EMITTANCE REDUCTION BY LONGITUDINALLY VARYING DIPOLE FIELD

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Abstract

We have studied the emittance reduction by a longitudinally varying magnetic field in a bending magnet. The radius of curvature is assumed to vary with polynomial of nth degree (n=1,2,3,4). The emittance is calculated numerically for minimum emittance and minimum emittance achromat condition. It was found that the emittance reduction rate is independent of the average field strength and the bending magnet length if the emittance is plotted by the maximum field strength normalized by the average field strength. The radius of curvature expressed by the second order polynomial is the most effective distribution and the emittance decreases to about 1/5 for minimum emittance condition and 1/3 for achromat condition. We applied the longitudinally varying field to an ultra-low emittance ring [1]. The emittance of 32 pm was obtained, however, the dynamic aperture is too small to inject and storage an electron beam. The detuned optics with the 63 pm emittance was designed and +3.6/-3.3 mm dynamic apertures were obtained. But the reduction of emittance from the original ultra-low emittace ring was only 20 pm and the application of this method to an ultra-low emittance ring was not so effective.

INTRODUCTION

The photon beam brightness is the most important parameter for synchrotron radiation sources. As the brightness is inversely proportional to the electron beam emittance, it is important for synchrotron radiation sources to have lower emittance. Accordingly it is not too much to say that the history of synchrotron radiation source is the history of lowering the emittance of electron storage ring.

The most simple and effective method to reduce the emittance is to design a storage ring that have small angle bending magnets as the emittance is proportional to the third power of bending angle. Under a constant bending angle, we can reduce the emittance by introducing transverse field gradient in bending magnets. However, reduction of the emittance is 50 % at most. Especially, this method becomes less effective as the emittance becomes small.

Recently the emittance reduction by longitudinally varying dipole fields is studied for a damping ring [2] and synchrotron radiation source [3]. Under a constant bending angle, this method has a possibility to improve the emittance.

We have studied the emittance reduction by a longitudinally varying dipole field. The result is applied to an ultra-low emittance ring and the effectiveness of the method is studied.

BASIC IDEA

Emittance of an electron storage ring is determined by the balance of excitation by radiation emission and damping by RF acceleration. If the amount of radiation excitation is small, the emittance becomes lower. Normally, as dipole fields are constant along the longitudinal direction, the energy spectrum of radiation is the same inside the bending magnets. But if the magnetic field is strong at smaller dispersion function and weak at larger dispersion function, the amount of radiation excitation becomes small, which results in a smaller emittance beam. Actually, since not only dispersion function but also betatron function relates to the radiation excitation, the problem is more complex. However, the basic mechanism is the simple one as described above.

The emittance of electron storage ring is expressed by [4]

$$\varepsilon_{x0} = \frac{C_q \gamma_0^2}{J_x} \frac{\oint H/\rho^3 ds}{\oint 1/\rho^2 ds}$$
$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc}$$
$$J_x = 1 - \frac{\oint \eta/\rho (1/\rho^2 + 2B'/B\rho) ds}{\oint 1/\rho^2 ds} \qquad (1)$$
$$H = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2 .$$

We can see from eq. (1) that, to be exact, the emittance is determined by the so-called H-function and a radius of curvature. Thus it is important to know the shape of the H-function to reduce the emittance. The H-functions for SPring-8 bending magnet are shown in Fig. 1 for the case of minimum emittance achromat condition (MEA) and minimum emittance condition (ME). We can see from Fig. 1 that for ME lattice, the radius of curvature should be large at the both ends of the magnet and small at the center of the magnet. For MEA, the curvature should be small at the entrance of a magnet and large at the exit.

