

# EXPERIMENTAL CHARACTERIZATION OF PERMANENT MAGNET HARMONIC CORRECTOR RINGS\*

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## Abstract

A total of three permanent magnet chicane magnets have been installed at the Advanced Light Source (ALS) at the Lawrence Berkeley National Laboratory. The magnet design incorporates an annular array of counter-rotating permanent magnet pairs (PMs) with supplemental fast trim coils (EMs). The purpose is to provide a fixed angular separation between two successive elliptically polarizing undulator (EPU) photon fans (with the PMs) and to correct steering perturbation resulting from EPU polarization state and gap changes (with the hysteresis-free EMs). This paper presents a method for fine tuning relative orientation settings of the rotors in the presence of initial uncertainty of the exact PM rotor geometrical and magnetization parameters by performing magnetic measurements with rotating coils. The measurement method will be developed and illustrated with experimental data from the measurement of a 16 cylinder permanent magnet harmonic corrector ring.

## INTRODUCTION

The Advanced Light Source (ALS) at Lawrence Berkeley National Laboratory uses the concept of Permanent Magnet Harmonic Corrector Ring (PMHCR) [1] within chicane magnets used to bend the electron beam to operate two insertion devices on a single straight section [2]. The PMHCR plus the EM trim coils provide adequate magnetic field harmonic correction in less than a second as needed by the insertion device operation mode, and avoid hysteresis phenomena inherent in conventional iron-core electromagnets.

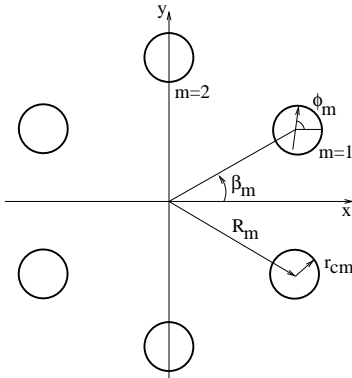


Figure 1: Permanent Magnet Harmonics Corrector Ring

A PMHCR is made of  $M$  permanent magnet cylinders as showed by figure 1:

\*This work was supported by the Director, Office of Science, U. S. Department of Energy under Contract No. DE-AC02-05CH11231.

- spaced uniformly in the azimuthal direction at nominal angles  $\beta_m = m\frac{2\pi}{M} + \beta_0$ ,
- with nominal magnetization direction  $\phi_m$ ,
- with nominal strength  $B_{rm}$ ,
- with nominal radius  $r_{cm}$
- located at nominal radius  $R_m$ ,
- and of nominal length  $L_m$

where  $m$  refers to the cylinder index ( $m = 1..M$ ),  $\beta_0$  is the offset angle that rotates all the cylinders to give the right orientation to the PMHCR harmonic correction.

The complex conjugate integrated magnetic field ( $I^*(\zeta)$ ) provided by a PMHCR is known [1] as:

$$\begin{aligned}
 I^*(\zeta) &= \int_{-\infty}^{\infty} B^*(\zeta) dZ & (1) \\
 &= \sum_{n=1}^{\infty} \sum_{m=0}^{M-1} \left[ \frac{L_m}{2} \frac{r_{cm}^2}{R_m^2} \left( \frac{r_p}{R_m} \right)^{n-1} \right. \\
 &\quad \times \left. n B_{rm} e^{i(\phi_m - (n+1)\beta_m)} \left( \frac{\zeta}{r_p} \right)^{n-1} \right] & (2) \\
 &= \sum_{n=1}^{\infty} C_n \left( \frac{\zeta}{r_p} \right)^{n-1}
 \end{aligned}$$

where  $\zeta = x + iy$  and where  $B^*(\zeta) = B_x - iB_y$  is the conjugate of the 2-D complex magnetic field density. In Eq. 1,  $Z$  is the longitudinal cartesian coordinate.

As demonstrated by the Eq. 2,  $I^*(\zeta)$  can be expanded as a power series of  $\zeta/r_p$  where  $r_p$  is the reference radius,  $n$  the harmonic order ( $n = 1$ : dipole,  $n = 2$ : quadrupole...) and  $C_n$  the complex harmonic coefficients.

