NUMERICAL STUDIES OF RESISTIVE WALL EFFECTS

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Abstract

In this paper we describe a new numerical code to calculate wake fields of resistive wall geometries. Our code is based on conformal implicit scheme. It allows to estimate wake fields of very short bunches taking into account transient resistive effects neglected in the European XFEL impedance budget so far.

INTRODUCTION

The achievement and preservation of the very small electron beam emittance with high peak current is one of the most actual challenges in modern accelerator for fundamental and applied sciences to reach the design goals of the projected facilities [1,2]. The physics of high energy small emittance electron beams is basically dominated by the interaction of the beam with surrounding structure through the excited electromagnetic fields [3]. These fields, known as the wake fields, have in general the transverse and longitudinal components which produce the transverse kick and extra voltage for the trailing charges in the beam. The analytical solutions for the wake fields are available for the structure with relatively simple geometry [4,5].

The real structure, that can include cavities, transitions, collimators, bellow etc, in general has a complicated geometry and composed of resistive material. Various Maxwell grid equation (MGE) based numerical codes have been developed to solve the 2D and 3D wake field problems in frequency and time domains [6-7] but usually without resistive wall losses. From existing numerical codes only CST Microwave Studio [8] can model structures with finite resistivity but the algorithm suffers from the numerical dispersion. To prevent the numerical dispersion in longitudinal direction, the dispersion-free numerical scheme is proposed, for example, in [9].

Based on it a new (longitudinally) dispersion-free algorithm is developed to evaluate the wake fields in structures with finite wall conductivity. The impedance boundary condition in this scheme is modelled by the one dimensional wire connected to boundary cells. A good agreement of the numerical simulations with the analytical results is obtained. The developed code allows to calculate wake fields of arbitrary shaped geometries with walls of finite high conductivity.

FORMULATION OF THE PROBLEM

Consider the ultrarelativestic charged particle bunch with longitudinal distribution ρ, moving along the azimuthally symmetric structure with the speed of the light c (Fig.1). The internal region of the structure Ω is bounded by the resistive infinite wall with conductivity κ. The problem is to calculate the electromagnetic fields $\vec{E}, \vec{H}$ induced by the bunch. It reads

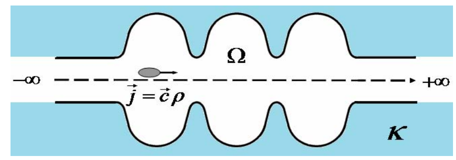

Figure 1: Charged particle moving through an accelerating structure supplied with infinite pipes.

$$\nabla \times \vec{H} = \frac{\partial}{\partial t}\vec{D} + \vec{j}, \qquad \nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B},$$
$$\nabla \cdot \vec{D} = \rho, \ \nabla \cdot \vec{B} = 0, \qquad \vec{H} = \mu^{-1}\vec{B}, \vec{D} = \varepsilon\vec{E},$$
$$\vec{E}(t=0) = \vec{E}_0, \quad \vec{H}(t=0) = \vec{H}_0. \tag{1}$$

The current density is $\vec{j} = \vec{j}_b + \vec{j}_c$, where $\vec{j}_b = \vec{c}\rho$ is the charge current and $\vec{j}_c = \kappa\vec{E}$ the current induced in the wall. The boundary conditions for electromagnetic fields are given by the continuity of tangential components of electric and magnetic fields on the boundary between the vacuum and the wall.

In accelerator applications, the studied structure is usually supplied by ingoing pipe and the well known analytical solution for ultra-relativistic beam in a perfectly conducting cylindrical pipe [4] can be used as initial field.

Let us consider an incident plane EM wave from vacuum on the conductor surface (fig. 2). For conducting media the relation between the incident angle and the transmitted one can be written in the form of Snell's law

$$\sin\phi_t = \frac{1}{n(\phi_0, \omega, \kappa)}\sin\phi_0 \tag{2}$$

where ϕ_0 and ϕ_t are the incident and transmitted angles, correspondingly, ω is the frequency of incident wave, n is a function of conductivity κ, angle ϕ_0 and frequency ω. The exact form of function $n(\phi_0, \omega, \kappa)$ can be found in [10]. For $\kappa >> \varepsilon_0\omega$ the incident waves propagating not parallel to the boundary surface are transmitted perpendicular to the boundary surface ($\phi_t \sim 0$), i.e. only tangential components of electric and magnetic fields survive in the conducting media.

