# LOAD CURVES DISTORTION INDUCED BY FRINGE FIELDS EFFECTS IN THE ION NANOPROBE

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*Abstract*

Nanoprobes are known to be high precision systems, which require preliminary modeling for thorough analysis of optimal working modes. One of most crucial characteristics of the special class of such beam lines is the so-called load curves (or surfaces). This paper investigates one of the types of intrinsic effects, i.e. fringe fields and their influence on load curves and surfaces, which make it possible to construct the purposeful search of optimal working regimes for nanoprobes. A number of different models for fringe field presentation are discussed in the paper. Analytical and numerical methods and tools are used for analysis and selection of optimal parameters for fringe field models.

## INTRODUCTION

Methodology of construction nanoprobes which ensures desired beam characteristics are given in [1, 2]. These papers deals with ideal piecewise continuous presentation of magnet fields, in another words in similar cases fringe field effects are often neglected. On the first steps, described in [1, 2] Sometimes this is a quite good approximation. But, it is well known that the piecewise model for field distribution dots not satisfy Maxwell's equations. That is why in this paper we consider influence of the fringe field effects. In many papers the problem of fringe field effects is studied in nonlinear approximation (see, for example, [4]). Unfortunately it is not possible to study influence of the fringe fields based only on experimental data. That is why we have to use some model approximations for the fringe fields. The corresponding functions should allow to approximate the experimental data for corresponding types of lenses. In this paper we consider several models of the fringe fields description in order to approach to real magnet fields in practice. A few optimal variants of a nanoprobe are considered in [1, 2] are used for investigation of fringe field effects. Here we use parameters of nanoprobe systems, which can be obtained in according to a methodology, described in [1, 2].

## MATHEMATICAL MODELS OF FRINGE FIELDS

This paper deals with investigation of relative fringe fields length and form influence on so-called "load curves" and on some beam characteristics. This influence is demonstrated using Kiev's nanoprobe parameters [3]. The selection of model functions is determined either experimental data for real lenses or fringe field forms, which have minimal effects.

### *Solution of Motion Equations*

In linear approximation one can write particle motion equations for quadrupole [1] in the form of

$$\begin{cases} x'' + k(s)x = 0, & x' = dx/ds, \\ y'' - k(s)y = 0, & y' = dy/ds. \end{cases} \quad (1)$$

These scalar equations can be rewritten in the vector form

$$\frac{d\mathbf{X}(s)}{ds} = \mathbb{P}(s)\mathbf{X}(s), \quad (2)$$

with $\mathbf{X}_0 = \mathbf{X}(s_0)$ as an initial data. The solution of (1) can be written using matrizant [1]

$$\mathbf{X}(s) = \mathbb{R}(s|s_0)\mathbf{X}_0. \quad (3)$$

Full matrizant for the nanoprobe $\mathbb{R}(s_N|s_0)$ could be presented using group property as the multiplication of partial matrizants corresponding to individual pieces of the whole system

$$\mathbb{R}(s_N|s_0) = \prod_{k=1}^{N} \mathbb{R}(s_k|s_{k-1}) \quad (4)$$

where $s_0$ and $s_N$ — initial and final values of variable $s$, which is measured along some trajectory. Equation (4) is known as exact presentation for any $N$ and does not depend on fragmentation way and satisfied Cauchy task.

### *Load Curves Construction*

Following [1] the full matrizant of the system $\mathbb{R}(s_N|s_0)$ could be written as $\mathbb{R}(s_N|s_0) = \mathbb{R}_g\mathbb{M}\mathbb{R}_a$, where the full beam line consists of "pre-distance" with the length $a$, focusing component — "objective" and "working distance" with the length $g$. As the additional condition we append "from point to point" term, which means $r_{12} = 0$, where $r_{12}$ — element of the full matrizant $\mathbb{R}(s_N|s_0)$. Using this condition it is possible to evaluate $g$. Load curve is the curve which satisfied the equation $m_{11} = m_{22}$, where $m_{ij}$ — elements of partial matrizant for the objective. As an example in this paper we consider the Kiev's nanoprobe system [3]. On Fig. 1 the load curve is presented as the dashed black line on Fig. 1, blue and red lines on it present curves correspond with 100 and 1000 times demagnification accordingly. Optimal solutions should belong to the load curve. On the Fig. 1 the best parameters could be found on the load curve and in exterior of set which is defined by blue or red line (against the demagnification).

