FINITE PLASMA WAKEFIELD ACCELERATOR

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Abstract

The analysis of a guided plasma wakefield accelerator is considered. It is shown by analytical and graphic resolution that the wakewave is propagative and that its phase velocity is greater than the light velocity. An expression of the potentials describing this system is also given: it appears that although the longitudinal wake field intensity is decaying, the transform ratio is still the same, so that the acceleration gradient is the one predicted by the infinite PWFA theory. We also notice that the focalisation is enhanced, especially for small bunches.

Introduction

This paper presents an investigation of the dynamics of a plasma waveguide accelerator. This is deeply inspired from the PWFA (Plasma WakeField Accelerator), developed by Chen et al. [1], Katsouleas et al. [2], J.J. Su et al. [3]. The PWFA principle is as following: Let us pose to consider an infinite plasma, in which the ions are not supposed to contribute to the motion. The injection of a relativistic electron bunch, properly shaped, will generate, because of its interaction with the medium, a steady wave which is in fact a plasma wakewave. This wave is the structure which will extract energy from the driving bunch and transfer it to a trailing group of electrons and then accelerate them.

A disadvantage of this previous scheme is that the electromagnetic energy flux occupies volumes exceeding the region where the trailing bunch is accelerated. Whereas considering a plasma waveguide configuration, where the plasma has a finite radial extent and is surrounded by a dielectric, reduces the energetical flux area and concentrates it into the useful regions.

This paper is divided into two sections: in the first one, we will determine the dispersion relation related to the guided configuration. The wakewave, generated by the interactions between the driving bunch and the plasma, is a travelling one, and can propagate with phase velocities $v_{ph} > c$ (where c is the light velocity). In the following section, we will draw up the expression of the fields characterising the wakewave and compare them with those obtained in the infinite configuration.

Dispersion relation

So, let us pose to consider the plasma wave guide configuration described in Fig. 1. The plasma column is assumed to be longitudinally infinite, but has a finite radial extent b. The plasma is surrounded by a dielectric which is limited at r=d by a conducting wall.

We will then inject through the plasma a relativistic electron bunch, whose charge density is properly defined by $\sigma(r,\xi)=f(r).g(\xi)$; ($\xi=v_b t-z$ is the longitudinal component, v_b is the bunch velocity and is supposed to be almost equal to the light velocity c). For our convenience, we will choose the radial and the longitudinal part of the charge density as: $f(r)=D$ =const., $0\leq r<a$; elsewhere, $f(r)=0$.

$g(\xi)=\xi$, $\quad -L\leq \xi\leq 0$; elswhere, $g(\xi)=0$.

where a is the radius of the bunch, L is its length.

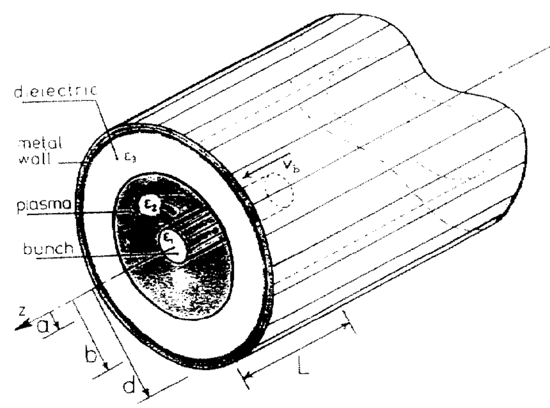

Fig. 1. Configuration of the plasma waveguide accelerator.

We will part this system into three sections:

-the region (1) is inside the bunch : $r<a$.

-the region (2) is the plasma area : $a\leq r\leq b$.

-the region (3) is inside the dielectric: $b\leq r\leq d$.

We will introduce the double Fourier Transform in z,t as: $f(k,\omega)=\iint e^{-i(kz-\omega t)}f(z,t)dzdt$; defining k and ω to be the transform variables.

By considering the equivalent dielectrics [4] (containing no free charge) and by combining the Maxwell, Poisson, motion and continuity equations, we obtain the dielectric constants for the three sections as following:

$\varepsilon_1=\varepsilon_0(1-\omega_p^2/\omega^2-\omega_{pb}^2/(\omega-v_b t)^2)$.

