PERSPECTIVES OF ULTRACOLD AND CRYSTALLINE ION BEAMS COOLED BY ELECTRONS

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For storage rings employing electron cooling there is increasing interest in ultracold or even crystalline beams, for which the mutual Coulomb repulsion between neighbouring ions plays a dominant role. We present theoretical results for the maximum cooling rates and the interplay of cooling with various heating mechanisms, like intrabeam scattering and collective instabilities. Conditions, under which ultracold or even "crystalline" beams can be expected, and some consequences on the Schottky signals are discussed.

Introduction

It has been shown both experimentally and theoretically that electron cooling becomes the better the colder the ion beam is[1,2]. It is therefore of interest to inquire in the ultimate phase space densities that can be achieved with electron cooling. We ignore the problems occuring during the initial stage of cooling, where the cooling rate is small, and discuss limitations that are expected for an equilibrium situation with the maximum theoretical cooling rate.
A major limitation is intrabeam scattering, whereas the longitudinal microwave instability can be damped, if the intensity remains below a certain limit. Results from the Novosibirsk NAP storage ring[4] have suggested that for very low intensity an "ultra-cold" beam exists, where intrabeam scattering is suppressed and ordering effects among individual protons play a significant role. It has been speculated that under such extreme conditions the electron cooled ion beam might undergoe even a phase transition to a crystalline state[5,6], similar to the recently published laser cooled ions in Paul traps[7,8]. Ordering effects in the beam are conveniently described by the coupling parameter Γ, which gives the ratio of the Coulomb energy between neighbouring ions (at distance a) to the average thermal energy

$$\Gamma = \frac{e^2 Z^2/a}{kT} \qquad (1)$$

For ordering effects to become noticeable one requires $\Gamma \gtrsim 1$, whereas crystallization is expected for $\Gamma \approx 150$. Under normal conditions in a storage ring Γ is of the order of 10^{-4} or below, mainly due to the relatively large thermal motion (i.e. $\Delta p/p$ and emittance).
The advantage of high-Z heavy ions for achieving large Γ seems obvious from Equ.(1)[5]. It is, however, necessary to examine more carefully the increase of a with Z and the heating effect of high-Z on the cooling electrons, which in turn has an effect on the minimum ion temperature.
We apply our results to the following reference case: U^{92+} at an energy of 50 MeV/u and storage ring parameters comparable with the ESR ($2R\pi \approx 10^2$m; $Q \approx 2.3$).

Electron Cooling Rates for Advanced Cooling Stage

Fast cooling[3] benefits from the fact that the very low parallel (to the magnetic field) temperature of the electron beam $kT_{\parallel e}$ determines the cooling rate rather than the relatively high transverse temperature $kT_{\perp e}$. If one assumes a strictly flattened electron distribution ($kT_{\parallel e} = 0$) the cooling rate increases with decreasing ion temperature. For a proper calculation of the cooling rate in an advanced stage of cooling it is then necessary to take into account that the velocity spread of the ions can be much smaller than that of the cooling electrons parallel to the magnetic field. This "slow ion" limit is the subject of the present study. It requires an extension of the plasma dielectric approach to the cooling force[9] by assuming a realistic non-zero parallel electron temperature $kT_{\parallel e}$. This "slow ion" limit is given by

$$\frac{\delta v_{\parallel i}}{\delta v_{\parallel e}} \equiv \frac{\Delta p}{p} \left(\frac{\beta^2 m_e c^2}{kT_{\parallel e}}\right)^{1/2} \lesssim 1 \qquad (2)$$

$$\frac{\delta v_{\perp i}}{\delta v_{\parallel e}} \equiv \left(\frac{\varepsilon}{\beta(m) kT_{\parallel e}} \beta^2 \gamma^2 m_e c^2\right)^{1/2} \lesssim 1 \qquad (3)$$

with β(m) the machine β-function at the cooler. Both the transverse and parallel cooling times have been found to saturate at values, which are approximately described by the expression[10]

$$\tau_{\parallel,\perp} \approx 3\,(kT_{\parallel e}/10^{-3}\text{eV})^{3/2}\,(n_e/10^8\text{cm}^{-3})^{-1}\,\gamma^2 A/Z^2 \quad [\text{msec}] \qquad (4)$$

where n_e is the electron density, and the cooler length has been assumed to fill 2 % of the ring circumference. In order to evaluate τ we need to know $kT_{\parallel e}$. A typical design goal is $kT_{\parallel e} \approx 10^{-4}$ eV; for $n_e \approx 10^8\text{cm}^{-3}$ and large Z this leads to an inconsistency in the correlations between ions and cooling electrons. In Ref. 10 it has been suggested to adopt an effective $kT_{\parallel e}$ in the presence of an ion beam, which is given by

$$kT_{\parallel e} \geq Z^{2/3}\, 10^{-4} (n_e/10^8\ \text{cm}^{-3})^{1/3} \qquad (5)$$