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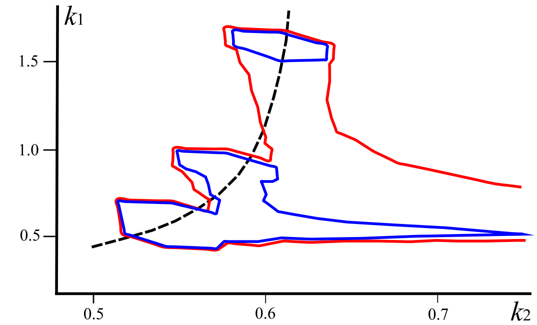


Figure 1: Example of load curve.

### *A Description of Fringe Fields Functions*

In this paper for necessary evaluations of the matrizant we use a piecewise approximation of a fringe field (see Fig. 2). This approach allows to use analytical formulae for matrizant for any model of the fringe field. Usually in

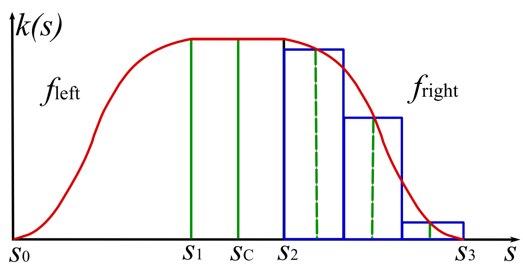


Figure 2: Piecewise continuous approximation of a fringe field.

practice $L_{\text{eff}}$ is used instead of the actual magnet length. This effective length could is introduced as

$$L_{\text{eff}} = \frac{1}{k_{\text{max}}} \int_{s_0}^{s_3} k(s)ds = \gamma L_{\text{iron}},$$

where $k_{\text{max}}$ — maximum focusing strength of the field, and $L_{\text{iron}}$ — real ("in iron") corresponding length of a magnet element.

Fringe fields are intrinsic effects in any type of beam forming and focusing systems. Usually every kind of beam lines has a lot of control elements, so it is necessary to take into account superposition of fringe fields for nearby magnets. This paper deals with ordinary sum of fringe fields functions without investigation of induced fields in nearby elements. We consider symmetrical fringe fields relatively the center $s_c = (s_2 - s_1)/2$ of each element in account of experimental data (see Fig. 2). Also $L_{\text{eff}}$ is supposed to be constant. Assuming fringe field as a piecewise function which can write

$$f(s) = k_{max} \begin{cases} f_{\text{l}}(s), & s \in [s_0, s_1), \\ 1, & s \in [s_1, s_2], \\ f_{\text{r}}(s), & s \in (s_2, s_3], \end{cases} \tag{5}$$

where $f_l(s)$ and $f_r(s)$ — left (input) and right (output) part of fringe field modeling function. In order to make $f(s)$ smooth in joints it is necessary to demand several additional conditions

$$\begin{gathered} f_{\text{l}}(s_0) = f_{\text{r}}(s_3) = 0, \quad f_{\text{l}}(s_1) = f_{\text{r}}(s_2) = f_0, \\ \frac{\mathrm{d}f_{\text{l}}(s_0)}{\mathrm{d}s} = \frac{\mathrm{d}f_{\text{l}}(s_1)}{\mathrm{d}s} = \frac{\mathrm{d}f_{\text{r}}(s_2)}{\mathrm{d}s} = \frac{\mathrm{d}f_{\text{r}}(s_3)}{\mathrm{d}s} = 0. \end{gathered} \tag{6}$$

