# LHC TRANSVERSE FEEDBACK DAMPING EFFICIENCY

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*Abstract*

A simulation model has been developed to predict the damping efficiency of the LHC transverse feedback system in the presence of coupled bunch instabilities and under realistic assumptions for the injection error. The model tracks both the centre of gravity of a bunch and the r.m.s beam size during and after injection. It includes the frequency characteristic of the transverse feedback system. Nonlinearities in the beam optics will cause the bunches to filament and lead to an increase of the transverse emittance after injection. The resistive wall instability reduces the effectiveness of the transverse feedback by slowing down the damping process. Possibilities for enhancing the performance of the feedback system by signal processing schemes are outlined.

## INTRODUCTION

The LHC, providing proton-proton collisions with two beams circulating in opposite direction, relies on entirely stable beams and preservation of the transverse beam size to ensure a high luminosity during Physics. The LHC transverse feedback system [1] is designed to damp dipole injection errors and provide feedback to cure coupled bunch dipole instabilities. At injection energy the rise time of the resistive wall instability, dominated by the impedance of the warm part of the machine, has been estimated at $18.5$ ms [2]. An injection error will quickly filament and lead to an increase in beam size without active damping. The estimation of the performance of the LHC transverse feedback system has previously relied on approximations of the filamentation process by an exponential law [3, 4]. In this approximation the overall damping time $\tau$ follows from

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm dec}} + \frac{1}{\tau_{\rm d}} - \frac{1}{\tau_{\rm inst}} \qquad (1)$$

where $\tau_{\rm d}$ is the damping time by active feedback, $\tau_{\rm inst}$ the rise time due to the resistive wall instability and $\tau_{\rm dec}$ the decay time constant of decoherence. For the specifications of the LHC transverse feedback system $\tau_{\rm d} = 3.6$ ms [1], $\tau_{\rm dec} = 68$ ms and an instability rise time $\tau_{\rm inst}$ of 14 ms were initially assumed. The overall damping time follows from Eq. (1) as $\tau = 4.5$ ms (50 turns).

## DECOHERENCE

In a perfectly linear machine, a single particle starting on a trajectory offset from the closed orbit, will continue to exercise transverse oscillations. A distribution of particles injected will maintain its r.m.s. size. The centre of gravity will continue to oscillate with constant amplitude about the closed orbit. In a real machine non-linearities in the lattice will eventually lead to decoherence. The dependence of the machine tune $Q$ on amplitude can be expanded into a power series

$$Q = \sum_{k=0}^{\infty} a_k r^k \ , \qquad (2)$$

with $r > 0$ representing the amplitude of oscillation. $k = 0$ represents the central tune in this model and $k = 2$ the octupolar term with a quadratic dependence of tune with amplitude. Sextupoles in a location with dispersion provide a change of tune with momentum. However, when the beam is centred, sextupoles do not provide a tune change with betatron amplitude. In the present simulation we combine the decoherence effects caused by octupoles and chromaticity with the active damping by the feedback. Other sources of decoherence are not considered.

In absence of feedback analytical formulas have been derived for the decoherence by chromaticity and octupoles [5, 6]. In the case of chromaticity alone and under the assumption of constant synchrotron frequency, full recoherence occurs after one synchrotron period. In practice octupoles and the fact that the synchrotron frequency itself depends on momentum prevents complete recoherence. When defining the parameters of the LHC transverse feedback system it has been assumed that all oscillations be damped during the initial decoherence and any effect of recoherence has not been taken into consideration. Note that the damping time aimed for is of the order of 1/4 of the synchrotron period.

At this point it is also worth noting that, taking into account a tune shift with amplitude, the decaying of the oscillation with the spiralling movement of particles in phase space is rather poorly described by an exponential law for the decay of the centre of mass motion. The exponential decay imposes a maximum reduction in oscillation amplitude at the very first turns, while according to the analytical expressions [6] the dependence with time follows a power law $-(t/T_0)^2$ for the first few turns. $T_0 = 88.9\ \mu$s denotes the revolution time. The decoherence becomes only visible when the outer regions of the bunches have sufficiently drifted away from the centre and start to deform the transverse bunch distribution. This is illustrated in Fig. 1. As in the derivation for the analytical expressions [6] we assume distributions in transverse and longitudinal phase space that are not correlated, i.e. particles are initially se-

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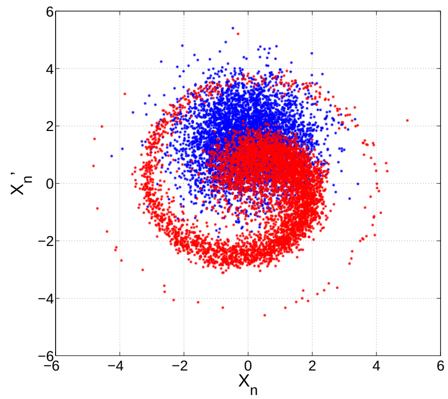


Figure 1: An injection error of $\Delta X'_{\rm n} = 1.5\,\sigma$ leads to a turn by turn filamentation of a bunch (blue, initial beam size $\epsilon_0$); an amplitude dependent tune spread of the individual particles is assumed. Filamentation after 750 turns is shown in red for $\mu = 10^{-4}$, $Q' = 2$, and $(\Delta p/p)_{\rm rms} = 0.44 \times 10^{-3}$. Without active damping the emittance increases to $\epsilon/\epsilon_0 = 2.125$.

lected to have randomly distributed transverse and longitudinal phase space coordinates. Gaussian distributions are used for both planes, longitudinal and transverse. Detailed analytical expressions for the decoherence of coherently kicked particles can be found in [5, 6].

