COLLECTIVE IONIZATION BY ATTOSECOND ELECTRON BUNCHES

A. Ogata*, T. Kondoh, K. Norizawa, J. Yang and Y. Yoshida
ISIR, Osaka University
Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Abstract

The formalism of the stopping power for cluster beams was adapted to the stopping power for short electron bunches by using the wake field of a medium characterized by a plasma frequency. It was shown that an electron bunch with the length of the order of attoseconds collectively loses its energy to excite a high-energy density state in the target.

The dynamics of intricate molecular and atomic processes can be elucidated by using electron bunches of the order of 100 fs in the pump-probe method[1]. However, direct usage of the energy of electron beams has been limited, because electron beams interact with matter in a more moderate manner than ion beams.

If the electron bunch length σ is of the order of attoseconds, its inverse is comparable to the ionization frequency, i.e., $c/\sigma \sim \omega_I = I/\hbar$, where c and I are the light velocity and the ionization potential of a target, respectively. This type of electron bunch allows collective ionization of the target, which will enable us to develop many different applications in the future.

The energy loss of a single electron in a bunch can be expressed as

$$\frac{dW}{d\tilde{z}}(\rho, \tilde{z}) = \frac{dW_1}{d\tilde{z}} + \frac{dW_2}{d\tilde{z}}, \tag{1}$$

where $\rho = (x^2 + y^2)^{1/2}$ and $\tilde{z} = z - vt$, which is defined relative to the position $(x, y, z) = (0, 0, vt)$ of the moving electron with a velocity v. The first term $dW_1/d\tilde{z}$ is the energy loss due to the direct interaction between the electron itself and the environment, that is termed as stopping power. If the electron density of the bunch is high, we also need to consider the second term $dW_2/d\tilde{z}$, which denotes the energy loss due to the fields caused by other electrons in the bunch. In this study we focus on this second term. Conventionally this type of energy loss has been studied in cluster beams, in which each cluster can be regarded as a ultra short bunch[2].

We adapt the formalism of the stopping power for cluster beams to that for short electron bunches by using the wake field of a medium characterized by a dielectric function $\epsilon(k, \omega)$[3]. If the frequency characterizing the bunch length is much greater than the highest resonant frequency, the dielectric function becomes a simple function of the plasma frequency[4]. Although our approach is more macroscopic than Bethe's[5], we can refine it to derive the same result as that obtained by Bethe[6].

* ogata@post.kek.jp

The following description is applicable to the energy region of electron beams in which neither density effect[7] nor radiation loss[5] is primary. A beam is assumed to be frozen, i.e., the spatial distribution of electrons in the beam is fixed throughout the interaction.

In this study, we have used the Coulomb gauge. If we do not take account of the transverse motion of an electron, we obtain a pure transverse vector potential in this gauge. However, because the transverse current is not negligible in relativistic beams[8], we consider the following equation to describe the longitudinal electric field accompanying an electron of the bunch in a medium,

$$\mathbf{E}_\parallel(\mathbf{r}, t) = -\nabla\Phi(\mathbf{r}, t) - \frac{1}{c}\frac{\partial \mathbf{A}_\parallel}{\partial t}. \tag{2}$$

We obtain the potentials in the linear approximation by performing a Fourier transform with wave number k and frequency ω[9]:

$$\begin{aligned}\Phi &= \frac{2e}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv}\omega d\omega J_0[\rho(k^2 - \omega^2/v^2)^{1/2}] \\ &\times \left[\mathrm{Re}\left(\frac{1}{\epsilon_\parallel}\right)\cos k\tilde{z} - \mathrm{Im}\left(\frac{1}{\epsilon_\parallel}\right)\sin k\tilde{z}\right], \end{aligned} \tag{3}$$

$$\begin{aligned}\mathbf{A}_\parallel &= \frac{2e}{\pi c^2}\int_0^\infty \frac{dk}{k}\int_0^{kv}\omega d\omega J_0[\rho(k^2 - \omega^2/v^2)^{1/2}] \\ &\times \left[\mathrm{Re}\left(\frac{k^2 - \omega^2/v^2}{k^2 - \epsilon_\perp\omega^2/c^2}\right)\cos k\tilde{z}\right. \\ &- \left.\mathrm{Im}\left(\frac{k^2 - \omega^2/v^2}{k^2 - \epsilon_\perp\omega^2/c^2}\right)\sin k\tilde{z}\right]. \end{aligned} \tag{4}$$