One can use asymptotic conditions instead of (6) in order to consider broader class of modeling functions for fringe fields

$$\begin{array}{llll} \lim\limits_{s\to+s_0} f_{\text{l}}(s) & = 0, & \lim\limits_{s\to-s_1} f_{\text{l}}(s) & = f_0, \\ \lim\limits_{s\to+s_0} \dfrac{\mathrm{d}f_{\text{l}}(s)}{\mathrm{d}s} & = 0, & \lim\limits_{s\to-s_1} \dfrac{\mathrm{d}f_{\text{l}}(s)}{\mathrm{d}s} & = 0, \\ \lim\limits_{s\to+s_2} f_{\text{r}}(s) & = 0, & \lim\limits_{s\to-s_3} f_{\text{r}}(s) & = f_0, \\ \lim\limits_{s\to+s_2} \dfrac{\mathrm{d}f_{\text{r}}(s)}{\mathrm{d}s} & = 0, & \lim\limits_{s\to-s_3} \dfrac{\mathrm{d}f_{\text{r}}(s)}{\mathrm{d}s} & = 0. \end{array} \tag{7}$$

Using the experimental data and requirements (6) functions we can approximate left part (right part could be found after mirroring left part relative to $s_c$) of real fringe fields for example with some approximation functions below:

– $f_{\text{l}}(s) = A\sin(\nu s + \psi) + B$ — trigonometric one,
– $f_{\text{l}}(s) = As^3 + Bs^2 + Cs + D$ — polynomial one.
We also consider more complex approximation functions satisfied (7)
– $f_{\text{l}}(s) = 1/\left[1 + \exp\left(1/P_5(s)\right)\right]$, where $P_5(s)$ is a five degree polynomial $As^5 + Bs^4 + Cs^3 + Ds^2 + Es + F$.

Efficiency of function approximation could be estimated using two equivalent norms (see Fig. 2):

I. $\| f - g \|_C = \sup_{s\in[s_0,s_3]} | f(s) - g(s) |$,

II. $\| f - g \|_{L_2} = \int\limits_{s_0}^{s_3} \Big( f(s) - g(s) \Big)^2 \mathrm{d}s.$

### *Fringe Fields "Sewing" and Superposition*

Using conditions (6) or (7) we can construct smooth approximation of a quadrupole gradient distribution along the optical axis. This approach (see Fig. 3) is used in a special software for fringe field modeling.

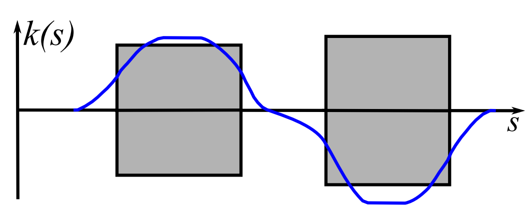


Figure 3: Example of fringe field superposition.

## LOAD CURVES WITH FRINGE FIELDS

The fringe field presentation and the corresponding beam line segmentation are presented on Fig. 2. The load curves should be be evaluated taking into account fringe fields. For this purpose the beam line have been separated on several parts. Each quadrupole could be splitted like on

Fig. 2. Using (4) full matrizant can be presented as a production of all partial matrizants. In this paper we exploit well-known matrizant presentation for a piece-wise model. "Height" of each rectangle step is defined by value of modeling fringe field function in the middle of correspondent interval.

## COMPUTER MODELING

A number of appropriate parameters of nanoprobe were received in [1, 2]. Using referred above methodology we could construct load curves for rectangle model in neighborhood of optimal parameters and with an allowance for fringe field effects. We can compare load curves behavior with and without fringe fields for parameters from [3].

### *A Load Curves Construction Algorithm*

1) Construct the load curve using ideal piecewise model.

2) Retrieve a few tens of points belong to load curve.

