

# A FOUR-DIMENSIONAL VLASOV SOLVER FOR MICROBUNCHING INSTABILITY IN THE INJECTION SYSTEM FOR X-RAY FELS

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## Abstract

The micro-bunching instability (MBI) is seeded by small charge-density fluctuations in the electron bunches, and is successively amplified by the combined effect of space charge and coherent synchrotron radiation, as the beam travels through magnetic compressors. The quantitative understanding of this effect demands for accurate numerical simulations. Here we report on the progress of an upgrading of a 2D Vlasov solver code toward a 4D grid-based Vlasov solver, including also the transverse dynamics. The goal is to provide an accurate characterization of the MBI seeded by random noise present in the bunch distribution. We also comment on the advantages of our procedure with respect to other approaches, e. g. macroparticle simulations.

## INTRODUCTION

The demand for high-brightness electron beams is crucial for the proper operation of the FEL-based next generation of synchrotron light sources. Because existing linac sources can provide electron beams with required emittance and energy spread, but not with the necessary peak current, some manipulations of the beam phase space dynamics are required in order to increase the peak current, which would ensure the necessary gain to bring the laser into saturation in a reasonable undulator length. Such a manipulation may occur through magnetic chicanes which became a common tool to all 4<sup>th</sup> generation light sources, proposed or under construction, to realize the electron bunch compression enhancing the peak current. Although they provide a very effective means to compress the bunch, their use can affect other qualities of the beam like transverse emittance and the uncorrelated energy spread. In particular, the finite dispersion generated by the bends in combination with collective effects may give rise to the so-called 'microbunching' instability (MBI) [1], which may result into large charge-density fluctuations downstream the compressor and eventually into an unacceptably large energy spread at the end of the linac. The instability is seeded by the unavoidable small perturbations of the beam density present at injection due, for example, to noise in the photo-gun laser or charge fluctuations caused by shot noise. Much effort has been devoted over the last few years to study and to characterize the collective effects in the injection systems for X-ray FELs and the basic physics is nowadays well understood [2]. Accurate numerical modelling of the MBI still poses some challenges, partly due to the high phase-space resolution necessary for an accurate

simulation. Within such a context we note that the use of macroparticle simulation methods, some of the most widespread tools for beam-dynamics studies, may be of limited use for a quantitative description of the MBI, unless the number of macroparticles used in the simulations is close to that of the electrons in the physical bunch. This fact occurs because of the concurrence of the sampling noise introduced by the use of macroparticles, and of the high sensitivity of the instability to small initial perturbations. A simulation employing  $N_{mp}$  randomly deposited macroparticles overestimates indeed the amplitude of the shot noise by a factor  $\sqrt{N/N_{mp}}$ , where  $N$  is the number of physical electrons per bunch. An effective alternative to macroparticles simulations is represented by codes that solve directly the Vlasov equation ruling the beam dynamics. The Vlasov Solvers (V-S) yield the evolution of the phase space beam density as a function defined on a grid. V-S are therefore immune from sampling noise problems and may therefore be used as a more effective tool to characterize the instability. A first successful demonstration of the use of V-S methods for single-pass systems was reported in [3, 4]. So far, however, the developed solvers treat two-dimensional problems and are therefore suited to model the longitudinal phase space only of beams with negligible transverse emittance. These methods have proved useful for both beam dynamics studies and lattice optimization when some heuristic and approximate account of the effect of the transverse dynamics on the longitudinal motion is included [4]. However, a more accurate characterization of the instability requires a complete description of the coupling between the longitudinal and horizontal motion. In this paper we report on the progress we have recently made toward the goal of developing a fully operational 4D V-S for application to peak current enhancement in Linac dedicated to FEL operation. The goal is to develop a solver with the capability of following the beam through the various elements of an actual lattice and to account for the collective effects associated with the onset of MBI, namely longitudinal space-charge, coherent synchrotron radiation, and possibly RF wakes. We report an outline of the core algorithm dedicated to the solution of the Vlasov equation modelling the beam propagation, a discussion of some of the technical problems and the numerical challenges posed by such a V-S, and preliminary results from some numerical tests.

## VLASOV EQUATION

The evolution equation that describes the beam dynamics of a charged beam density distribution  $\Psi$  is the Vlasov

equation

$$\frac{\partial \Psi}{\partial s} = H\Psi \quad \text{with} \quad \Psi|_{s=0} = \Psi_0 \quad (1)$$

where  $s$  refers to the propagation coordinate.