Because the momentum transfer from the projectile electron to a media electron is relativistic, we can assume $\epsilon_\parallel = \epsilon_\perp$[10]. The procedure to derive the longitudinal field for ion beams has been presented in [8, 9, 11]; hence we have omitted the derivation of the following result for ion beams:

$$\begin{aligned}\mathbf{E}_\parallel(\rho, \tilde{z}) &= -\frac{2\pi e^4 NZ}{mv^2}\cos\left(\frac{\omega_p\tilde{z}}{v}\right) \\ &\times \left[\log\frac{2mv^2\gamma^2}{I} - \beta^2\right] K_0\left[\frac{\omega_p\rho}{v}, \frac{v}{v_F}\right], \end{aligned} \tag{5}$$

where N, Z and v_F respectively denote the atomic density, the atomic number and the Fermi energy of the medium, $\beta = v/c \sim 1$, $\gamma = 1/(1-\beta^2)^{1/2}$, m is the electron mass, e is the electron charge, and $\omega_p = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency when the medium is regarded as an electron gas of density n.

Because our projectiles are electrons, the terms in the first square bracket must be changed. We write

$$\mathbf{E}_{\parallel}(\rho,\tilde{z}) = -\frac{2\pi e^4 NZ}{mv^2}\cos\left(\frac{\omega_p \tilde{z}}{v}\right) \times \left[\log\frac{(mc^2)^2\gamma^2}{2I^2} + \frac{1}{8}\right] K_0\left[\frac{\omega_p\rho}{v}, \frac{v}{v_F}\right]. \quad (6)$$

The difference is due to the facts that in the case of electron projection, the maximum energy loss in any collision is $mv^2/4$ and not $mv^2/2$, and that we have to take into account the Mott scattering cross section for identical particles with a 1/2 spin.

The function contained in eq. (6),

$$K_0(\xi,\eta) = \int_0^{\eta} \frac{yJ_0[\xi y]}{1+y^2}dy, \quad (7)$$

gives the transverse dependence of the field[11], which approaches the modified Bessel function $K_0[\xi]$ for a large value of ξ. We first consider the case in which all the electrons are lined along the beam axis directed by $\rho = 0$. When $\xi = 0$, i.e., $\rho = 0$, we have $K_0(0,\eta) = [\log(1+\eta^2)]/2$.

We now assume specific values for some of the parameters. First, assume $\omega_p = 10^{16}\mathrm{s}^{-1}$. This is the typical value of ω_p for metals, which corresponds to a plasmon energy of $\sim$7eV and a plasma skin depth c/ω_p of $\sim$ 30nm. The Fermi velocity of copper is $\sim 1.57\times10^6\mathrm{ms}^{-1}$. If we consider this as the typical value for metals, we have $v/v_F \sim c/v_F \sim 20$ in eqs. (6-7).

The energy loss denoted by the second term of eq. (1) is proportional to the longitudinal electric field or the longitudinal wake field:

$$\frac{dW_2}{d\tilde{z}} = e\mathbf{E}_{\parallel}(\rho,\tilde{z}), \quad (8)$$

On the other hand, the stopping power, or the first term of eq. (1) is proportional to the field existing at the origin:

$$\frac{dW_1}{d\tilde{z}} = e\mathbf{E}_{\parallel}(0,0) = \frac{2\pi e^4 NZ}{mv^2}\left[\log\frac{(mc^2)^2\gamma^2}{2I^2} + \frac{1}{8}\right]. \quad (9)$$