$\varepsilon_2=\varepsilon_0(1-\omega_p^2/\omega^2)$ and $\varepsilon_3=\varepsilon_0$

where $\omega_p^2=n_0e^2/m\varepsilon_0$= plasma frequency;

and $\omega_{pb}^2=Ne^2/m\varepsilon_0 .f(r)g(\omega,k)$ = beam plasma frequency .

The mixing of the Maxwell equations, adding the double Fourier Transform, leads to the well known propagative equation for the longitudinal components:

$[\nabla_\perp^2-(k^2-\omega^2/c^2\,\varepsilon_i)]E_{zi}=0$,where i=1,2,3. (1)

$\nabla_\perp^2$ is the transverse cylindrical laplacian $=1/r\partial_r(r\partial_r)$. The three mediums are assumed to be uniform and isotropic, so the axial propagation constant k must be the same into the three sections in order to satisfy the boundary condition.

The three solutions of eq. (1) must be matched at the boundaries. The boundary conditions are:

at r=a : $E_{1z}(r=a) = E_{2z}(r=a)$ (2)

$\varepsilon_1(r=a).E_{1r}(r=a) = \varepsilon_2(r=a).E_{2r}(r=a)$ (3)

at r=b: $E_{2z}(r=b) = E_{3z}(r=b)$ (4)

$\varepsilon_2(r=b).E_{2r}(r=b) = \varepsilon_3(r=b).E_{3r}(r=b)$ (5)

at r=d: $E_{3z}(r=d) = 0$ (6)

at r=0: $E_{1z}(r=0)$ is finite. (7)

Let's try to establish the dispersion relation by setting up the field components:

- for $0 \leq r < a$: because of eqs. (1), (7), we have

$E_{1z} = A \cdot J_0(\beta_1 r)$, where J_0 is the Bessel Function of zero order and $\beta_1^2 = \omega^2\varepsilon_1/c^2 - k^2 > 0$.

- for $a \leq r < b$: eq. (1) leads to: $E_{2z} = F \cdot I_0(\beta_2 r) + D \cdot K_0(\beta_2 r)$, where I_0 , K_0 are the modified Bessel Functions of zero order, and $\beta_2^2 = k^2 - \omega^2\varepsilon_2/c^2 > 0$.

- for $b \leq r \leq d$: eqs. (1) and (6) leads to

$E_{3z} = G[\,I_0(\beta_3 r) \cdot K_0(\beta_3 d) - I_0(\beta_3 d) \cdot K_0(\beta_3 r)\,]$, where $\beta_3 = k^2 - \omega^2\varepsilon_3/c^2 > 0$ and A,F,D,G constants to be determined.

The fields satisfying the conditions (2), (3), (4), (5), we can eliminate the unknown constants and establish the dispersion relation:

$-\beta_1(a).J_0(\beta_1 a)/J_1(\beta_1 a) = [(\beta_2/\beta_3)L.M + P.Q] / [L.S/\beta_3 + P.T/\beta_2]$ (8)

$\beta_3^2 - \beta_2^2 = \omega^2/c^2[\varepsilon_2 - \varepsilon_3]$ (9)

$\beta_2^2 - \beta_1^2 = \omega^2/c^2[\varepsilon_1 - \varepsilon_2]$ (10)

where $L = I_1(\beta_3 b)K_0(\beta_3 d) + I_0(\beta_3 d)K_1(\beta_3 b)$,

$M = I_0(\beta_2 a)K_0(\beta_2 b) - I_0(\beta_2 b)K_0(\beta_2 a)$,

$P = I_0(\beta_3 b)K_0(\beta_3 d) - I_0(\beta_3 d)K_0(\beta_3 b)$,

$Q = I_0(\beta_2 a)K_1(\beta_2 b) + K_0(\beta_2 a)I_1(\beta_2 b)$,

$S = I_1(\beta_2 a)K_0(\beta_2 b) + K_1(\beta_2 a)I_0(\beta_2 b)$,

$T = I_1(\beta_2 a)K_1(\beta_2 b) - K_1(\beta_2 a)I_1(\beta_2 b)$.

The two parts of eq. (8) are plotted in Fig. 2. The graphic resolution leads to: $\beta_1 a = \rho_n$; where $\rho_n = \pi/2 + n\pi$. At the first order (for n=0), $\beta_1 a = \pi/2$. The dispersion relation is propagative and is expressed as following:

$\omega^2 = \omega_p^2 + k^2c^2 + (\pi c/2a)^2$. (11)

Hence the phase velocity v_{ph} of the wave is greater than c. The cut off is $[\omega_p^2 + (\rho_n/a)^2]^{1/2}$; there won't be any propagation below this value.