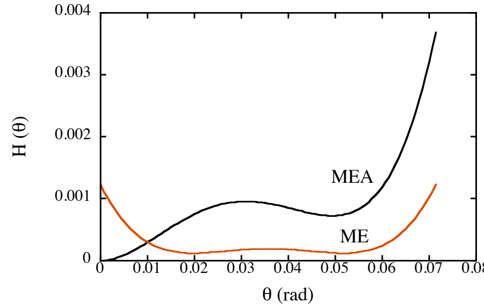

Figure 1: H-functions of SPring-8 bending magnet for minimum emittance (ME) and minimum emittance achromat (MEA) condition.

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We assumed the radius of curvature as follows.

$$\rho = \rho_a |s - s_0|^n + \rho_0 \qquad (2)$$

n = 1,2,3,4 $s_0 = 0$ for MEA, $s_0 = L_m/2$ for ME

where L_m is the length of a bending magnet. Using eq. (2), we calculated the emittance numerically. In calculation, a bending magnet is sliced into small pieces. In a small piece, the field strength is assumed to be constant and Twiss parameters are calculated applying transfer matrix to these small pieces. Slice width is decreased until the emittance value converges.

We need to determine concrete parameters to calculate the emittance. We chose two typical examples. A 6 GeV, 88 bending magnets and 0.5 T average dipole field machine as a representative of high energy machine, and a 2 GeV, 24 bending magnets, 1.5 T dipole field machine as a low energy one. We calculated the emittance for these two machines varying the polynomial order from the first to the fourth order. As an example, the calculated emittance of ME and MEA in case of second order polynomial is shown in Fig. 2 as a function of maximum dipole field strength. For both cases, the emittance decreases exponentially and approaches the limiting value. This tendency is the same for all polynomial order. The maximum dipole field strength was normalized with the average one and the normalized emittance was plotted. The results were shown in Fig. 3 for the case of second order polynomial. If the polynomial order is the same, normalized emittances only depend on B/B_0. It does not relate to the energy, the average dipole field strength, and the magnet length. Normalized emittance of ME and MEA for four polynomial orders is shown in Fig. 4.

Emittance saturates as the maximum dipole field strength increases and the saturation begins faster for higher polynomial order. But the second order polynomial has the smallest emittance. The emittance decreases to about 1/5 for ME case and 1/3 for MEA case.

Figure 5 shows the numerator of eq. (1) $I_5 = \oint H/\rho^3 ds$ and denominator $I_2 = \oint 1/\rho^2 ds$ for the fourth order polynomial. Though the denominator increases monotonically with the maximum dipole field, the numerator initially decreases and has minimum value at certain field strength and gradually increases. This is the cause of the emittance saturation.

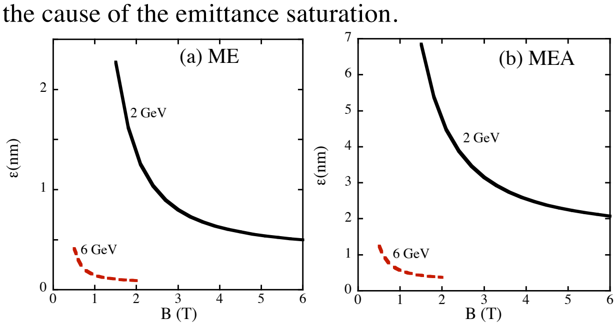

Figure 2: Emittance of ME and MEA for the case of second order polynomial.

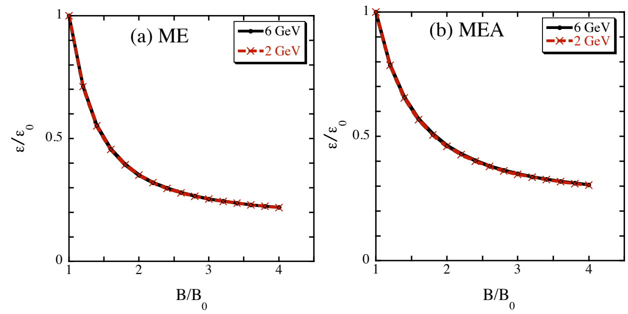

Figure 3: Normalized emittance of ME and MEA for the case of second order polynomial.