The longitudinal wake field along the beam axis $\rho = 0$ is shown in Fig. 1. An electron in the bunch trailing the injected electron loses its energy, if the wake field is negative,

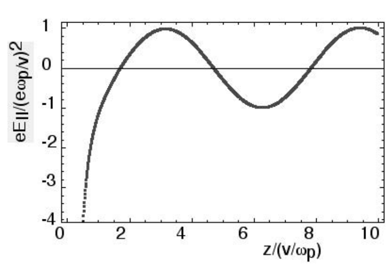

Figure 1: The $\tilde{z}$ dependence of the wake field of a single electron.

and vice versa. Figure 1 shows that if the distance between the driving electron and a trailing one is less than $\sim 2c/\omega_p$ (it is $\sim$60nm in the present case) the trailing electron loses its energy. As the distance between the electrons increases, there is an alternate loss and gain of energy[12].

In order to derive the total energy loss/gain in the bunch as a whole, we have to sum up the total change in energy for all the electrons. As shown in Fig. 1, the energy change of a trailing electron depends on the distance between the electrons in a pair. Thus, the distribution of the mutual distances between the electrons in the bunch is required to calculate the total energy change. Let us assume that the electrons in the bunch are Gaussian distributed longitudinally with a deviation σ. Statistics tells us that the mutual distances x of the pair of electrons in the bunch also make Gaussian distribution, in which the deviation is 2σ[13]:

$$f(x) = \frac{1}{2\sigma\pi^{1/2}}\exp\left[-\frac{x^2}{4\sigma^2}\right]. \quad (10)$$

If a bunch consists of N electrons, the total number of pairs is given by

$$\binom{N}{2} = \frac{N!}{2!(N-2)!} = \frac{N(N-1)}{2}. \quad (11)$$

This value approaches $N^2/2$ when $N \gg 1$.

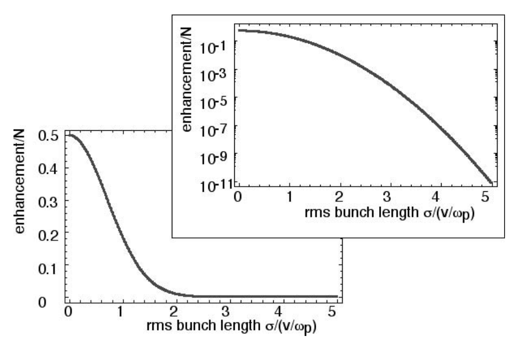

Figure 2: Enhancement of the stopping power by the collective effect as a function of the bunch length σ. Enhancement is given by the value in the figure times N, the number of electrons in the bunch.

The collective energy reduction of a bunch due to the wake field then becomes

$$\left[\frac{dW_2}{d\tilde{z}}\right]_C = \frac{N(N-1)}{2}\int_{-\infty}^{\infty} f(\tilde{z})\frac{dW_2}{d\tilde{z}}d\tilde{z}. \quad (12)$$

If we neglect the wake, the energy reduction of a bunch consisting of N electrons is simply given by

$$\left[\frac{dW_2}{d\tilde{z}}\right]_I = N\frac{dW_1}{d\tilde{z}}. \quad (13)$$

The enhancement ϵ due to the collective effect defined by

$$\epsilon = \left[\frac{dW_2}{d\tilde{z}}\right]_C \Big/ \left[\frac{dW_2}{d\tilde{z}}\right]_I, \quad (14)$$

is shown in Fig. 2 as a function of the bunch length using both linear and logarithmic scales. Because $f(x)$ approaches the delta function as σ approaches 0, the limit on enhancement is given by $(N-1)/2 \sim N/2$.

