

# A KINETIC MODEL OF MULTIPACTION FOR SRF CAVITIES FOR ACCELERATOR DRIVEN SUB-CRITICAL SYSTEM (ADSS)\*

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## Abstract

This work simulates multipaction in a 700 MHz elliptical SRF cavity. The cavity design is optimized using SUPERFISH. Then the electromagnetic field is re-computed with FEMLAB, a package that uses the finite element method, to obtain a more accurate field-mapping, and to make the field values available for computation of multipaction. In the multipacting subroutine, electrons are assumed to be released into the system from various points with different initial parameters. The electron trajectories are tracked until they hit the cavity surface. Leap-frog scheme is used to solve the Lorentz force equation for primary electrons, as it is easy to use and is accurate up to the second order. The position, velocity, phase and kinetic energy of primary electrons at each time step are calculated and stored. An interpolation function is used to calculate secondary emission yield (SEY) at different impact energies. With the emission of secondary electrons, their trajectories too are tracked along with primary electrons, in order to identify parameters responsible for multipaction. By repeating this process for large number of electrons, the multipacting trajectories and field levels are identified.

## INTRODUCTION

The performance of superconducting cavities, couplers and ceramic windows might be greatly affected due to multipacting. Under the influence of a radio-frequency (RF) field, electrons are accelerated and repeatedly impact the walls of the cavity. Primary electrons impinging on the wall with appropriate kinetic energy may release secondary electrons from the walls. The secondary emission yield ( $\delta$ ) is defined as the number of secondary electrons generated per incident electron. If  $\delta > 1$ , an avalanche of electrons may be created inside the structure. This can lead to electric breakdown or even thermal breakdown of a superconducting structure.

The Accelerator and Pulse Power Division of BARC is involved in the development of superconducting cavities for Accelerator Driven Sub-critical System (ADSS). A high current proton linac (1 GeV, 30 mA) is an integral part of ADSS [1]. RF superconducting elliptical cavities will be used to accelerate protons up to 1 GeV. At such high field levels, it is necessary to design the cavity so as to minimize multipacting. At IIT Kanpur, a numerical code has been developed to investigate the occurrence of multipacting in a superconducting cavity. The code is validated by using it to compute electron trajectories within a TESLA cavity, and comparing with published

results. This code is then applied to a single cell prototype cavity designed by BARC which is operated at 700 MHz. Multipacting trajectories and field levels are identified through simulation studies.

## NUMERICAL COMPUTATION

At first, the prototype 700 MHz cavity is designed using 'SUPERFISH', which is a 2D finite difference based simulation code. For ease of integration with the multipacting subroutine designed in-house, the electromagnetic field was recalculated using the finite element based 'FEMLAB'. The axial symmetry of the cavity structure was used to reduce computation. The problem geometry was plotted using a combination of exact equations for line segments, arc of ellipse, etc as this would help in identifying points on the cavity surface with greater accuracy. As the field is highly non-uniform in certain regions, third order basis elements are selected; a highly refined finite element mesh is used to obtain an accurate field map. Finally, the fields obtained from 'SUPERFISH' and 'FEMLAB' are compared.

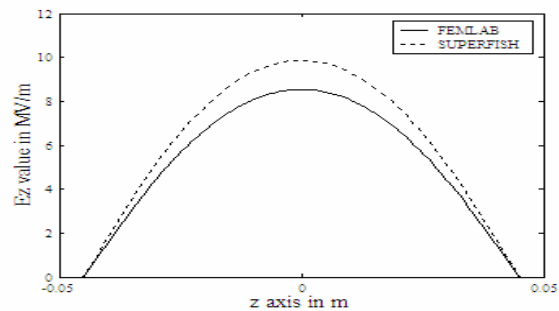


Figure 1: Axial electric field values along the beam axis computed by SUPERFISH and FEMLAB.

Figure 1 shows the  $E_z$  vs.  $z$  plot, where  $z$  is the beam axis of the cavity obtained and  $E_z$  the electric field in the direction of the  $z$ -axis. It is evident that the field plots obtained from both FEMLAB and SUPERFISH are close, except at peak  $E_z$  values. The finite element based computation is expected to give better field resolution in regions of high non-uniformity.

Once the field map is obtained, the trajectory described by the electrons under the influence of the electromagnetic field is generated. Leap Frog scheme is used to integrate the equations of motion for electrons. This method is used because it is simple to implement and it gives accurate results for second order equations. Owing to the high frequencies involved, it is imperative that a small enough time-step be chosen for convergence of solution. Especially, for H-field dominated regions, a

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very small time step of the order  $10^{-12}$  becomes necessary. For E-field dominated regions, larger time steps suffice and thus help in reduction of computation time.