The formal solution of eq. (1) can be written, for a small propagation interval  $\Delta s$  and if  $H$  is not explicitly a time dependent operator, as

$$\Psi = \exp(H\Delta s) \Psi_0 \quad (2)$$

representing the Vlasov evolution operator. Actually, in writing eq. (2), we are neglecting any contribution due to time ordering corrections, that arises whenever the operator  $H$  is explicitly time dependent and does not commute with itself at different times. With these assumptions we neglect third order terms in the integration step  $\Delta s$ , the symplecticity is however automatically preserved by the exponential form of the evolution operator.

The operator  $H$  encloses the physical properties of the propagation problem and contains the beam dynamics. In absence of any collective effects generated by wake fields, it is a differential operator, describing the beam propagation through magnetic lens systems [5]. If we include the wake fields effects,  $H$  becomes an integral operator and eq. (2) becomes non linear [3]. For example, in case of beam transport through a bending magnet of radius  $R$  and length  $L$ , in a 4D beam dynamics, we can write

$$H = \frac{x}{R} \frac{\partial}{\partial z} - \theta \frac{\partial}{\partial x} + \left( k_\beta^2 x - \frac{\delta}{R} \right) \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \delta} \frac{e^2 N}{E_0 L} \int_{-\infty}^{\infty} W(z' - z) \rho(z') dz' \quad (3)$$

with  $x$  and  $\theta$  the transverse coordinates,  $z$  and  $\delta$  the longitudinal ones,  $k_\beta$  the magnet focusing strength,  $N$  the number of particles in the bunch,  $\rho(z)$  the projection of the density distribution  $\Psi$  on the longitudinal axis, and  $W(z)$  the longitudinal wake function, providing, for the problem we are considering, the CSR wake field that we write as [6]

$$W(z) = \frac{1}{4\pi\epsilon_0} \frac{2}{(3R^2)^{1/3}} \frac{\partial}{\partial z} z^{-1/3} \quad z > 0 \quad (4)$$

Accordingly, we neglect the screening effect of conducting walls and we consider only the steady state radiation from an ultra-relativistic particle in a long magnet [7].

Equations analogous to eq. (3) can be written for the quadrupoles, drifts and RF cavities, which are the beam transport devices used for our study of the MBI. Furthermore, the longitudinal space charge wake field, of the kind [8]

$$\frac{1}{4\pi\epsilon_0\gamma^2} \left( 1 + 2 \log \frac{b}{a} \right) \delta' \quad (5)$$

with  $\delta'$  the derivative of the Dirac delta function, has been used in the drift sections by considering a transversally uniform bunch density with circular cross section  $a$  and a circular beam pipe of radius  $b$ . This wake field is supposed to be the main cause leading to the MBI [9].

05 Beam Dynamics and Electromagnetic Fields

Once the expression of the operator  $H$  is known for each device of the bunch compressor, we can write the explicit solution of eq. (2). In order to do that, we observe that any expression of  $H$  of the kind of eq. (3) can be always written as the sum of the beam transport matrix without collective effects  $A_0$ , and the non linear term due to the wake field, that is  $H = A_0 + F(z)\partial/\partial\delta$ .

With this assumption, the operator  $H$  consists of two parts that we decouple by using the operator splitting technique

$$e^{H\Delta s} = e^{\frac{1}{2}\Delta s A_0} e^{\Delta s F(z)\frac{\partial}{\partial\delta}} e^{\frac{1}{2}\Delta s A_0} \quad (6)$$

Such a decoupling realizes a kind of interaction picture in which the collective effects are separated from the ordinary transport part whose action on the beam distribution function is known exactly. We have indeed

$$e^{\frac{1}{2}\Delta s A_0} \Psi_0 = \Psi \left( R \left( -\frac{\Delta s}{2} \right) X_0 \right) \quad (7)$$

with  $R(s)$  the transport matrix of the element and  $X_0$  the initial coordinates vector. The action of the non linear part can be evaluated in an analogous way and it is readily understood since the exponential operator accounting for its effect is just a shift operator in the coordinate  $\delta$ , providing a translation of the same coordinate by  $\Delta s F(z)$  in the distribution function, i. e.

$$e^{\Delta s F(z)\frac{\partial}{\partial\delta}} \Psi(z, \delta, x, \theta) = \Psi(z, \delta + \Delta s F(z), x, \theta) \quad (8)$$

This is the way we transport the distribution function through the compressor devices in our V-S.

## NUMERICAL RESULTS

The simulation code TEO [3] was upgraded to the 4D domain. The initial e-beam distribution  $\Psi$  was sampled on a uniform dense Cartesian grid  $(z_i, \delta_j, x_k, \theta_l)$ . The beam was then advanced by a discrete step  $\Delta s$  along the beam line using the operator method described in the previous section. For each grid point the advanced distribution  $\Psi'$  was obtained by numerical interpolation of the distribution  $\Psi$  at the beginning of the step using the method of local characteristics. Namely,  $\Psi'(z_i, \delta_j, x_k, \theta_l) = \Psi(\hat{z}_i, \hat{\delta}_j, \hat{x}_k, \hat{\theta}_l)$ , where the origin of the characteristic  $\hat{z}_i, \hat{\delta}_j, \hat{x}_k, \hat{\theta}_l$  was obtained by applying the exponential operator as in (8). Lagrange polynomials of the 5<sup>th</sup> order were used for the 4D off-grid interpolation of the distribution.