The physics behind Equ.(5) is that the collective binding energy of the Debye-screening cloud[11] is thermalized as an effective temperature. For protons and $n_e = 10^8\text{cm}^{-3}$ this yields just the design $kT_{\parallel e}$, whereas for U^{92+} there is an effective increase to $2\cdot10^{-3}$ eV. There results a cooling time of $\tau_{\parallel,\perp} \approx 0.3$ msec ($\gamma \approx 1$), if $\delta v_i/\delta v_{\parallel e} = 0.5$. For the U^{92+} reference case this is equivalent to $\Delta p/p = 10^{-4}$ and $\varepsilon = 10^{-6}$ m-rad. For still slower ions we ignore the weak gradient and take 0.3 msec as cooling time limit. A particular feature of the "slow ion" limit is that the transverse cooling rate is as fast as the longitudinal and practically independent of the orientation of $\delta\vec{v}_i$ with respect to the magnetic field (contrary to fast ions, where also transverse "antifriction" occurs for certain directions).
Combining the Z-dependence in Equ.(5) with Equ.(4) we find the following (weak) scaling of the cooling time:

$$\tau_{\parallel,\perp} \approx 0.23\ A/Z\ [\text{msec}] \qquad (6)$$

rather than $\sim A/Z^2$, if Equ.(5) is ignored.

Intrabeam Scattering Limit to Cooling

We are interested in the equilibrium distribution as a result of cooling and various heating effects. The most obvious sources of heating are intrabeam-scattering and the longitudinal microwave instability [12]. With decreasing $\Delta p/p$ the Keil-Schnell-limit is reached only, if the transverse emittance ε is not too small, otherwise the longitudinal heating by intra-beam-scattering stops the cooling. We assume that ε and $\Delta p/p$ are small enough in order to apply the "slow ion" limiting expression for the cooling time according to Equ.(4). For each value of $\Delta p/p$, ε we can calculate N by keeping in mind that $\tau_{IBS} \sim N^{-1}$ and requesting $\tau_{IBS} \geq \tau_{cooling}$ for all directions.

Results for the ESR lattice and the U^{92+} reference case are shown in Fig. 1, which indicates that an emittance exceeding 10^{-7} m-rad allows to go below the Keil-Schnell-limit. The latter has been calculated for

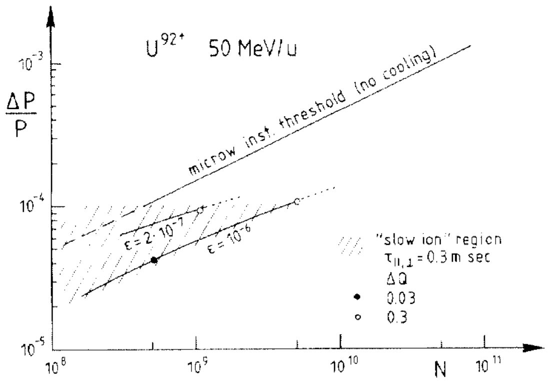

Fig. 1: "Slow ion" region with maximum cooling rate balanced against intrabeam scattering rate.

a Gaussian momentum distribution, which has a much more favourable stability limit than a parabolic one. We note, however, that the longitudinal microwave instability is not of real concern, if we consider the possibility of damping it by the friction effect of the cooling force. One finds that a cooling rate of approximately double the microwave growth rate is sufficient for this frictional stabilization[13]. We write the microwave growth rate in the convenient form

$$\tau_m^{-1} = \tau_o^{-1}\, n\, \frac{\mathrm{Re}Z}{\mathrm{Im}Z}\, \frac{\Delta p}{p} \left(\frac{N}{N_{th}}\right)^{1/2} \tag{7}$$

with τ_o the revolution time, n the harmonic and N_{th} the threshold intensity without cooling. With the cooling rate

$$\tau_{\parallel}^{-1} \gtrsim 2\,\tau_m^{-1} \tag{8}$$

we can readily calculate, up to which N there is stability. Assuming (for 50 MeV/u) ReZ/n = 10 Ω, ImZ/n = 3 kΩ and that the maximum growth rate occurs at n ≈ 150 (cutt-off mode number) we calculate $(\Delta p/p)/N_{th}^{1/2}$ from the Keil-Schnell criterion. We thus obtain, from Equ.(7) and (8) and τ = 0.3 msec, that frictional stabilization occurs for

$$N \lesssim 10^{12} \tag{9}$$

covering all intensities of practical interest. We expect that the much slower transverse resistive instability is likewise frictionally damped, hence we only deal with the limit posed by intrabeam scattering.

