

APPLICABILITY OF STOCHASTIC COOLING TO SMALL ELECTROSTATIC STORAGE RINGS

Håkan Danared

Manne Siegbahn Laboratory

Frescativägen 28, S-104 05 Stockholm, Sweden

Abstract

Several small electrostatic storage rings have been built during recent years, or are being built, for experiments mainly in atomic and molecular physics. One example is the DESIREE double electrostatic storage ring under construction at the Manne Siegbahn Laboratory. As in larger magnetic rings used for the same purpose, beam cooling could improve experimental conditions considerably in electrostatic rings. Although electron cooling of 20 keV protons has been demonstrated at the KEK electrostatic storage ring, electron-cooling times become unrealistically long for slow, heavy ions. The rates of stochastic cooling, on the other hand, are at a first glance unrelated to the beam energy. Furthermore, the low particle numbers expected for many heavy molecules seem to make stochastic cooling attractive, theoretical rates being inversely proportional to particle numbers. In this paper, the rates of stochastic cooling for slow heavy particles are investigated with respect to, mainly, the bandwidths and signal strengths that can be expected at the low particle velocities that are of interest at, e.g., DESIREE, and some numerical examples are presented.

INTRODUCTION

Small electrostatic storage rings have been built during recent years, or are being built, as tools for atomic and molecular physics. Also biophysics is targeted in these rings because of their ability to store useful beams of particles essentially without upper mass limit. This is because, with injection from a source on a given electrostatic potential, the magnetic fields in a magnetic storage ring must increase in proportion to the square root of the ion mass while the electric fields in an electrostatic storage ring become independent of ion mass.

One example of an electrostatic ring is DESIREE (Double ElectroStatic Ion Ring ExpEriment) at the Manne Siegbahn Laboratory [1]. This device consists of two

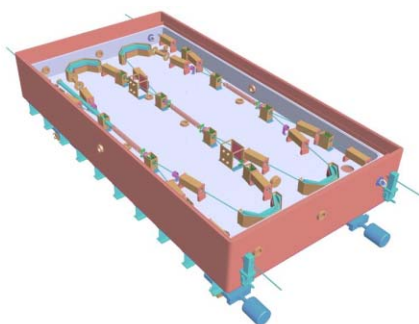


Figure 1. Schematic drawing of DESIREE.

rings side by side with a common straight section for studies of interactions between positive and negative ions in a merged-beam configuration. A schematic view of DESIREE is shown in fig. 1. The rings are housed in a common cryostat and will be cooled down to around 10 K in order to allow molecular ions to relax to their vibrational and rotational ground states and to prevent that externally produced cold ions get excited by black-body radiation from the vacuum-chamber walls.

The maximum particle energy in DESIREE will be determined by the voltages of the injector platforms, the highest of which will be 100 kV. The maximum particle energy is thus 100 keV per charge state. The maximum energy in other electrostatic rings is similar, as seen in table 1.

Table 1. Maximum equivalent injector-platform voltage, or particle energy per charge state, for some electrostatic storage rings.

Ring	Max. voltage (kV)	Ref.
ELISA, Århus	25	[2]
KEK ring, Tokyo	30	[3]
TMU ring, Tokyo	30	[4]
DESIREE, Stockholm	100	[1]
CSR, Heidelberg	300	[5]

BEAM COOLING AT LOW ENERGIES

Beam cooling is in principle of great interest for these small electrostatic rings. If ions are injected from a platform, the longitudinal energy spread of the stored ions will normally be quite small, so cooling would be more important in the transverse degree of freedom. A small transverse emittance for example improves count rates in experiments where the stored ions interact with laser beams, and in DESIREE the merged-beam experiments would benefit from cooled beams. If the particles are accelerated in the ring, the momentum spread becomes larger, and also longitudinal cooling can be of interest.

At the low energies relevant for these rings, electron cooling can be applied to light ions. This has been demonstrated in the electrostatic ring at KEK [3], where an electron cooler was installed, and where cooling of 20 keV protons was observed. At CRYRING, weak effects of cooling have been observed for heavier ions at similar velocities, such as water-cluster ions with 73 mass units at 18 keV per nucleon [6]. However, the electron current in the electron cooler becomes very low at these low energies, making cooling times long. Protons at 20 keV require electrons of about 100 eV for cooling, and with a

gun perveance of typically $2\text{--}3 \mu\text{A}/\text{V}^{3/2}$, the electron current becomes approximately $100 \mu\text{A}$. Cooling times are then in the order of tens of seconds.

