\alpha$. This relation should hold for all values of α ,since the transformation ratio does not depend on charges of the driving and accelerated bunches. The right hand side is minimal at $\alpha = 1$, so we obtain $k \leq 2$.

It can be shown [7,8] that for the symmetric driving bunch the single-mode approximation yields the transformation ratio also equal to 2 . And this is achiveved in case when maximal deceleration is experienced by average particle of the bunch. In particular, for a uniform bunch of duration T with account of the single excited mode, according to (2) we obtain:

$$\begin{aligned} V^-(\tau) &= -\frac{U_o}{T\omega_o}\sin\omega_o\tau \qquad 0<\tau<T, \\ V^+(\tau) &= -\frac{2U_o}{T\omega_o}\sin\frac{\omega_o T}{2}\cos\omega_o\left(\tau-\frac{T}{2}\right) \qquad \tau>T \end{aligned} \qquad (6)$$

Here $V^-(\tau)$ corresponds to the loss function inside the bunch, $V^+(\tau)$ - behind the bunch . Fig.1 shows dependences of acceleration rate and transformation ratio on the bunch duration at the given number of particle in it. At $T=2\pi/\omega_o$ the bunch does not excite the consider mode, since all energy emitted on this mode of the forepart of the bunch is spent for acceleration of its back part. Maximal transformation ratio is 2 being achieved at $T\omega_o = \frac{\pi}{2}(1+4n)$.

However, because of the bunch self-acceleration, of practical significance is the first peak (n=0), when the bunch duration is

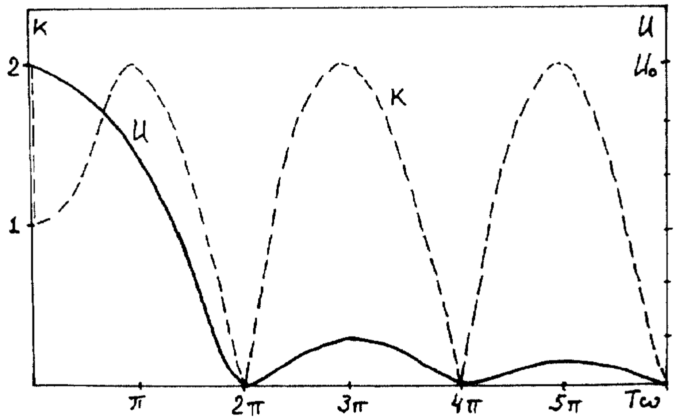

Fig.1

half as large as the period of the excited mode. In this case the acceleration rate falls off nearly twice. Although for real structure the account of all modes leads to a decrease in k (~1.5), conditions can be found at which it will be higher than 2. If frequencies of excited modes relate as $\omega_n=(2n+1)\omega_0$ and coefficients U_n coincide, then such structure will yield a transformation k=8.(1+1/3+1/5+...)/π [9].

As an asymmetric bunch we'll consider a driving bunch with linear growth of current density $P(\tau)= 2\tau/T^2$. For the loss function we obtain

$$V^-(\tau) = -\frac{2U_0}{T^2\omega_0^2}\left(1-\cos\omega_0\tau\right)$$

$$V^+(\tau) = \frac{2U_0}{T^2\omega_0^2}\left[T\omega_0\sin\omega_0(\tau-T)+\cos\omega_0\tau-\cos\omega_0(\tau-T)\right] \qquad (7)$$

Fig.2 presents dependences of acceleration rate and transformation ratio on bunch duration at a given number of particles in it.

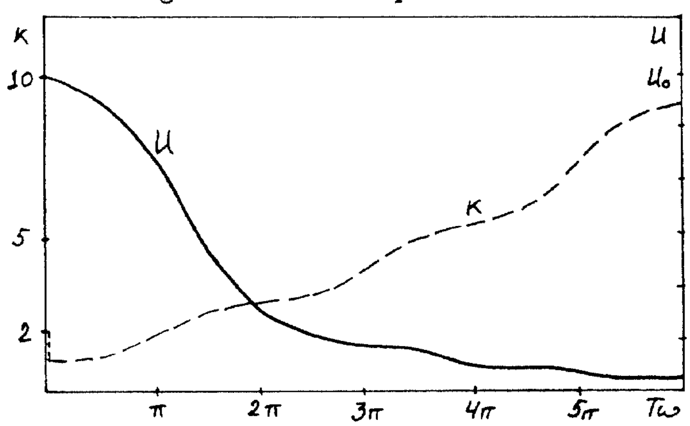

Fig.2

At $T=2\pi N/\omega_0$ (N-is integer) a transformation ratio is $k=\pi N$. Schematically, the loss functions $V(\tau)$ at N=3 are shown in Fig.3a. One can see that the transformation ratio is proportional to the number of identical minima inside the bunch (in the limit $V^-(\tau)$=const).

Thus, for single-mode structures we can formulate the following statement: electromagnetic waves with a maximal transformation ratio are generated by a driving bunch all particles of which lose the same energy.