From Fig. 2 it can be observed that if $\sigma = 3c/\omega_p$ or ∼100nm at $\omega_p = 10^{16}\mathrm{s}^{-1}$, the enhancement is $\sim 10^{-4}N$. For a bunch with a charge is 1pC, the enhancement is 625. The range of a single electron of 10MeV in copper is 6.9mm, which can be reduced to $\sim 10\mu$m. If the beam were frozen, a total beam energy of 10^{-7}J would be absorbed in a cylinder having a volume of $3\times10^{-18}\mathrm{m}^{-3}$. The energy density would then become $3 \times 10^{12}\mathrm{Jm}^{-3}$, which corresponds to a high-energy density state[14]. However, further dynamic study is necessary to clarify the validity of the assumption of the frozen beam.

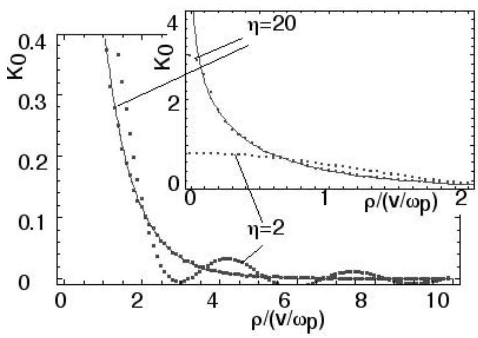

Figure 3: Plots of eq.(7) (dotted lines) for two values of $\eta = v/v_F \sim c/v_F$ and the corresponding Bessel function (solid line).

We now consider the case in which the bunch has a finite transverse size. In eq. (6), the value of K_0 determines the ρ dependence of the wake, which is shown in Fig. 3 for two values of $\eta = v/v_F \sim c/v_F$. This implies that the approximation by the Bessel function is valid except the neighborhood of the projectile in the present case. It also implies that the transverse bunch size should be less than $\sim 2c/\omega_p$ to allow collective ionization of a target ; that is, if $\omega_p = 10^{16}\mathrm{s}^{-1}$, the bunch width should be less than ∼60nm.

This is a very severe restriction. However, the bunch width can be reduced by the lens effect of a plasma[15]. In vacuum, the space-charge force balances the pinch force of the bunch current. In a plasma, the plasma electrons escape and the remaining ions neutralize the space charge of the bunch electrons. The remaining pinch force causes a reduction in the bunch width.

Calculation of the plasma lens effect is beyond the scope of this study. However, we here provide an approximate estimation. The transverse beam size σ_W obeys the envelope equation and is given by

$$\frac{d^2\sigma_W}{dz^2} + K\sigma_W = \frac{\epsilon^2}{\sigma_W^3}, \qquad (15)$$

where ϵ is the emittance. For the matched beam condition, the beam size is given by $\sigma_W^2 = \epsilon/K^{1/2}$. We can approximate the plasma focusing strength as $K = 2\omega_p^2/(\gamma c^2)$[15]. By assuming $\gamma = 50,\ \omega_p = 10^{16}\mathrm{s}^{-1},\ \epsilon = 10^{-6}$m, we can obtain a matched beam size of $\sigma_W \sim 1$nm. This estimate shows that the plasma has a strong lens effect.

Yudin[16] has shown that the enhancement has cutoffs at

$$\epsilon_1 = n_b\left(\frac{v}{\omega_I}\right)^3, \text{ and } \epsilon_2 = \frac{\hbar v}{e^2}\frac{mv^2}{\hbar\omega_I}, \qquad (16)$$

where n_b is the electron density in the beam. These values are $\epsilon_1 \sim 1.5 \times 10^4$ and $\epsilon_2 \sim 3 \times 10^4$ at $v = c$, I =10eV and $n_b = 2 \times 10^{21}\mathrm{cm}^{-3}$. In our numerical example, the enhancement was 625, which is well below these limits.

In summary, we have observed that collective energy loss occurs in an attosecond electron bunch, and the loss in energy is proportional to the square of the number of electrons in the bunch. We have presented some results for the case in which the bunched electrons are frozen along the beam axis. The transverse size of the bunch must also be small to allow the collective ionization. The plasma lens effect would decrease the bunch width to satisfy this condition.

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