In accelerators the spectrum of EM fields excited by the bunch interaction with surrounding structure extends to the frequencies $\omega_b \sim c/\sigma$, where σ is the rms bunch length. The vacuum chambers usually are made from materials with high conductivity and the condition $\kappa >> \varepsilon_0\omega$ is satisfied. As an example, for stainless steel ($\kappa = 1.4 \cdot 10^6 \,\Omega^{-1} m^{-1}$) and for a bunch with rms length of 25 μm, $\kappa / \varepsilon_0\omega \sim 10^4$.

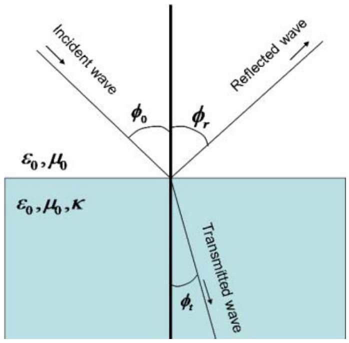

Figure 2: Transmission and reflection of EM wave on vacuum-conductor boundary surface.

Hence, in media with high conductivity only tangential components of the fields should be taken into account and they has to be coupled to the full three dimensional field in the vacuum.

In vacuum the full three dimensional grid space is used. Propagation of the tangential components of each boundary cell in the conducting media is described by one dimensional (1D) set of Maxwell's equations. The numerical algorithm based on this model is given in the next section.

A SCHEME WITH CONDUCTIVITY

In this section we describe an implicit algorithm for electromagnetic fields computation that includes the boundaries with finite conductivity. The propagation of the tangential field in the conductor is described by 1D electromagnetic problem discretized as shown in Fig.3.

The excitation source of EM field in conducting media is the tangential field in boundary cell. Following the matrix notation of the finite integration technique (FIT) [11] the implicit 1D scheme reads

$$\widehat{\mathbf{e}}_s^{\,n+1} = \mathbf{A}\mathbf{e}_s^{\,n} + \mathbf{B}\mathbf{P}_s \frac{\widehat{\mathbf{h}}_s^{\,n+1} + \widehat{\mathbf{h}}_s^{\,n}}{2}$$

$$\widehat{\mathbf{h}}_s^{\,n+1} = \widehat{\mathbf{h}}_s^{\,n} + \Delta\tau \mathbf{P}_s^{*} \frac{\widehat{\mathbf{e}}_s^{\,n+1} + \widehat{\mathbf{e}}_s^{\,n}}{2} \qquad (3)$$

The two-banded matrix $\mathbf{P}_s$ plays the role of discrete differential operator. The matrices $\mathbf{A}$ and $\mathbf{B}$ are diagonal with entries

$$a_{ii} = e^{-\kappa Z_0 \Delta\tau}, \; a_{00} = \mathrm{e}^{-0.5\kappa Z_0 \Delta\tau}, b_{ii} = \frac{(1 - a_{ii})}{\kappa Z_0}. \qquad (4)$$

The boundary conditions at the interface read

$$\widehat{e}_{sN}^{\,n} = 0, \; \widehat{h}_{s0}^{\,n} = \widehat{h}_{s0}^{\,n+1} = \widehat{h}_{\varphi}^{\,n+0.5}, \qquad (5)$$

where $\widehat{h}_{\varphi}^{\,n+1/2}$ is magnetic field in the vacuum cell to which belong the wire.