The quality of the correction provided by the PMHCR depends on how accurately their magnetic characteristics are known. We presents here how these characteristics could be determined by means of rotating coil magnetic measurements while the PMHCR is already assembled.

## CHARACTERIZATION METHOD

This section is focused on the determination of the parameters of one cylinder with index  $m$ . One PMHCR cylinder with index  $m$  is completely determined by 6 parameters (see Eq. 2):  $L_m$ ,  $r_{cm}$ ,  $R_m$ ,  $B_{rm}$ ,  $\phi_m$  and  $\beta_m$ .

## General Method

The only degree of freedom used in our characterization method is  $\phi_m$  which is used to isolate the effect of the cylinder  $m$  from the others in Eq. 2.

Two measurements are taken for 2 different values of  $\phi_m$  ( $\phi_m^0$  and  $\phi_m^1$ ) and lead to two sets of complex harmonic coefficients (respectively  $C_n^0$  and  $C_n^1$ ).  $\phi_m^0$  and  $\phi_m^1$  are not known but we accurately measure their difference  $\delta\phi_m$ :

$$\delta\phi_m = \phi_m^1 - \phi_m^0 \quad (3)$$

Then, by calculating the difference between the 2 sets of harmonic ( $\Delta C_n$ ) we obtain a formula in which remains only the influence of the cylinder  $m$ .

$$\begin{aligned} \Delta C_n &= C_n^0 - C_n^1 \\ &= L_m \frac{r_{cm}^2}{R_m^2} \left( \frac{r_p}{R_m} \right)^{n-1} n B_{rm} \sin \left( \frac{\delta\phi_m}{2} \right) \\ &\times e^{i(\phi_m - (n+1)\beta_m + \frac{\delta\phi_m}{2} - \frac{\pi}{2})} \end{aligned} \quad (4)$$

Eq. 4 is the core relation of the proposed method, all the formulae needed to determine the parameters are deduced from it.

## Parameters Determination

The harmonic difference modulus is a function of 4 of the wanted parameters and its argument is a function of the remaining parameters:

$$|\Delta C_n| = f(L_m, r_{cm}, R_m, B_{rm}) \quad (5)$$

$$\text{Arg}[\Delta C_n] = g(\phi_m, \beta_m) \quad (6)$$

From Eq. 5 and Eq. 6, 4 parameters can be determined with 2 values of  $n$ : the 2 angles, and only 2 cylinder parameters. This requires some assumptions about the 2 others that we will develop later.

**Formulae for angles** From Eq. 4, we can deduce the two following relations:

$$\beta_m = \text{Arg} \left[ \frac{\Delta C_n}{\Delta C_{n+1}} \right] \quad (7)$$

$$\phi_m = \text{Arg} [\Delta C_n] + (n+1)\beta_m - \frac{\delta\phi_m}{2} + \frac{\pi}{2} \quad (8)$$

**Formulae for 2 cylinder parameters** From Eq. 4, the only parameter of the modulus that depends on powers of  $n$  is  $R_m$  which can be isolated and calculated with:

$$R_m = \frac{n+1}{n} r_p \left| \frac{\Delta C_n}{\Delta C_{n+1}} \right| \quad (9)$$

Then from the three remaining parameters to determine, we can assume that both the cylinders length ( $L_m$ ) and radius ( $r_{cm}$ ) can be mechanically measured with an accuracy close to the micron range. This accuracy enables us to consider these two mechanically measured parameters as data

in the next formula allowing us to more precisely determine  $B_{rm}$ .

$$B_{rm} = \frac{1}{n L_m \sin \left( \frac{\delta\phi_m}{2} \right)} \frac{R_m^2}{r_{cm}^2} \left( \frac{R_m}{r_p} \right)^{n-1} |\Delta C_n| \quad (10)$$

## SENSITIVITY TO ERRORS

We develop here some calculations to determine the sensitivity of the parameters we propose to use to characterize to measurement errors. As the previous relations are deduced from the  $\Delta C_n$ , we propose to investigate the influence of the harmonic differences modulus ( $|\Delta C_n| = d_n$ ) and argument ( $\text{Arg}[\Delta C_n] = \theta_n$ ) errors on the parameters. The errors are defined respectively as  $\epsilon_{d_n}$  and  $\epsilon_{\theta_n}$ , respectively for the modulus and the argument (see Eq. 11).

