THE EFFECT OF HEAD-ON BEAM-BEAM COMPENSATION ON THE STOCHASTIC BOUNDARIES AND PARTICLE DIFFUSION IN RHIC*

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Abstract

To compensate the effects from the head-on beam-beam interactions in the polarized proton operation in the Relativistic Heavy Ion Collider (RHIC), an electron lens (e-lens) is proposed to collide head-on with the proton beam. We used an extended version of SixTrack for multiparticle beam-beam simulation in order to study the effect of the e-lens on the stochastic boundary and also on diffusion. The stochastic boundary was analyzed using Lyapunov exponents and the diffusion was characterized as the increase in the rms spread of the action. For both studies the simulations were performed with and without the e-lens and with full and partial compensation. Using the simulated values of the diffusion an attempt to calculate the emittance growth rate is presented.

INTRODUCTION

To maintain the beam lifetime and polarization in the proton (pp) run in the Relativistic Heavy Ion Collider (RHIC) the current working points for the proton beams are (28.685, 29.695) and (28.695, 29.685) for the two RHIC rings, which are constrained between $2/3$ and $7/10$ betatron resonances lines. It has been shown by both experiments and simulations that when the fractional betatron tune is close to $2/3$ the beam lifetime is strongly affected and when the vertical tune is close to $7/10$, both the luminosity lifetime and the proton polarization are reduced. In the 2008 RHIC pp run, the bunch intensity had reached 1.7×10^{11} protons in the Blue ring, however to increase the bunch intensity beyond 2.0×10^{11} there will be not enough tune space between $2/3$ and $7/10$ resonances to hold such large tune shift and tune spread generated by the head-on beam-beam interactions.

One solution is to adopt head-on beam-beam compensation [1, 2] where the proton beam collides head-on with a low energy electron beam. In this article, we report the results from the study of stability of a single proton in the presence of head-on beam-beam compensation in RHIC. We will first introduce the parameters of the proton and electron beams and the lattice for this study. Then, we calculate and compare the Lyapunov exponent and the action diffusion and compare the results between the cases with and without head-on beam-beam compensation. With the diffusion results we estimate the emittance growth rate and the lifetime of the proton beam under different bunch intensities and e-lens configurations.

BEAM PARAMETERS

For the RHIC pp run, the two proton beams collide at IP6 and IP8. In the current design, the RHIC e-lenses are close to the crossing point IP10 however in the following simulations, for simplicity, we assume the e-lenses are exactly located at IP10 [2].

The proton beam has an energy of 250 GeV and transverse rms emittance of 2.5 mm mrad. The linear chromaticities are corrected to $Q'_{x,y} = +1$ and the multipole magnetic field errors in the triplet quadrupoles and separation dipole magnets in the IRs are included in the lattice. The beta function at IP6 and IP8 is 0.5 m and at the e-lens location (IP10) is 10 m. The bunch intensity is varied between $N_p = 1.3 \times 10^{11}$, which was the intensity during the 2006 p-p run, and $N_p = 3.0 \times 10^{11}$ protons/bunch.

For the best head-on beam-beam compensation we assume that the electron beam have the same transverse Gaussian profile as the proton beam at IP10 and, for the full head-on beam-beam compensation the electron particle density is twice of that of the proton bunch intensity, while for the half head-on beam-beam compensation the electron particle intensity is the same as the proton intensity per bunch.

SIMULATION RESULTS

Stochastic Boundaries

In order to support the emittance growth rate calculation we investigated the stochastic boundary of the system. We used two particles with a small difference in the initial position in the 6D phase space and track both for 10^6 turns calculating the Lyapunov exponent at every 500 turns, that measures how fast these particles are diverging from each other. In Figure 1 are the results of the tracking for one of the working points (above the diagonal) studied for on momentum particles, the case of off momentum particles and the other working point (below the diagonal) have no particular difference. Particles labeled as stable in the plots means that up to 10^6 turns their trajectories appear to be regular. The particles represented by red dots have chaotic trajectories and we observe that almost all particles, including the ones in the core, are chaotic. The fact that there is almost no regular motion indicates that the core particles experience diffusion and that emittance growth should be observed in simulations. The use of the head-on compensation does not result in an improvement neither intensify the chaotic behavior of the particles.