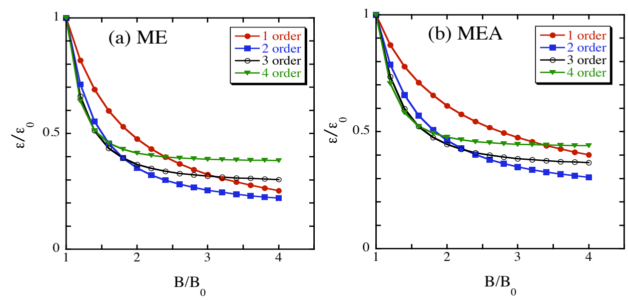

Figure 4: Normalized emittance of ME and MEA with different field distribution.

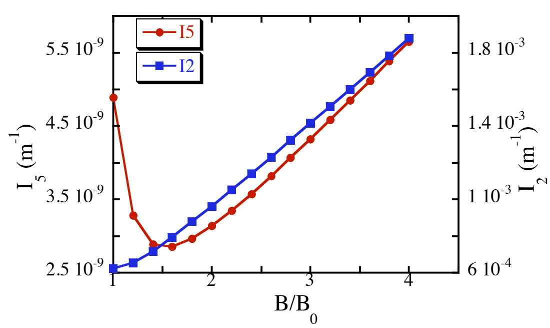

Figure 5: I_5 and I_2 as a function of B/B_0 for the fourth order polynomial.

APPLICATION TO ULTRA-LOW EMITTNCE RING

We applied the longitudinally varying dipole field to the ultra-low emittance storage ring. The ultra-low emittance ring has the same circumference as that of SPring-8 storage ring and consists of 24 ten-bend achromat cells. The natural emittance is 83 pm and the electron energy is 6 GeV. We applied the varying dipole field to this ring. The average field strength is set to 0.8 T. The maximum field strength of 2T is chosen to get a 70 % emittance reduction.

We designed two optics modes. One is a low emittance mode that is anticipated to have small dynamic aperture. The other is a high emittance mode that has a larger dynamic aperture than that of low emittance mode.

Low Emittance Optics

Betatron and dispersion functions of low emittance optics are shown in Fig. 6. The dispersion function is very small and even the maximum value is 0.017 m. Emittance is 32 pm, which is 61 % emittance reduction, however, the sextupole strength is 5 to 11 times stronger. Dynamic apertures were calculated and are shown in Fig. 7. The aperture is so small even after improvement that it is difficult to realize such a ring.

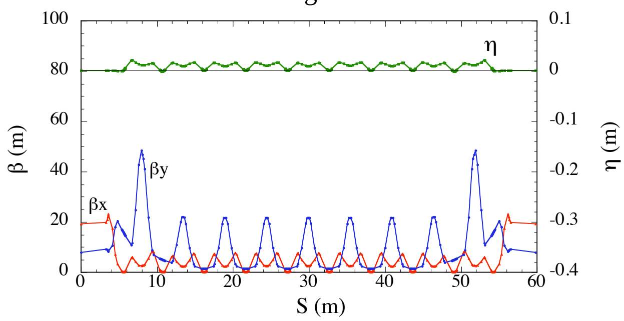

Figure 6: Betatron and dispersion functions in a cell for low emittance optics.

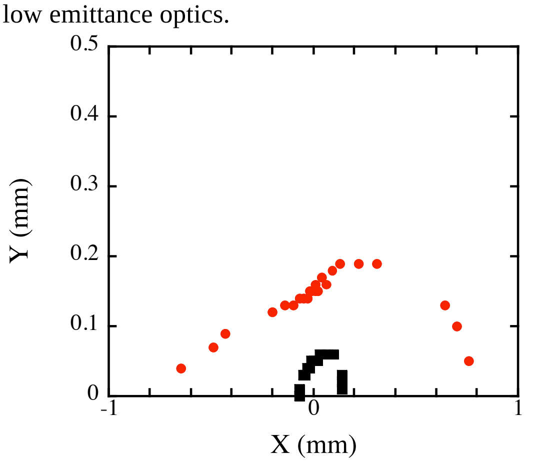

Figure 7: Dynamic aperture before harmonic correction ■ and after harmonic correction ● for low emittance optics.