Wakefields

The above dispersion relation will be helpful for the determination of the wakefield $\mathbf{W} = \mathbf{E} + \mathbf{v_b} \wedge \mathbf{B}$ generated by the driving bunch. This wakefield could be simply expressed by considering the perturbed vector and scalar potentials $\mathbf{A}$ and Φ, related by $\mathbf{E} = -\nabla\Phi - 1/c(\partial\mathbf{A}/\partial t)$. According to Ref. [1], [2], [3], we derive: $W_{//} = \partial_\xi(A_z - \Phi)$ and $W_\perp = \partial_r(A_z - \Phi)$, where $\mathbf{W} = \mathbf{W}_{//} + \mathbf{W}_\perp$; (the longitudinal component of the field $W_{//}$ contributes to the accelerating process, while the transverse part of the wakefield $W_\perp$ characterises the focalisation). One sets up the following equations:

$(\nabla_\perp^2 - k_p^2)(A_{iz} - \Phi_i) = -4\pi en(r,\xi)$, where i=1, 2. (12)

$\nabla_\perp^2(A_{3z} - \Phi_3) = 0$, inside the dielectric. (13)

and $\partial^2_\xi{}^2 n(r,\xi) + k_p^2 n(r,\xi) = k_p^2\sigma(r,\xi)$ (14)

where n is the plasma density perturbation and $k_p = \omega_p/c$.

Equation (14) is easily solved : $n^{+(-)}(r,\xi) = (\rho/e)f(r)G^{+(-)}(\xi)$, where +(-) means respectively 'outside' (inside') the bunch.

$G^-(\xi) = -(\rho e/k_p)(k_p\xi - \sin k_p\xi)$ for $-L<\xi<0$ (inside the bunch).

$G^+(\xi) = (2\rho\pi e/k_p)\cos k_p\xi$ for $\xi<-L$ (outside the bunch).

We also have to consider the boundaries conditions for the resolution of eqs. (12), (13):

at r=a : $(A_{1z} - \Phi_1)_{r=a} = (A_{2z} - \Phi_2)_{r=a}$.

$\varepsilon_1(a) \times \partial_r(A_{1z} - \Phi_1)_{r=a} = \varepsilon_2(a) \times \partial_r(A_{2z} - \Phi_2)_{r=a}$.

at r=b : $(A_{2z} - \Phi_2)_{r=b} = (A_{3z} - \Phi_3)_{r=b}$.

$\varepsilon_2(b) \times \partial_r(A_{2z} - \Phi_2)_{r=b} = \varepsilon_3(b) \times \partial_r(A_{3z} - \Phi_3)_{r=b}$.

Therefore the potentials are expressed as following:

$(A_{1z} - \Phi_1) = D^*.G(\xi)\{-I_0(k_p r)[K_1(k_p a) + T.I_1(k_p a)] + 1/k_p a\}$, $0 \leq r < a$

$(A_{2z} - \Phi_2) = D^*.G(\xi)\, I_1(k_p a)\,[K_0(k_p r) - T.I_0(k_p r)]$, $a \leq r < b$

$(A_{3z} - \Phi_3) = D^*.V.G(\xi)\, I_1(k_p a)\, Ln(r/d)$, $b \leq r < d$

where $V = [\varepsilon_2 Ln(b/d)k_p b\, I_1(k_p b) - I_0(k_p b)]^{-1}$,

$T = -\,[K_0(k_p b) + \varepsilon_2 k_p b\, Ln(b/d)\, K_1(k_p b)]/V$, $D^* = 4\pi.eDa/k_p$.

The expressions of the longitudinal and transverse wakefield components are straightforwardly established from the above potentials. For large 'b' ($k_p b >> 1$), we obtain the potentials corresponding to infinite plasma case. These fields are plotted Fig. 3,4,5 (full line for the bounded plasma , broken line for the infinite plasma).