Ultracold "Linear Chain" Beam

Experiments at the Novosibirsk NAP ring have suggested that intrabeam scattering is suppressed, if the number of particles in the beam is low enough.

A critical intensity can be defined by requesting that in the absence of thermal motion the ions lie on the equilibrium orbit with regular spacing[6] ("linear chain"). This strictly linear arrangement is stable only, if the Coulomb repulsion for a small lateral displacement is compensated by the lattice restoring force. This defines a minimum ion spacing

$$L \geq L_o \equiv \left[(4Z^2R^2r_p)/(A\beta^2\gamma^3Q_o^{\,2})\right]^{1/3} \tag{10}$$

with r_p the classical proton radius, R the machine radius and Q_o the machine tune (in the absence of space charge). For our reference case this yields L_o = 35 µm, or

$$N \leq N_o = 3\times 10^{6} \tag{11}$$

as total number of particles. In such a linear chain ions no longer pass each other, and intrabeam scattering has no meaning. A condition for this to exist is that the thermal energy of ions is much lower than the mutual Coulomb repulsion, i.e. Γ >> 1. With the ion parallel temperature given by

$$kT_{\parallel i} = E_o\left[\frac{eV}{u}\right] A \left(\frac{\Delta p}{p}\right)^2 \quad [eV] \tag{12}$$

and the coupling parameter expressed in the form

$$\Gamma \cong \frac{Z^2/A}{3\,kT(eV)} \cdot \frac{1}{a(\mu m)} \tag{13}$$

where a is the ion spacing, we readily find for our

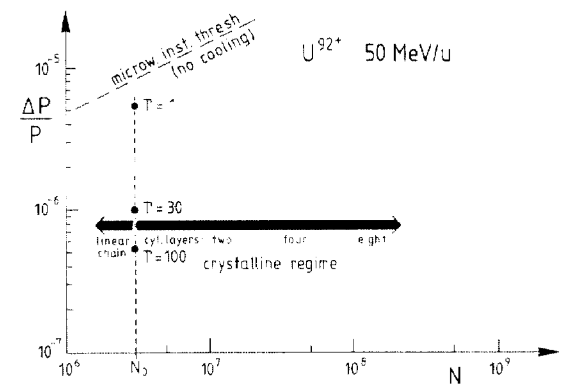

Fig. 2: Critical number of particles N_o at transition from "linear chain" to "crystalline" regime indicating different coupling parameters.

reference case and a = L_o that Γ > 1 is equivalent to $kT_{\parallel i} \geq 1/3$ eV, or $\Delta p/p < 5\cdot 10^{-6}$ (see Fig.2). The parallel equilibrium temperature of the ions with the cooling electrons has been calculated by means of the Fokker-Planck equation in the limit of weak coupling[9]:

$$T_{\parallel i} = \frac{\pi}{4}\left(T_{\parallel e}\, T_{\perp e}\right)^{1/2} \tag{14}$$

We adopt this formula for further estimates also in the strong coupling case, since we are not aware of a consistent theory applicable to it.

Employing Equ. (5) and $kT_{\perp e}$ = 0.1 eV we find $kT_{\parallel i} = 10^{-2}$ eV and Γ ≈ 30. Using the Z^2/A dependence of Equ.'s (5), (10) and (13) we obtain (for the same kinetic energy)

$$\Gamma \sim Z/A^{2/3} \tag{15}$$

This indicates only a 2.5 times larger Γ for U^{92+} as compared with protons.

We note that ignoring Equ.(5) and using $kT_{\parallel e} = 10^{-4}$ eV would raise the Γ to 135 for our reference case.

Crystalline Beams

By adding particles above the limit N_o of Equ.(9), the linear chain must break up into a helical arrangement[6]. A further increase of N saturates the particles on a cylindrical shell, until a new helix is made up on axis, etc. Computer simulation has shown that such a regular structure occurs for Γ ≳ 150.

The average distance between neighbouring ions follows from balancing the externally applied focusing force against the defocusing force of a uniformly filled cylinder. In this cold-beam limit there are no betatron oscillations, hence the machine tune Q_o is depressed to nearly zero due to space charge. This force balance leads to an average density n_i of ions, which we convert into an average distance a by using $n_i = (4/3\ a^3\pi)^{-1}$. Comparison with Equ. (8) readily shows that $a = (3/8)^{1/3}\ L_o \approx 0.72\ L_o$ which yields $a \approx 25\ \mu m$ and $n \approx 1.5\ 10^7\ cm^{-3}$ for our reference case. Adopting $kT_i = 10^{-2}$ eV as in the previous section, we find a coupling parameter of $\Gamma \approx 50$, which is only a factor of 3 below the value, at which crystallization is expected theoretically to occur.