For a chosen field level, electrons are launched within the cavity from different points on the cavity wall and are tracked until they impinge on the cavity wall. If the electron hits the cavity wall at a proper phase angle, then the electron is tracked again for next impact. Each electron is tracked for 20 impacts. The final impact energy and phases are calculated after each impact. At each impact, the secondary yield emission is computed, based on the secondary emission yield (SEY) graph.

### Code Verification

For the verification of the developed code, a TESLA cavity is analysed and the trajectories verified with those reported to have been obtained from Multipac, a simulation software toolbox for analyzing electron multipacting in axi-symmetric structures [3].

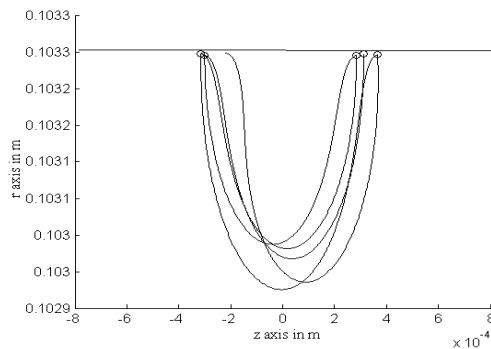


Figure 2 : Electron trajectories in a TESLA cavity

Figure 2 shows trajectories in a TESLA cavity when peak electric field is 47.5MV/m. The trajectories shown above are similar to first order 2 point multipaction trajectories, but the impact energy is below multipacting range. The trajectory shapes are similar to the one calculated by the Multipac program [3]. The dimensions of the trajectories are almost same in both cases. The average impact energy calculated by our code is 34.49 eV whereas that calculated by Multipac is 31.8 eV.

Calculations are carried out at several field levels. For each field level, starting electrons are tracked until they impinge on the wall. Depending on their energy at impact, the secondary emission yield (SEY) is calculated from the SEY vs. Impact energy graph for Niobium shown in Fig.3, and taken from [3].

Two counter functions are defined and updated as the computation proceeds [3]. The primary counter function counts the total no of primary electrons tracked and the secondary counter function tracked the total no of secondary electrons released. If for a particular field level, secondary counter function is largely greater than primary counter function we assume multination is likely to occur in the structure at that particular field level.

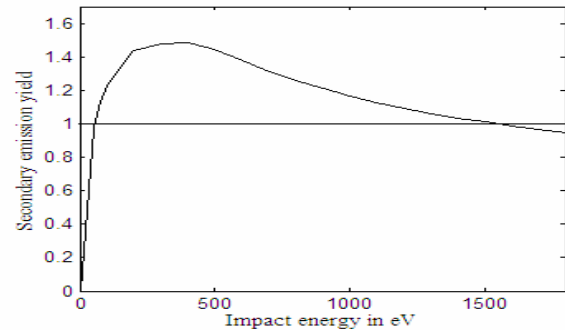


Figure 3 : Secondary emission yield curve

## SIMULATION RESULTS

The cavity under analysis is an elliptical cavity designed for 700 MHz with  $\beta=0.46$ . In reality, computations showed that the actual eigenfrequency of the cavity is 708.27 MHz.

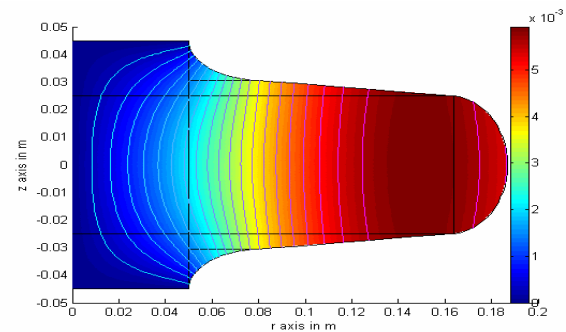


Figure 4: Cavity with magnetic field plot (Hphi in A/m)

Figure 4 shows the 700 MHz cavity structure along with magnetic field (Hphi) plot where magnetic field is lowest at the axis and highest near the equator. In the leap frog scheme, the computational time-step plays a crucial role in achieving solution convergence, especially in regions of non-uniform field.

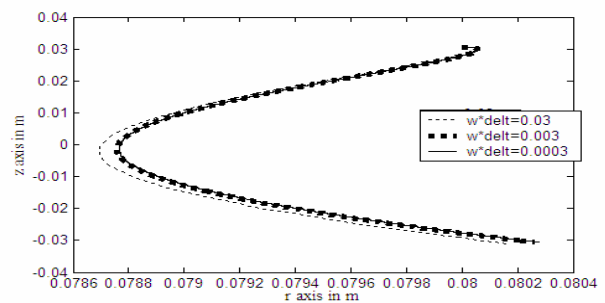


Figure 5: Trajectory computation as a function of time step: delt is the computational time-step

Figure 5 shows trajectory plots in regions of high magnetic field, computed with different time step ( $\Delta t$ ) and angular frequency ( $\omega$ ) combinations. Since, for all the above time-steps, the solution is seen to converge, the largest among them ( $6.82e-12s$ ) is selected for calculation in order to reduce computation time.