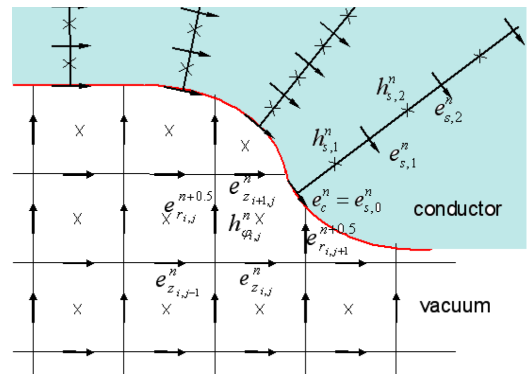

Figure 3: Vacuum grid with 1D conducting lines at the boundary.

The implicit scheme in vacuum region reads

$$\widehat{\mathbf{e}}_r^{\,n+0.5} = \widehat{\mathbf{e}}_r^{\,n-0.5} - \Delta\tau\, \mathbf{M}_{\varepsilon_r^{-1}} \mathbf{P}_z^{*} \widehat{\mathbf{h}}_{\varphi}^{\,n},$$

$$\widehat{\mathbf{h}}_{\varphi}^{\,n+0.5} = \widehat{\mathbf{h}}_{\varphi}^{\,n} + \frac{\Delta\tau}{2} \mathbf{M}_{\mu^{-1}} \left[\mathbf{P}_z \widehat{e}_r^{\,n+0.5} - \mathbf{P}_r \widehat{e}_z^{\,n} + \widehat{\mathbf{e}}_c^{\,n} \right],$$

$$\mathbf{W} \frac{\widehat{\mathbf{e}}_z^{\,n+1} - \widehat{\mathbf{e}}_z^{\,n}}{\Delta\tau} = \mathbf{M}_{\varepsilon_z^{-1}} \left[\mathbf{P}_r^{*} \widehat{\mathbf{h}}_{\varphi}^{\,n+0.5} + \widehat{\mathbf{j}}_z^{\,n+0.5} \right]$$

$$\widehat{\mathbf{h}}_{\varphi}^{\,n+1} = \widehat{\mathbf{h}}_{\varphi}^{\,n+0.5} + \frac{\Delta\tau}{2} M_{\mu^{-1}} \left[\mathbf{P}_z \widehat{e}_r^{\,n+0.5} - \mathbf{P}_r \widehat{e}_z^{\,n+1} + \widehat{e}_c^{\,n+1} \right], \qquad (6)$$

where $\mathbf{W} = \mathbf{I} + \frac{\Delta\tau^2}{4} \mathbf{M}_{\mu_{\varphi}^{-1}} \mathbf{P}_r \mathbf{M}_{\varepsilon_r^{-1}} \mathbf{P}_r^{*}$ and $\widehat{\mathbf{e}}_c^{\,n}$, $\widehat{\mathbf{e}}_c^{\,n+1}$ are the voltages at the conductive boundary.

The stability condition of the above introduced scheme is $\Delta\tau \le \Delta z$. With the time step $\Delta\tau$ equal to longitudinal mesh step Δz , the scheme does not have dispersion in longitudinal direction. The transverse mesh and mesh step in conductor can be chosen independently from stability considerations.

The above scheme can be easily generalized for higher order azimuthal harmonics and 3D case. Such generalization and an explicit variant of TE/TM scheme will be published in [12,13].

NUMERICAL EXAMPLES

As the first test we calculate the steady state wake of the Gaussian bunch with rms length σ=1mm in round pipe of radius a=1 cm and of conductivity κ=1e5 S/m. To obtain the steady state solution we have calculated 2m of the pipe and subtracted the wake of the first meter. Fig. 4 shows convergence of the loss factor to the analytical value 1.31 V/pC. Fig. 5 compares the analytical and the numerical wakes for mesh resolution of 10 points on σ. In this case the error in loss factor is about 3%.

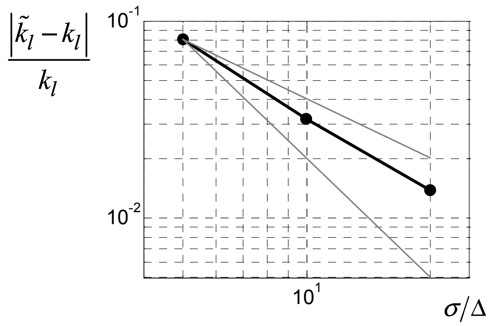

Figure 4: Convergence of the loss factor.