These estimates clearly show that the heating of electrons by ions depresses quite sensitively the Γ in the equilibrium case. Taking the initial $kT_{\parallel e} = 10^{-4}$ eV, we would obtain $\Gamma \approx 220$ instead, hence a consistent theory of the equilibrium temperature at large Γ in the presence of a magnetic field is quite necessary.

Lattice Considerations

Assuming that the cooling leeds to $\Gamma >> 1$, which requires that $Q \to 0$, the next question is to investigate the stability of such an ordered beam with respect to the storage ring focusing lattice. We have found that the (zero space charge) phase advance σ_o per super-period of the lattice should not be too large, if a matched solution is to exist. We have not succeeded to find matched envelope solutions, if σ (including space charge) was to cross the stop-bands at multiples of 180° during the transition $Q \to 0$. This means that crossing of Q = 3 rsp. 2 rsp. 1 and 2 in a lattice with 6 rsp. 4 rsp. 2 super-periods could not occur.

We have then performed computer simulations with binary Coulomb-forces between pairs of ions (molecular dynamics calculations) to check on the stability. In Fig.3 we show the result for a fictitious AG lattice with Q_o = 1 and 6 super-periods neglecting bending magnets. The number of particles was adjusted in such a way as to obtain one cylindrical layer around a helical structure on axis with Γ = 120 at start. There is a modest heating of the order of 10 % per revolution. In a weak focusing lattice this heating is practically absent[14].

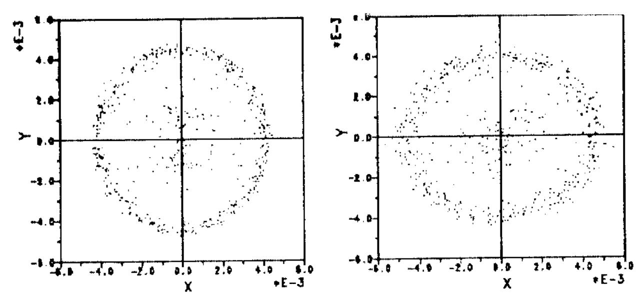

Fig. 3: Computer simulation of crystalline structure at start (left) and after 10 revolutions (right) with σ_o = 60° AG focusing (scale in cm).

A completely different picture arose for Q_o = 2.5, i.e. σ_o = 150°. There was a large emittance growth factor by over two orders of magnitude for 10 revolutions. Our interpretation of this strong coherent heating effect is the envelope instability, which has been studied extensively in connection with high-current beam transport[15]. Calculation of KV-envelopes for this case indeed indicates strong instability for $1.125 > Q > 0$, except for a small stable gap around $Q \approx 0.6$. Hence, in such a system a beam with Γ comparable with or larger than unity cannot exist. The envelope instability disappears, if σ_o is below 90°.

Schottky Signal

We have calculated the Schottky signal spectrum for fixed N and $\Delta p/p$ decreasing towards the microwave instability threshold of a Gaussian momentum distribution. Near the threshold the signal power is concentrated in two sharp peaks at the fast und slow plasma wave frequencies, with decreasing total power per Schottky band. Further studies are necessary to exploit, how this suppressed signal can be used to diagnose the momentum distribution.

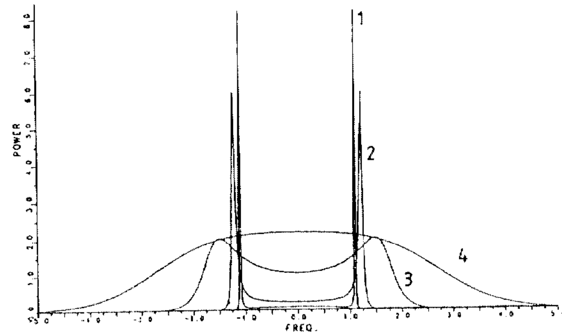

Fig. 4: Schottky spectrum of Gaussian distribution with increasing $\Delta p/p$: (1) near microwave stability threshold $(\Delta p/p)_{th}$, (2) $\sqrt{2}$times, (3) $\sqrt{8}$ times and (4) $\sqrt{40}$ times threshold.

Conclusion

We have determined cooled beam equilibria in the "slow ion" region, for which maximum cooling rates have been given. For U^{92+} and 50 MeV/u, as an example, one obtains $5 \cdot 10^8$ particles with $\Delta p/p = 3 \cdot 10^{-5}$ and $\varepsilon \approx 10^{-6}$m-rad.

Ordered (ultra cold) beams have been found consistent with theoretical cooling limits for values of Γ of the order of 10^2, with details depending on the heating of electrons. For $N > 3 \cdot 10^6$ a 1d "linear chain" ordering is expected to change into 3d crystalline ordering. This is, however, stable only in rings with sufficiently many superperiods. Bending effects require separate study for the 3d crystalline case.

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