$$\Delta C_n^{\text{real}} = (d_n + \epsilon_{d_n}) e^{i\theta_n + \epsilon_{\theta_n}} \quad (11)$$

If  $\epsilon_{\beta_m}$  is the error on  $\beta_m$  defined by Eq. 7 then we get:

$$\epsilon_{\beta_m} = \beta_m^{\text{Real}} - \beta_m = 2 \epsilon_{\theta_n} \quad (12)$$

Then, we can deduce the error ( $\epsilon_{\phi_m}$ ) performed on the cylinder orientation angle ( $\phi_m$ ):

$$\epsilon_{\phi_m} = \phi_m^{\text{Real}} - \phi_m = (2n+3)\epsilon_{\theta_n} + \frac{\epsilon_{\delta\phi_m}}{2} \quad (13)$$

$\epsilon_{\delta\phi_m}$  represents the error performed on the measurement of the cylinder magnetization orientation  $\phi_m$ .

The same calculation can be led for  $\epsilon_{R_m}$ :

$$\begin{aligned} \epsilon_{R_m} &= R_m^{\text{Real}} - R_m \\ &= r_p \left( \frac{n+1}{n} - \frac{R_m}{r_p} \right) \frac{\epsilon_{d_n}}{d_{n+1} + \epsilon_{d_n}} \end{aligned} \quad (14)$$

Two remarks can be formulated concerning the minimization of  $\epsilon_{R_m}$  (See Eq. 14).

- The harmonic order ( $n$ ) can be chosen to reduce the difference in the parenthesis which depends on the ratio between  $R_m$  and  $r_p$ ,
- getting  $d_{n+1}$  as big as possible will minimize the effect of the error on the harmonic difference modulus ( $\epsilon_{d_n}$ ).

We can also evaluate the error through dimensionless variables which could be more convenient for the next calculations. We then define  $k_{d_n}$ :

$$k_{d_n} = \frac{\epsilon_{d_n}}{d_n} \quad (15)$$

$$d_n^{\text{Real}} = d_n + \epsilon_{d_n} = d_n(1 + k_{d_n}) \quad (16)$$

In the most unfavorable case we take  $k_{d_n} = \text{Max}[k_{d_n}, k_{d_{n+1}}]$ :

$$k_{R_m} = \frac{\epsilon_{R_m}}{R_m} \quad (17)$$

$$= \frac{1}{R_m} \left( \frac{n+1}{n} r_p \frac{d_n(1+k_{d_n})}{d_{n+1}(1-k_{d_n})} - R_m \right) \approx 2k_{d_n} \quad (18)$$

We can finally deduce  $k_{B_{rm}}$  with the assumptions that possible errors on the mechanical measurements of  $L_m$  and  $r_{cm}$  are too small to disturb the determination of  $B_{rm}$ .

$$k_{B_{rm}} = \frac{\epsilon_{B_{rm}}}{B_{rm}} \approx (1 + 4k_{d_n})(1 + 2k_{d_n})^{n-1} - 1 \quad (19)$$

## EXPERIMENTAL DATA

We have begun testing the method presented with a 16 cylinder PMHCR whose nominal parameters have been designed as:

- $R_m = 68 \text{ mm}$ ,  $r_{cm} = 9.5 \text{ mm}$ ,  $L_m = 6.25 \text{ mm}$ ,
- $\beta_m = m\frac{\pi}{8} - \frac{\pi}{16}$ ,  $B_{rm} = 1.04 \text{ T} \pm 2\%$

To tune the  $\phi_m$  angles without the method we are proposing, we have to rely on the magnetization orientation marked on the cylinder ( $\pm 1^\circ$  accuracy). Moreover as these cylinders are tuned manually with a  $360^\circ$  graduated disk, it adds an additional error of  $\pm 0.5^\circ$ . If it is performed without the magnetic measurement method, the  $\phi_m$  are then tuned with only a  $\pm 1.5^\circ$  accuracy ( $\pm 26 \cdot 10^{-3} \text{ rad}$ ).