Cooling times become even longer for heavier ions, both because they are slower, such that electron currents become still lower, and because of the higher inertia of heavier ions. With singly charged ions injected from an ion source on a given electrostatic potential, electron-cooling times therefore increase quadratically with the ion mass. The rates of stochastic cooling, on the other hand, are at a quick glance unrelated to beam energy. Also, the low particle numbers expected for many heavy molecules seem to make stochastic cooling attractive since theoretical cooling rates are inversely proportional to the number of particles. It thus seems worth while to investigate if stochastic cooling is applicable to small electrostatic storage rings.

STOCHASTIC COOLING

Cooling Rates

The theoretical rate for stochastic cooling can be written as [7]

$$\frac{1}{\tau} = \frac{W}{N} [2g - g^2(M + U)],$$

where W is the bandwidth of the cooling system, N is the number of stored particles, g is the gain of the system (proportional to the electronic gain), M is the mixing factor, and U is the noise-to-signal ratio. The so-called bad mixing between pickup and kicker is neglected here. It is seen that the optimum gain is obtained for $1/g = M + U$, although the gain would have to be adjusted during the cooling process to keep it at the optimum. If this can be achieved, the cooling rate is

$$\frac{1}{\tau} = \frac{W}{N} \frac{1}{M + U}.$$

The mixing factor tells how well the beam distribution is randomized again between kicker and pickup once the cooling system has acted on it with the kicker. Optimum mixing exists at $M = 1$, and at larger values mixing is slower. The mixing factor can be written as

$$M = \frac{T_s}{\Delta T} = \frac{1}{2WT\eta \Delta p/p},$$

where $T_s = 1/(2W)$ is the sampling time in the pickup, ΔT is the spread in revolution time among the beam particles, T is the revolution time itself, η is the frequency-slip factor, and $\Delta p/p$ is the relative momentum spread.

The noise-to-signal ratio U depends both on the beam and on the pickup (for the signal strength) and on the electronics (for the noise) and is discussed in the next section.

To be specific, we concentrate on transverse cooling in the following, since this is the most interesting case for DESIREE.

Noise-to-Signal Ratio

For the calculation of the transverse pickup signal, the transverse Schottky current is written as

$$\begin{aligned} I_{\perp}(t) &= Zq \sum_{a=1}^N A_a \cos(Q_a \omega_a t - \varphi_a) \sum_{n=-\infty}^{\infty} \delta\left(t - \frac{\theta_{0,a}}{\omega_a} - \frac{2\pi n}{\omega_a}\right) \\ &\approx \frac{Zq\omega_0}{2\pi} \sum_{a=1}^N A_a \cos(Q_a \omega_a t - \varphi_a) \\ &\quad \times \sum_{n=-\infty}^{\infty} \exp(jn(\omega_a t - \theta_{0,a})), \end{aligned}$$

where a particle passing through the pickup is represented by a delta function in time multiplied by its transverse displacement due to the betatron oscillations, and where the summations are made over all turns from plus to minus infinity and over all N particles in the beam. In this expression, Zq is the particle charge, A_a is the betatron amplitude of particle a , Q_a its tune, ω_a its revolution frequency, φ_a the phase of its betatron oscillation and $\theta_{0,a}$ is its position (phase) relative to the pickup at $t = 0$. The average revolution frequency is ω_0 .

A difficulty with stochastic cooling of slow particles is that the signals induced in pickup structures also will be slow, and that the system bandwidth thus becomes limited. The optimum pickup structure may thus be different compared to systems operating at relativistic energies. For DESIREE, which consists of two rings inside a single cryostat, there is also not much space for large pickup and kicker structures. Therefore, we here choose look at signals from electrostatic pickups. Such pickups are also simple enough to allow the calculation of noise-to-signal ratios analytically from first principles.

The output from an electrostatic pickup is a voltage which is equal to the charge induced on the pickup electrodes divided by their capacity to earth. A transverse pickup measures the differential signal between opposite electrodes, and with particle velocity v , length of pickup electrodes l , distance between opposite electrodes $2d$ and capacity C of an electrode relative to earth, the signal voltage becomes

$$U(t) = \frac{\Delta Q(t)}{C} = \frac{I_{\perp}(t)l}{dvC}.$$

The power density $s_{\text{sign}}(\omega)$ is the Fourier transform of the autocorrelation function of $U(t)$ which gives

$$\begin{aligned} s_{\text{sign}}(\omega) &= \frac{l^2}{d^2 v^2 C^2} \frac{(Zq)^2 \omega_0^2}{8\pi} \\ &\quad \times \sum_{a=1}^N A_a^2 \sum_{n=-\infty}^{\infty} (\delta[\omega - (n + Q_a)\omega_a] + \delta[\omega - (n - Q_a)\omega_a]) \\ &= \frac{l^2}{d^2 v^2 C^2} \frac{N(Zq)^2 \omega_0^2}{8\pi} \langle A^2 \rangle \\ &\quad \times \sum_{n=-\infty}^{\infty} \left[\frac{1}{n+Q} f_0\left(\frac{\omega}{n+Q}\right) + \frac{1}{n-Q} f_0\left(\frac{\omega}{n-Q}\right) \right], \end{aligned}$$

In the last expression, $\langle A^2 \rangle$ is the mean square value of the oscillation amplitudes, f_0 is the distribution of revolution frequencies in the beam normalized to unity, and all particles are approximated to have the same tune Q . It is seen that the signal spectrum consists of sidebands to all multiples of the revolution frequency, separated from these by $\pm Q\omega_0$.