The first tests of the Vlasov solver have been performed in order to verify if the 4D dynamics of the simple transport without wake fields is well represented. In Fig. (1) we have used a magnetic bunch compressor with parameters close to those of SPARX BC2 [10] and have compared the transverse emittance with that obtained with the transport matrix method. The agreement is excellent. Also with the longitudinal dynamics there is total agreement. One problem arising in the use of a Vlasov solver is related to the high resolution of the grid necessary to evidence the MBI.

D05 Code Developments and Simulation Techniques

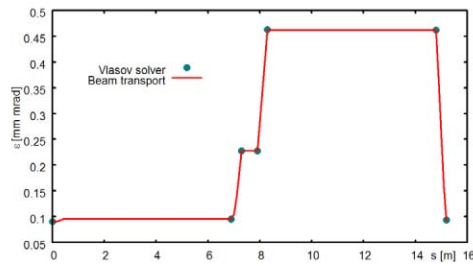


Figure 1: Emittance comparison between the Vlasov solver and the beam transport matrix.

If we consider, for example, the longitudinal phase space at the entrance of the compressor, as shown in Fig. (2), we can see that there is a strong correlation between the variables  $z - \delta$ . The same happens in the transverse plane. Using an uniform orthogonal grid, a large fraction of the mesh points ( $> 90\%$ ) corresponds to empty regions in the phase space, and therefore represents a waste of memory and computational resources. Studying the MBI in 2D (longitudinal phase space only) we have found that grids of the order of  $1000 \times 500$  nodes were needed in order to follow the instability at short wavelengths, say  $10 \mu\text{m}$  and below. In 4D a very coarse grid with a hundred points per direction correspond to  $100^4$  mesh nodes, and requires typically 2 GB of memory space at runtime. Preliminary investigations show that high resolution in the longitudinal phase space  $z - \delta$  calls for a comparable resolution in the transverse space  $x - \theta$ , because the two spaces are strongly coupled by the dynamics. It is clear that increasing by a factor 10 the grid resolution is not feasible, since it would correspond to a factor of  $10^4$  increase in the computational resources. Presently we are considering two options for addressing this problem: the distribution support could be remapped to a uniform orthogonal space using a coordinate transformation similar to that presented for a 2D solver in [4], or the phase space could be divided into parallel slices which closely follow the distribution support. The first option would require a more complex redefinition of the Hamiltonian operator, and could introduce numerical noise due to the remapping of the longitudinal coordinate. Therefore we are currently implementing the second option which requires just a careful book keeping of grid, without any changes to the computational kernel.

On the basis of the considerations just presented, it is worthwhile to note that Vlasov solvers for particle accelerator problems require large computational resource and, at least with present computers, it seems very hard to address the full 6D problem at a resolution high enough to resolve the CSR dynamics at very short wavelengths.

## CONCLUSIONS

We have discussed an accurate analysis of the microbunching instability. Macroparticle codes suffer by intrinsic numerical noise problems, while the possible limita-

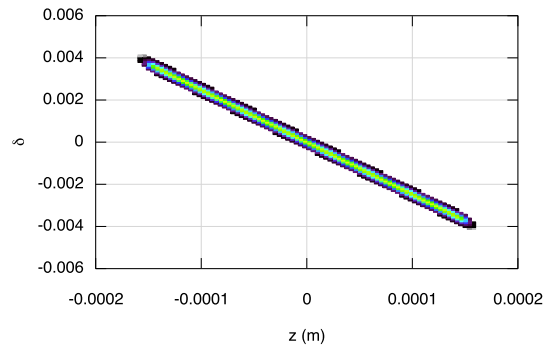


Figure 2: Longitudinal phase space at the entrance of the compressor. Linear color scheme.

tions of the V-S method is due to the grid number of points which may become prohibitively large with many useless points, since the phase space is strongly correlated. The essence of the instability itself is such a correlation, which gives rise to bunching and hence to coherent emission. The MBI dynamics share strong analogies with the FEL dynamics itself, whose final effect on the beam is that of creating a large (uncorrelated) energy spread. The interplay between FEL and saw-tooth type instabilities (a different flavour of MBI) has shown that they are competing effects [11], therefore MBI dampers based on FEL heater devices have been proposed [9]. A quantitative understanding of the MBI will therefore provide elements to design such a tool to inhibit the growth of the instability.

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