According to Eq. 4, the maximum dipole and quadrupole component variation we could expect to measure ( $\delta\phi_m = \pi$ ) at a  $25 \text{ mm}$  reference radius ( $r_p$ ) are:

- Dipole:  $dc_1 = |\Delta C_1| = 1.27 \cdot 10^{-4} \text{ T.m}$ ,
- Quadrupole:  $dc_2 = |\Delta C_2| = 0.94 \cdot 10^{-4} \text{ T.m}$

### Expected Measurement Sensitivity

This PMHCR has been measured with a 400 hundred-turn rotating coil which largest radius is  $25 \text{ mm}$ . The reproducibility of the measurement bench could be evaluated to  $\pm 4 \cdot 10^{-7} \text{ T.m}$  for the harmonic modulus ( $|C_n|$ ) and  $2 \cdot 10^{-3} \text{ rad}$  for their arguments. From these reproducibility measurements we can deduce how the harmonic differences are affected and we get:  $3 \cdot 10^{-6} \text{ T.m}$  for  $\epsilon_{d_n}$  and  $2 \cdot 10^{-2} \text{ rad}$  for  $\epsilon_{\theta_n}$ .

The  $\phi_m$  angles are measured with a 4000 points per turn encoder which leaves us with an uncertainty on  $\delta\phi_m$  of  $1.6 \cdot 10^{-3} \text{ rad}$ . We can then deduce how accurately the parameters can be determined with the Eq. 12, 13, 14 and 19. By working with the dipole and the quadrupole components, we obtain:

- $\epsilon_{\beta_m} = 4 \cdot 10^{-3} \text{ rad}$ ,  $\epsilon_{\phi_m} = 11.6 \cdot 10^{-3} \text{ rad}$ ,
- $\epsilon_{R_m} \approx 0.5 \text{ mm}$ ,  $\epsilon_{B_{rm}} = B_{rm} * k_{B_{rm}} = 0.13 \text{ T}$

These errors are not small enough to obtain a sharp parameter characterization but it should allow us to check the order of magnitude of the measured parameters.

### Parameters Measurements

We measured the parameters for 2 diametrically opposed cylinders (m=2 and m=10) and we worked on the dipole and quadrupole differences.

Table 1: Parameter comparison: design vs measurements

|      |          | $\beta_m \text{ (rad)}$ | $R_m \text{ (mm)}$ | $B_{rm} \text{ (T)}$ |
|------|----------|-------------------------|--------------------|----------------------|
| m=2  | Design   | 0.59                    | 68                 | 1.04                 |
|      | Measured | 0.568                   | 69.7               | 1.01                 |
| m=10 | Design   | -2.55                   | 68                 | 1.04                 |
|      | Measured | -2.61                   | 70.8               | 1.12                 |

Concerning  $\phi_m$ , we took 2 successive measurements with 2 different  $\delta\phi_m$  to validate Eq. 8 and obtained:  $\phi_{10}^{Meas1} = 0.87 \text{ rad}$  and  $\phi_{10}^{Meas2} = 4.26 \text{ rad}$ . The difference between these measurements ( $3.39 \text{ rad}$ ) has to be compared with the real rotation applied to the cylinder ( $3.28 \text{ rad}$ ).

The results from table 1 and the previous experiment shows that the measured values are of the right order of magnitude. However these parameter measurements are not accurate enough (especially for  $R_m$ ) to provide a better harmonic content than the one obtained with the imperfections [1].

## CONCLUSION

We proposed here a method that would lower the PMHCR correction error levels since this method is meant to determine the uncertainties about the parameters of assembled PMHCR. The measurements showed earlier could be more accurate if obtained from an adapted rotating coil. Indeed, the rotating coil we used had a  $25 \text{ mm}$  measurement radius compared to the  $55 \text{ mm}$  bore radius of the PMHCR we studied. However, even with a small radius rotating coil, it was possible to lower by a factor of 2 the accuracy on the magnetization orientation ( $\phi_m$ ) and have a reasonable accuracy on the other parameters determination ( $\beta_m$ ,  $R_m$  and  $B_{rm}$ ) to verify their related formulae.

The next step of this work is to characterize completely the PMHCR with a rotating coil sensor dedicated to it. Once these parameters will be known, the final step would be to generate a proper correction scheme and apply it to determine the magnetic field the PMHCR should provide. Then a magnetic measurement will check how pure the correction provided by the corrected PMHCR is.

## REFERENCES

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