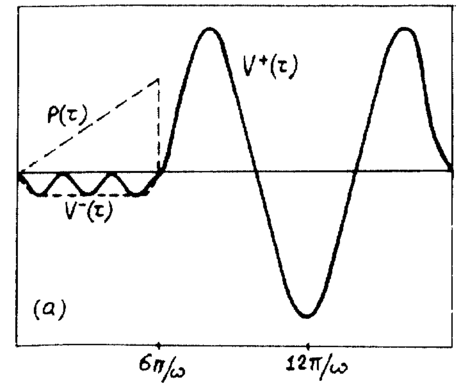

Fig.3

Indeed, let two arbitrary point-like bunch- Q_1 and Q_2 move along the same trajectory in the structure. Then the energy lost by particles of each bunch will be

$$V_1^- = U_0$$
$$V_2^- = \frac{Q_2}{Q_1}U_0 + 2U_0\cos\omega_0\tau_0 = \alpha V_1^- \qquad (8)$$

While maximal energy acquired by the test charge will be

$$V^+_{max} = 2U_0\sqrt{1+\alpha^2-2\alpha\cos\omega_0\tau_0} \qquad (9)$$

For the transformation ratio we then have

$$K = \begin{cases} 2\sqrt{1+\alpha^2-2\alpha\cos\omega_0\tau_0} & \alpha\le 1 \\ \frac{2}{\alpha}\sqrt{1+\alpha^2-2\alpha\cos\omega_0\tau_0} & \alpha\ge 1 \end{cases} \qquad (10)$$

Maximum in the right-hand side is achieved at $\alpha=1$, here $k=2(1+Q_2/Q_1)^{1/2}$. Note, that the ratio Q_2/Q_1 cannot exceed 3.

We can show that this is valid for N+1 bunches if for N bunches this condition holds (N is any integer). In this case a transformation ratio is

$$K = 2\sqrt{1+\sum_{n=2}^{N}\frac{Q_n}{Q_1}}\,, \qquad (11)$$

where Q_n is total charge of the n-th bunch.

Consider driving bunches whose particles would lose the same energy while flying through the decelerating structure.

From (2) it follows that there is no existent continuous driving bunch whose all particles would have lost the same energy. However the loss function inside such bunch can be parametrized as $V^-(\tau) = V_0(1-\exp(-\alpha\tau))$. At $\alpha\to\infty$, $V^-(\tau)$ is constant. Driving bunch distribution should then satisfy the integral equation (2). This equation can be solved by means of the Laplace transformation, so for the bunch distribution $P(\tau)$ may be find [8]

$$P(\tau) = P_0\left[(\alpha^2+\omega_0^2)e^{-\alpha\tau}+\omega_0^2(\alpha\tau-1)\right] \qquad (12)$$
$$0\le\tau\le T$$

In the limit $\alpha\to\infty$ such bunch yeilds a transformation $k=(1+(\omega_0T)^2)^{1/2}$.

For a sequence of N identical point-like driving bunches the condition of equality of particle energy losses holds if their sequence intervals τ_n satisfy the relation $\sin \omega_o \tau_n = 1/\sqrt{n}$ (n=1,2...) (Fig.4).

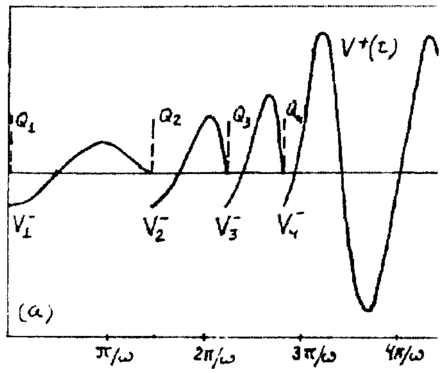

Fig.4

In this case a maximal transformation ratio $k = 2\sqrt{N}$ is achieved.

The driving bunch nonstandart distributions considered above offer certain difficulties in their production and acceleration. It is natural therefore to consider a sequence of N point-like bunches with increasing number of particles in them. Here particles of bunches lose the same energy when their recurrence period is $\tau_o = \pi/\omega_o$, and the number of particles in the n-th bunch is $N_n = N_1(2n-1)$. Schematically the loss function are shown in Fig.5

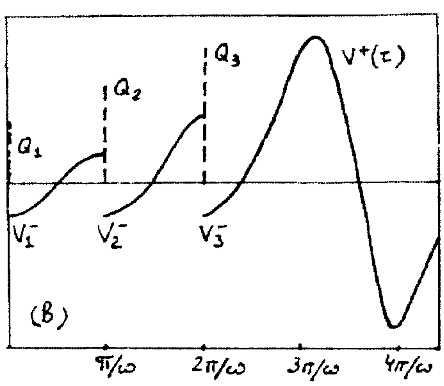

Fig.5

The transformation ratio according to (11) is $k = 2N$.

The above -obtained formulae for transformation ratios contain various parameters of bunches. Fig,6 presents dependences of the number of particles participating in wakefield generation on the transformation ratio at the given acceleration gradient.

Worsening in the parameters of piece-linear bunch at small k is caused by the influence of the bunch forepart where particles lose different energies.

One can see that most optimal is the sequence of point-like bunches. The advantage of the bunch sequence is also that the total number of particles participating in energy exchange may be considerably increased with a reasonable number of particles left in each bunch.

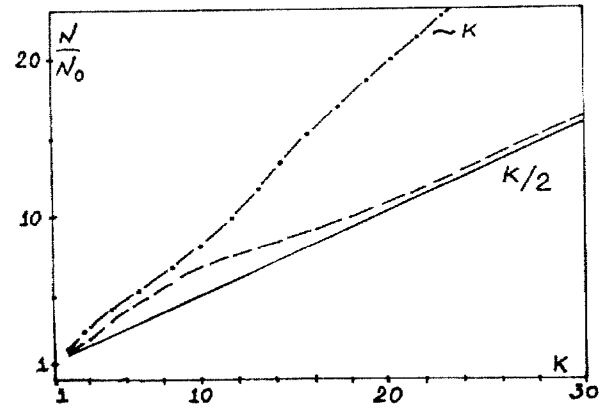

Fig.6 -.-.-.- - linear bunch,---- -piece-linear bunch,—— -sequence of point-like bunches.

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