High Emittance Optics

We designed high emittance optics to obtain a larger dynamic aperture. Optical functions are shown in Fig. 8. Emittance is 63 pm and it is 76 % of the original emittance. Sextupole strength is the same order as that of ultra-low emittance ring. Dynamic aperture is small (+3 mm/-3.3 mm), but it is possible to inject and storage an electron beam.

Emittance reduction is only 24 % and this shows that this method is not so effective for ultra-low emittance storage ring.

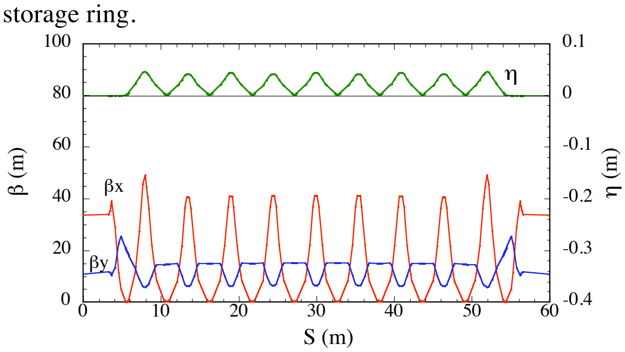

Figure 8: Betatron and dispersion functions in a cell for high emittance optics.

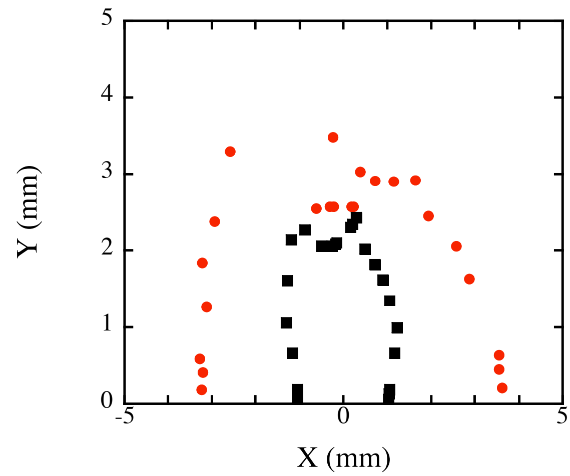

Figure 9: Dynamic aperture before harmonic correction ■ and after harmonic correction ● for high emittance optics.

CONCLUSIONS

Emittance reduction by a longitudinally varying dipole field was studied for minimum emittance and minimum emittance achromat condition. Dipole fields were assumed to vary with the first to the fourth order polynomials. It was found that the emittance reduction rate is independent of the average field strength and bending magnet length if the emittance is plotted by the maximum field strength normalized by the average field strength. The emittance decreases exponentially with the maximum field strength and begins to saturate as the field strength increases. This is more characteristic for the higher order polynomial. Most effective field distribution is the second order polynomial. The emittance decreases to about 1/5 as the maximum field strength increases to four times the average field strength for the minimum emittance condition and 1/3 for the minimum emittance achormat condition.

Bending magnets with the longitudinally varying field were applied to the ultra-low emittance ring. A ten-bend achromat lattice with 32 pm emittance was designed. The dynamic aperture is so small that it is difficult to realize such a ring. We designed a higher emittance lattice to get a larger dynamic aperture and a detuned lattice with 68 pm emittance was obtained. The dynamic aperture was calculated. The horizontal apertures are +3.6 mm and -3.3 mm. These values are small, but it is not difficult to inject a beam and storage it. But the emittance reduction is small compared to the expected one. This shows that the application of the longitudinally varying field to the ultra-low emittance ring is not so effective.

REFERENCES

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[4] M. Sands, SLAC-121, 1970.