Here, $s_{\text{sign}}(\omega)$ is in V^2 per unit of angular frequency, so to get V^2/Hz , the expression must be divided by 2π . Integrating over one sideband and multiplying by four because of the two sidebands at each n and because negative frequencies are seen as positive in a measurement, the integrated power in watts over one harmonic of the revolution frequency is obtained as

$$S_{\text{sign}} = \frac{N(Zq)^2 \lambda^2 \langle A^2 \rangle}{d^2 C^2},$$

where λ is introduced as the ratio between pickup length l and ring circumference.

The noise from a good charge-sensitive amplifier with GaAs FETs has a power density (reduced to the input of the amplifier) in the order of $s_{\text{noise}} = 1 \text{ nV}^2/\text{Hz}$. This is the voltage noise from the FETs themselves.

Integrating also the noise over one Schottky band, i.e., multiplying s_{noise} by $\omega_0/(2\pi)$, the noise-to-signal ratio becomes

$$U = \frac{\omega_0 s_{\text{noise}} d^2 C^2}{2\pi N(Zq)^2 \lambda^2 \langle A^2 \rangle}.$$

Bandwidth

Before the cooling time can be calculated, it is also necessary to know what bandwidth can be expected. In the case of an electrostatic pickup, the time it takes for a particle to pass through the pickup gives an upper frequency limit which is approximately the pickup length divided by the particle velocity. As a result, the maximum bandwidth is

$$W = \frac{\omega_0}{2\pi\lambda}.$$

In addition, there is no advantage in making the length of the pickup small compared to its radius because of the longitudinal extent of the charge distribution from a particle passing through. For the same reason, there is nothing to be gained by using, e.g., stripline pickups instead of electrostatic pickups. This is in contrast to existing stochastic-cooling systems with relativistic particles where the charge distribution becomes Lorentz contracted, allowing higher bandwidths.

An alternative method that could be worth investigating is cooling on a single harmonic with a resonant pickup. This sacrifices bandwidth, but possibly this loss can be regained from a better signal-to-noise ratio.

RESULTS

With the above expressions for mixing factor, noise-to-signal ratio and bandwidth, the cooling time is found to be

$$\tau = \frac{\pi N \lambda^2}{\omega_0 \eta \Delta p/p} + \frac{s_{\text{noise}} d^2 C^2}{(Zq)^2 \lambda \langle A^2 \rangle}.$$

As a first numerical example we take $1 \mu\text{A}$ of protons stored at 100 keV in DESIREE, where each ring has a circumference of 8.7 m . Assume $\lambda = 0.01$, $\eta = 1$, $\Delta p/p = 1 \times 10^{-4}$, $s_{\text{noise}} = 1 \text{ nV}^2/\text{Hz}$, $d^2/\langle A^2 \rangle = 4$ and $C = 100 \text{ pF}$. The theoretical cooling time is then 150 s . We have chosen $\Delta p/p$ somewhat large in relation to what one can expect with injection from a platform at a static voltage, and this keeps the mixing relatively good. As a result, the second term in the expression above dominates. It is independent of beam current and particle species, apart from the charge state, and only noise and pickup sensitivity is important.

Noise can possibly be reduced somewhat by cooling the preamplifiers—the interior of DESIREE is already at around 10 K —but finding a pickup structure that has higher sensitivity or higher bandwidth, and at the same time is small enough to fit into DESIREE or a similar ring and is universal in the sense that it is not limited to a particular particle species or a particular beam velocity seems to be a challenge.

As a second example we consider a heavy biomolecule with mass $10\,000$ produced in an electrospray ion source. These typically get multiply charged from the source but are produced in small quantities. We thus take $N = 1 \times 10^5$ and charge state $Z = 10$, but keep the other parameters unchanged. This gives a theoretical cooling time of 5.5 s . The higher charge state gives a stronger signal, and the cooling time now gets its largest contribution from the mixing term.

CONCLUSION

It is concluded that stochastic cooling times in small electrostatic storage rings become unrealistically long for singly charged ions because of low bandwidths and poor signal-to-noise ratios. But stochastic cooling of multiply charged large biomolecules could be feasible, even if cooling times in practice become somewhat longer than the theoretical values calculated here.

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