## SIMULATION MODEL

The numerical model describes the dynamics of an ensemble of $10^4$ particles, and tracks their transverse phase space coordinates $x$ and $x'$ turn by turn. To simulate LHC injection scenarios all 288 injected bunches of the longest LHC batch ($4 \times 72$ bunches) and part of the circulating beam, $3 \times 72$ bunches, must be tracked at the same time to realistically predict emittance increase due to injection kicker ripple. Coordinates are normalised for tracking

$$X_{\rm n} = \frac{x}{\sigma_s};\ X'_{\rm n} = \frac{\alpha_s\, x + \beta_s\, x'}{\sigma_s} \tag{3}$$

where $\alpha_{\rm s}$ and $\beta_{\rm s}$ are the optics Twiss parameters at the position $s$, and $\sigma_{\rm s} = \sqrt{\beta_{\rm s}\epsilon_0}$ is the initial r.m.s. beam size. As a result, all particles with the same oscillation amplitude will describe circles in the $(X_{\rm n}X'_{\rm n})$-plane. The emittance for a particle with betatron amplitude equal to one $\sigma_{\rm s}$ is therefore defined by the unit circle

$$\left(X_{\rm n}^{(\sigma)}\right)^2 + \left(X_{\rm n}^{\prime(\sigma)}\right)^2 = 1\,. \tag{4}$$

The instantaneous emittance of the distribution of particles, also referred to as *statistical* emittance is given by

$$\frac{\epsilon}{\epsilon_0} = \frac{1}{2}\left(\left\langle X_{\rm n}^2\right\rangle - \left\langle X_{\rm n}\right\rangle^2 + \left\langle X_{\rm n}^{\prime 2}\right\rangle - \left\langle X'_{\rm n}\right\rangle^2\right)\,. \tag{5}$$

Centres of gravity are calculated at the LHC injection kickers (MKI), the two beam position monitors of the transverse feedback (PU1, PU2) and at the damper kickers (DK). In the case of an ideal feedback system, each bunch is corrected by a kick

$$\Delta X'_{\rm n} = g\tilde{X}_{\rm n}^{\rm PU12}\,. \tag{6}$$

$g = 2\,T_{\rm rev}/\tau_{\rm d}$ is an adjustable feedback gain and $\tilde{X}_{\rm n}^{\rm PU12}$ represents the combined signal of PU1 and PU2. In the actual feedback path there is however a frequency dependence with a roll-off of gain (6 dB/octave), $-3$ dB at 1 MHz. If a single bunch is oscillating and neighbouring bunches are perfectly on the closed orbit, the frequency dependence of the feedback gain will actually spread the oscillation to adjacent bunches before the feedback eventually damps down the oscillation completely. In practice, once the lower frequency contents of the injection error is damped, one can switch in a digital filter that boosts the high frequency gain to obtain an overall flat gain with frequency. The actual power amplifier gain versus frequency curve [1] is modelled in the simulation code using a 81 tap FIR filter (40 MHz samples).

For the tune dependence we follow the notation of [5, 6] and identify $a_0 = Q_\beta$, $a_1 = 0$, and $a_2 = \mu$ in (2) and the average Q on turn $k$ becomes

$$Q[k] = Q_\beta - \mu(r[k])^2 + Q'\delta[k]. \tag{7}$$

$\delta[k]$ is the turn dependent longitudinal relative momentum deviation from the synchronous particle. The full set of parameters used for the simulation are summarized in Table 1.

Table 1: Simulation parameters

| | | |
|---|---|---|
| Number of particles per bunch | $N_{\rm b}$ | $10^4$ |
| Number of bunches | $k_{\rm b}$ | $(72 \times 7)$ |
| Number of turns | $N_{\rm turn}$ | 2500 |
| Feedback gain | $g$ | 0.05 |
| Chromaticity | $Q'$ | 2 |
| Octupoles | $\mu$ | $10^{-4}$ |
| r.m.s momentum spread | $\Delta p/p$ | $0.44 \times 10^{-3}$ |
| Betatron tune | $Q_\beta$ | 59.31 |
| Synchrotron tune | $Q_{\rm s}$ | $5.5 \times 10^{-3}$ |
| Instability coupling factor [7] | $K$ | $1.187 \times 10^{-3}$ |