Conclusion

The main difference between the infinite and the guided PWFA is that the first is a standing wave accelerator while the other is propagative. By comparing these two cases, it appears that the intensity of the transverse wakefield of the bounded configuration is somewhat enhanced, especially outside the bunch (for radii r>a). For small bunches, the increase of the focalisation is perceptible inside and outside the beam as well. The fact the electromagnetic energy flux is concentrated in a limited volume might explain this enhancement. The intensity of the longitudinal field for the guided case is decaying compared with the infinite one. Anyhow, the transform ratio R, characterising the ability of the wave to transfer an energy amount from the diving bunch to the trailing one, doesn't change whatever the configuration is. So the previous acceleration expectations are still on. Anyhow, further investigations have to be pursued.

Acknowledgment

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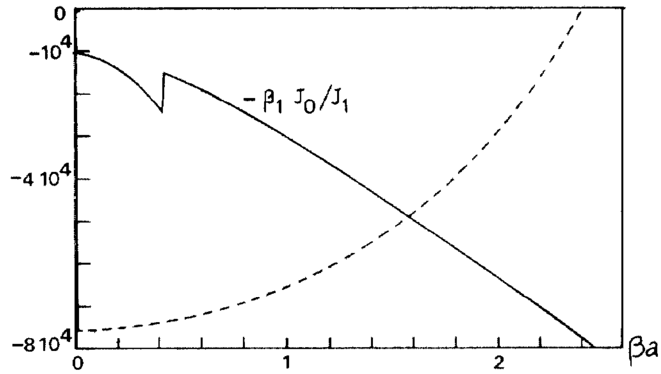

Fig.2. Graphic resolution of eq.(8): left(right) part vs. $\beta_1 a$ ($\beta_3 a$); left part :full line; right part: broken line. Intersection point=$\pi/2$.

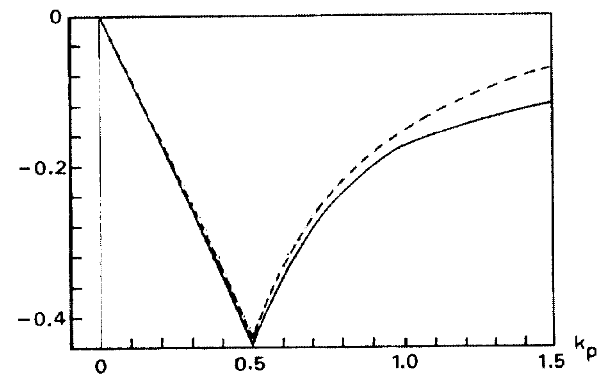

Fig.3.Radial part of the normalised transverse fields versus $k_p r$ ($a \leq r \leq c$). k_p=1.88 $10^4 m^{-1}$; γ_{ph}=1.34c; $k_p a$=0.5; $k_p b$=0.8; $k_p d$=1.5.

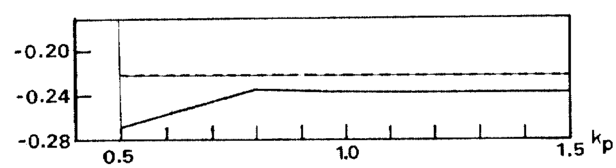

Fig.4. Radial part of the normalised transverse fields versus $k_p b$ ($a \leq b \leq c$). k_p=1.88 $10^4 m^{-1}$; γ_{ph}=1.34c; $k_p a$=0.5; $k_p r$=.0.8; $k_p d$=1.5.

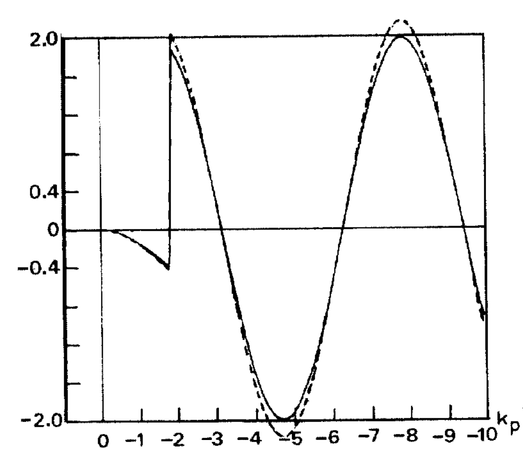

Fig 5. Normalised longitudinal fields versus $k_p \xi$. k_p=1.88. $10^4 m^{-1}$; γ_{ph}=1.34c; $k_p L$=-1.88; $k_p r$=16.9 10^{-3} ; $k_p a$=0.5; $k_p b$=0.8; $k_p d$=1.5 .