3) Split the real magnet field of each quadrupole lens on the left, center and right parts (see Fig. 2).

4) Approximate the left and right units with appropriate functions.

5) Divide the left and right parts into tens of intervals.

6) Compute partial matrizants for the left and right units using splitting like on Fig. 2 and the matrizant group property (4).

7) Calculate the full matrizant for objective using (4). The center part matrizant for each lens could be found exploiting well-known analytic matrizant for ideal piecewise model.

8) Construct the load curve for obtained objective matrizant point by point varying magnet excitations $k_1$ and $k_2$ (see Fig. 1) in neighborhood of retrieved points from the step 2.

In this paper we consider equal left and right parts of fringe field for each lenses as part of $L_{\text{eff}}$. We examined $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ parts of $L_{\text{eff}}$ for right and left fringe fields. It is not necessary to use the step 1 point of algorithm during fringe field growthing and it is required to retrieve points from previous step in step 2 to avoid redundant time and computational costs.

### *The Load Curves Distortion*

As an example we investigated parameters for Kiev nanoprobe [3]. On Fig. 4 we can see load curve change when increasing fringe field part of $L_{\text{eff}}$ from zero to $\frac{1}{8}$. Dashed line is corresponding load curve when left and right parts of fringe field are equal zero. Blue line conforms to $\frac{1}{32}$ part of $L_{\text{eff}}$ for left and right parts, red line — to $\frac{1}{16}$. On Fig. 5 dashed line is again conforms to load curve with zero fringe field parts, green line — to $\frac{1}{8}$ part of $L_{\text{eff}}$ for left and right fringe fields, blue line — to $\frac{1}{4}$ and red line to $\frac{1}{2}$.

In the beginning of load curve almost there is no impact of fringe fields (when $k_1 \le 0.5$ and $k_2 \le 0.5$), but while increasing $k_2$ one can see that load curves are deform to

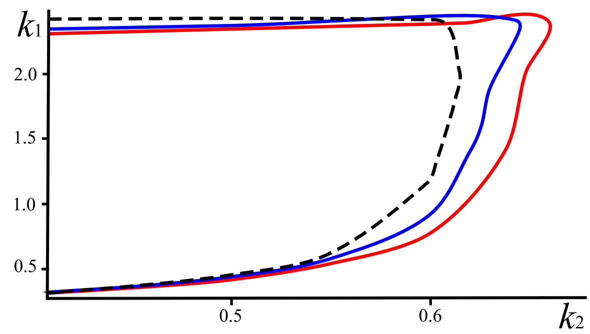


Figure 4: The load curves distortion for fringe field parts are equal $\frac{1}{32}$ and $\frac{1}{16}$.

right when fringe field part is growing from $0$ to $\frac{1}{2}$. Top part of load curves is turning down while fringe field parts is growing.

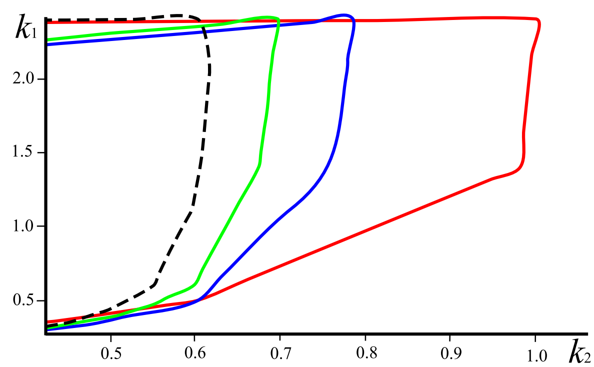


Figure 5: Load curves distortion with fringe field parts are equal $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$.

## CONCLUSION

As we can see using plots, fringe field effects could significantly change load curves. Optimal systems which parameters received without taking into account of fringe fields effects should be reviewed because of fringe field impact. Fringe field are the intrinsic effects so they have to be include in estimation of optimal nanoprobe parameters.

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