## SIMULATION RESULTS

Running the model with the ideal feedback system and an initial displacement of $3.3\,\sigma$ reveals that the value for the damping time of the feedback system $\tau_{\rm d}$ corresponds to the expected 3.4 ms or 39 turns. Enabling the FIR filter for the power amplifier drop of gain with frequency shows a slower damping ($\tau_{\rm d}$ = 7 ms, 79 turns) for the first and last bunches in the batch. This is due the fact that the transition from zero output signal to the required kick strength is limited by the rise time of the power amplifier, i.e. 10-90% rise time of 350 ns. With the actual phase compensating filter in place the damping rate is reduced to half its nominal value for the first and last bunches. The situation can be improved by interpolating in the signal processing between

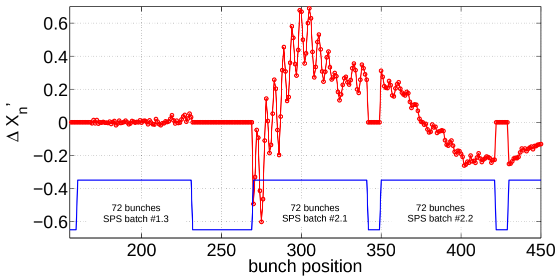


Figure 2: Residual kick error of the injection kicker which causes the injected bunches to oscillate. Signal processing in the feedback path can be used for interpolation between consecutive batches.

the gap of batches to move the time point where 50% of the kick strength is reached into the middle of the gap between the batches. A simple way to do this is to set the first 17 samples of the empty bunch places to the value of the last bunch of the circulating beam and the second 17 samples (the gap is 38 empty bunch places wide) to the first value measured on the injected batch (Fig. 2). This can be done, as all values are in any case stored for one turn and one can anticipate during the gap what will come.

A further check of assumptions of the decoherence time $\tau_{\mathrm{dec}}$ was done by turning on the octupoles in the model and inhibiting the feedback system ($g = 0$). Instabilities were not included. After an injection displacement of $1.5\ \sigma$, which is the maximum allowed kick error by the MKI, an e-folding time constant of 664 turns was obtained. The comparison with the expected value (750 turns) gives good agreement for injection errors smaller than $1.5\ \sigma$. Fig. 3 compares different assumption for the parameters of Eq. (7).

Turn-by-turn bunch oscillations for two different simulations (including instabilities, MKI injection error assumed) are shown in Fig. 4. The top graph indicates the case of active damping ($g = 0.15$) whereas the lower image reveals an increasing oscillation when the feedback is turned off. An amplitude growth becomes apparent after $\approx 250$ turns with an instability rise time $\tau_{\mathrm{inst}} \approx 16$ ms.

## CONCLUSION

A model has been developed that predicts the expected damping efficiency for the LHC transverse feedback system. Tune spread due to octupoles and chromaticity were included as sources for beam filamentation. The resistive wall instability is shown to be well controlled by the damper. The simulation model can now be used to optimise the signal processing in the feedback system.

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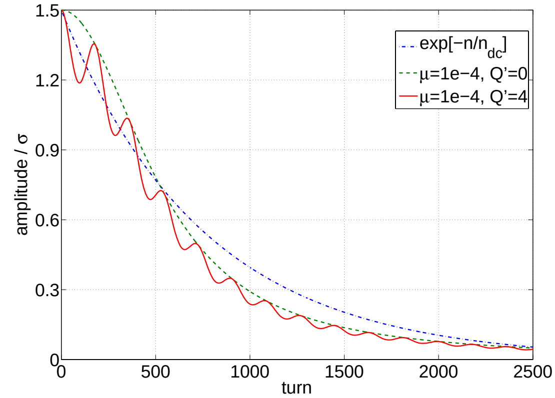


Figure 3: The centre of gravity motion (envelope) of a filamenting bunch without active damping shows different decaying characteristics. A first approximation describes an exponential decaying amplitude (blue, dash-dotted) with parameter $\tau_{\mathrm{dc}} = 750$ turns. The green curve (dashed) accounts for a detuning proportional to $r^2$, e.g. octupolar fields, with $\mu = 10^{-4}$. In case of a non-zero value for the chromaticity and non-vanishing momentum spread the decay is modulated by recoherence (red, solid). Analytical expressions were used [6].

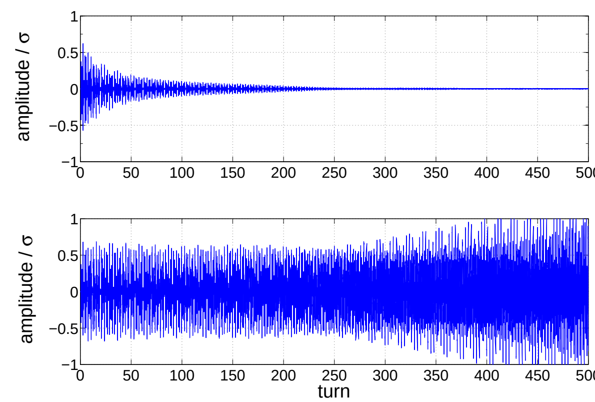


Figure 4: Turn-by-turn bunch oscillation amplitudes with feedback on (top) and off (bottom).

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