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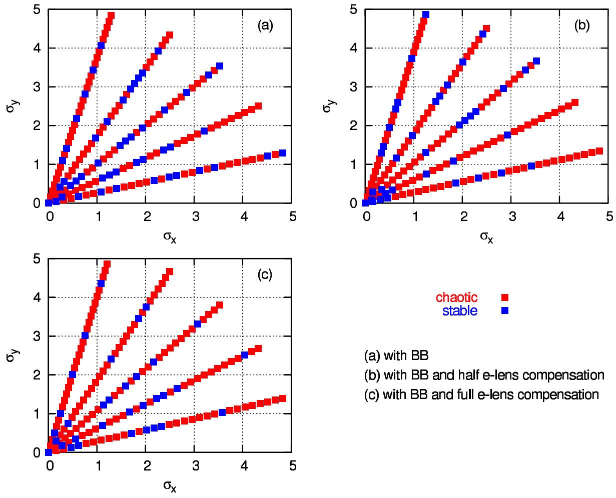

Figure 1: Chaotic behavior of the particles for the working point above the diagonal (0.685,0.695). All three plots show that almost all particles, up to 5 σ, are chaotic.

Action Diffusion

We used SixTrack to calculate the trajectories of 64 particles with the same initial action for 5×10^4 turns and for each turn we calculate the rms spread (σ_J) of the action for all the particles. We then average the rms action values over 500 turns to eliminate the short-term fluctuations due to phase-space orbit deviations from the linear model that can be caused by weak resonances [5]. The diffusion coefficient is then defined as

$$D(J) = lim_{N\to\infty} \frac{\langle J(N) - J(0)\rangle^2}{N} = lim_{N\to\infty} \frac{\sigma_J^2(N)}{N} \tag{1}$$

To calculate its values we fit a line ($\sigma_J^2(N) = a + bN$) over the rms action values for 10 points spaced by 5000 turns, so that we have $D(J) = b$. Once the diffusion points are calculated we fit the result using the expression $D(J) = AJ + BJ^2 e^{CJ}$. This fitting function behaves like a parabola for the inner-most particles in the bunch which is in agreement with the calculation for diffusion due to beam-beam effects [6] and for the tails it has a more linear behavior to account for diffusion due to the lattice non-linearities. In Figures 2 and 3 there are all the fittings of the diffusion coefficient for different bunch intensities and working points.

From Figures 2 and 3 we observe that, even though the compensation improves the diffusion at the core it often increases the diffusion for particles beyond 4σ.

Emittance Growth Rate and Lifetime

Emittance growth due to the non-linearities of the beam-beam interaction is a major problem in hadron colliders. Random fluctuations of the tune, closed orbit and other parameters are also some of the possible effects that can cause to emittance growth especially for tunes close to strong resonance lines. Assuming that the diffusion in action is a

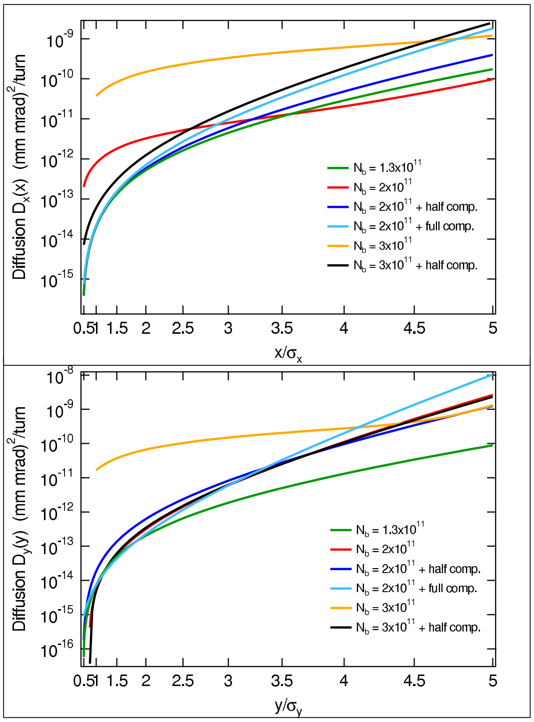

Figure 2: Fitting of the diffusion simulation results for the working point above the diagonal.

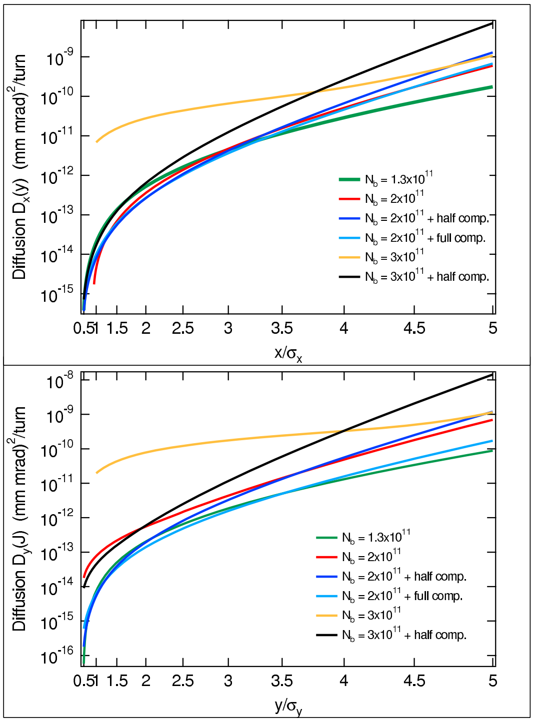

Figure 3: Fitting of the diffusion simulation results for the working point below the diagonal.