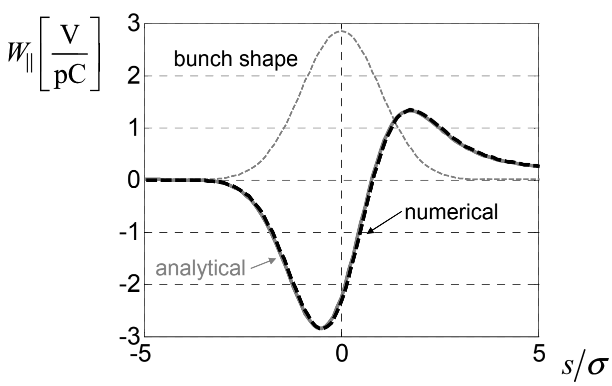

Figure 5: Comparison of numerical and analytical wakes.

As the second test we calculate a wake of finite length resistive cylinder. It has radius a=1cm, length b=10cm and conductivity κ=1e4 S/m. For the Gaussian bunch with σ=0.025 mm the analytical results of the paper [14] could be used. The loss factor reads

$$k_l = \frac{cZ_0 g \Gamma(3/4)}{4\pi^2 a\sqrt{2\kappa Z_0} s_g^{3/2}} K_l\left(\frac{\sigma}{s_g}\right), \; s_g = \sqrt{\frac{g}{2Z_0\kappa}}, \qquad (7)$$

where function K_l is given by Eq. (5.3) from [14].

Fig. 6 shows the numerically obtained wake (black dashed line) and the analytical steady state wake [3, 4, 15] (solid gray line). The numerically obtained loss factor is equal to 58 V/pC and coinsides with that given by Eq.(7) (57 V/pC). The steady state loss factor is equal to 16 V/pC and underestimates the energy loss.

Third test is wake potential calculation of tapered collimator (Fig.7) with parameters a_1=17mm, L_1=200 mm, a_2=10mm, L_2 =100mm, a_3=6mm and conductivity κ=1e4 S/m. For a Gaussian bunch σ=50 µm numerically obtained loss factor for conductive walls (270 V/pC) is two times larger than for perfectly conducting walls (133 V/pC) and cannot be obtain as direct sum of the geometrical and the steady-state solution.

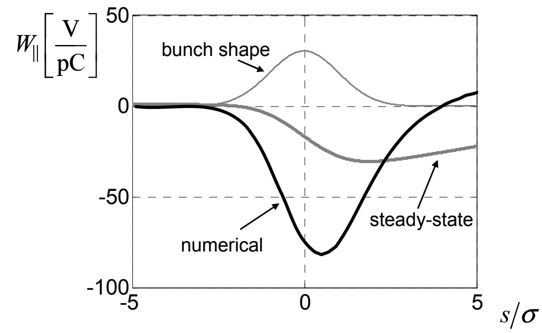

Figure 6: Comparison of transient and steady-state wakes.

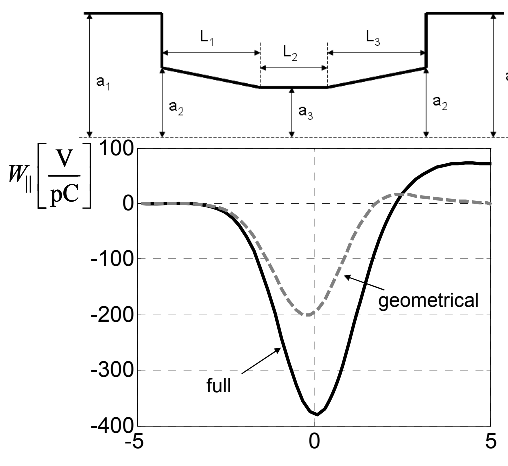

Figure 7: Comparison of wakes “with” and “without” resistivity.

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