Cavities may not be prone to multipacting at all field levels. In order to find the multipacting-prone levels, the analysis must be repeated at several field levels. We choose field levels from 0.5 MV to 10 MV. Enhanced counter functions normalised with respect to the primary counter function, are calculated and plotted as a function of field level  $E$ . If the normalised counter function is greater than 1, then it indicates that multipaction occurs at that particular field level.

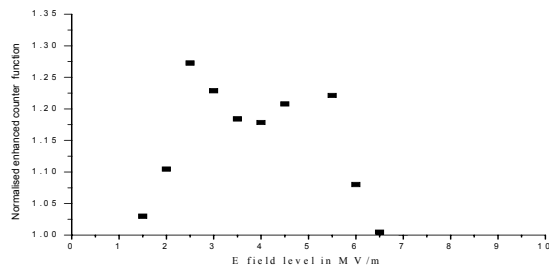


Figure 6: Normalised enhanced Counter Function for various field levels.

Figure 6 shows the values of the normalised enhanced counter function at several field levels. Multipacting is seen to occur at lower field levels ranging from 1.5MV to 6.5MV. Maximum multipaction occurs at  $E=2.5MV/m$ . Other multipacting peaks are also observed in 5.5 MV/m. The cavity may therefore be diagnosed to be prone to mutipacting at several field levels. In case of 2 point multipacting, order of multipaction denotes the number of half cycles the particle takes to impinge the wall. The analysis indicates that multipacting trajectories are resonant, i.e. time taken to impinge is half integer multiples of the RF periods. The order of multipaction achieved is seen to be different for different field levels. Usually the order of multipaction decreases with increase in field level. This is evident in Fig.7, which shows the number of full cycles required for impact by multipacting electrons at different field levels. Near the cavity equator, 2 point multipacting is seen to occur. However, evidence of 1 point multipacting was not found.

Figure 8 shows a 2 point multipacting trajectory at field level  $E=5$  MV/m for 4 consecutive impacts. The electron repeats its trajectory after every three half cycles. Also, the dependence of the initial kinetic energy of electrons on its ability to produce multipacting is studied. Figure 9 shows that the trajectory shape is independent of initial kinetic energy, and trajectory dimensions differ only slightly.

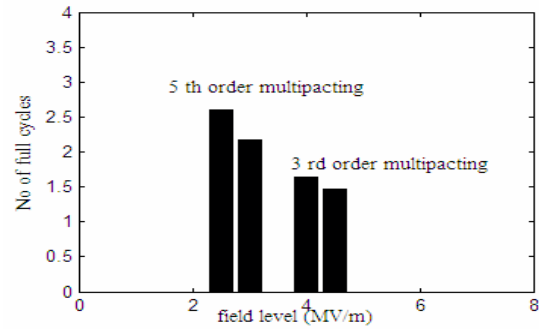


Figure 7: Number of full cycles required for impact at different field levels.

### CONCLUSIONS

In this work, a code has been developed in-house to study mutipacting in a cavity. The code was verified by analysing a TESLA cavity and comparing the results with published literature. The code was then used to test an RF cavity designed for 700 MHz. With the developed code, we were able to conclude that the designed cavity is likely to be multipacting-prone. It was also possible to identify the multipacting sites and field levels, as well as the order and type of multipacting expected.

As a next step, the requisite cavity will be redesigned to make it multipacting-free as far as possible. This code may also be extended for 3 dimensional cavity structures and other RF components like couplers, windows etc.

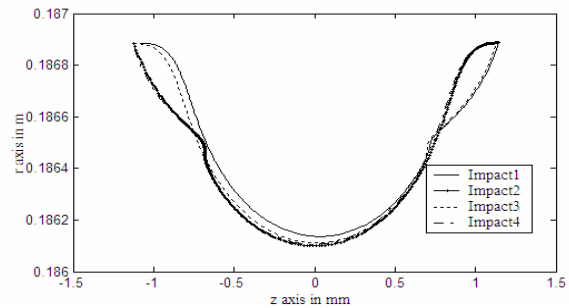


Figure 8: Trajectory plots for successive impacts

### REFERENCES

- [1] R. C. Sethi, BARC internal report (2000).
- [2] G Devanz, Proc. 7 th Eur. Part. Accel. Conf, European Physical Society, Geneva, 2000, p, 1366.
- [3] P.Yla Oijala, Jani Lukarinen, Seppo Jarvenpaa and Marko Ukkola, 'Multipac 2.1 manual'
- [4] W.Hartung, F Krawczyk and H Padamsee, SRF Conference, 2001, KEK.
- [5] H. Padamsee, Cornell University Ithaca, New York, "RF Superconductivity for Accelerators", Wiley-Interscience Publication (1998).
- [6] A. Roy, J Mondal, K. C. Mittal and R C Sethi, INPAC internal report, 2005.