Markov process and that the drift coefficient is half of the derivative of the diffusion coefficient (this is true for most Hamiltonan systems), we can describe the evolution of the phase-space distribution $\rho(J,t)$ by a Fokker-Planck equation. Assuming that the phase-space distribution is Gaussian in the (x,x') space, the emittance growth rate for each plane can be calculated as

$$\left\langle \frac{d\varepsilon}{dt} \right\rangle = \frac{1}{4\varepsilon_0} \frac{\int_0^{J_{cut}} D(J)\rho(J,t)dJ}{\int_0^{J_{cut}} \rho(J,t)dJ} \qquad (2)$$

where J_{cut} is a cut at 3 times the rms emittance and assuming that the derivative of the distribution function for $J > J_{cut}$ is close to zero, this is, the derivative $\frac{\partial\rho(J,t)}{\partial J} \approx 0$ at the tails of the distribution.

Figure 4 shows the emittance growth rate calculated for various configurations (different bunch intensity, and with and without half and full beam-beam compensation). The growth rate for the case of the working point above the diagonal with bunch intensities of $N_b = 2 - 3 \times 10^{11}$ and no compensation reflects the fact that at this intensities the horizontal tune is close or crosses the $2/3$ resonance which is known to deteriorate the beam quality, once the beam-beam head-on compensation in turned on the emittance growth rate is reduced to a value comparable to the one measured in 2006 since there is no crossing of the $2/3$ line anymore. For the case with the tunes below the diagonal with the highest bunch intensities and no compensation the same resonance is crossed but this time in the vertical plane, leading to an increase in the emittance growth rate as well. However it appears that the vertical crossing of this resonance is not as strong as the horizontal. The configuration with the lowest bunch intensity (left side of the plots in Figure 4) and the cases with compensation (right side of the plots in Figure 4) show that, within the error, the emittance growth rates are the same which show that the compensation brings down the growth rate to values which were already shown experimentally to be acceptable during the polarized proton run of 2006 and 2008 ($\frac{d\varepsilon_N}{dt} \approx 0.0002$ mm mrad). The drawback is that, although the calculated values for the emittance growth are close to the measured ones, the error bars are large due to the difficulty to calculate the diffusion from the simulation results, and so the accuracy of the comparison in within a factor of 2-3.

CONCLUSIONS

In our beam-beam simulations, without and with an e-lens, we find that almost all particles are chaotic, even those in the beam core. Based on this observation we investigated the emittance growth by calculating the diffusion coefficient at selected phase space locations. From the diffusion calculation we observe that the e-lens is able to almost restore the diffusion levels at the core however at the cost of increasing the diffusion for particles beyond 4σ. From the emittance growth rate calculation this reduction of the diffusion in the beam core appears as a reduction in the emittance growth rate for the cases with half or full compensation, however the error bars, due to the difficulty to have an accurate value for the diffusion coefficient, are too large to enable a close comparison between the cases with lower growth rate. In all cases we used an ideal e-lens that had the same transverse profile as the proton beam, is not affected by the proton beam, and has no position or current errors.

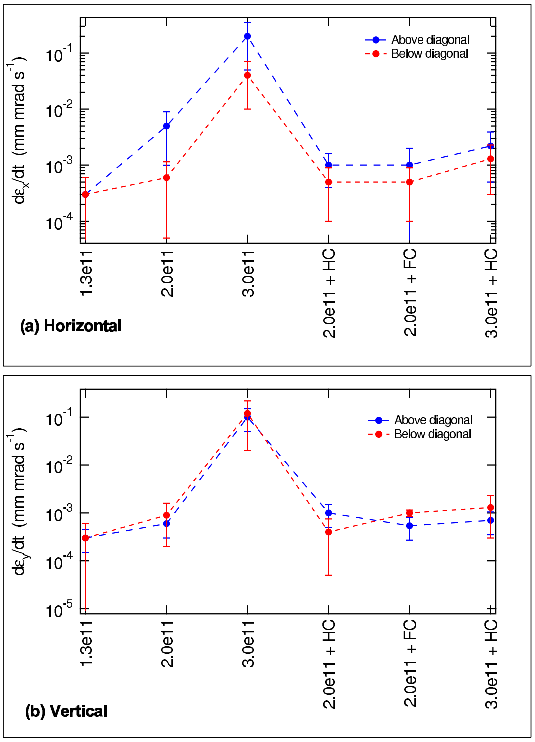

Figure 4: Emittance growth rate calculated using the diffusion results for each working point .

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