ON THE OPTIMAL NUMBER OF EIGENVECTORS FOR ORBIT CORRECTION*
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Abstract
The singular value decomposition method is widely used for orbit correction in the storage rings. It is a powerful tool for inverting of the usually rectangular response matrices, which usually have rectangular form. Another advantage is flexibility to choose number of eigenvectors for calculation of required strengths of orbit correctors. In particular, by reduction in number of eigenvectors one can average over ensemble the noise in the beam position monitors. A theoretical approach as well as experimental results on the NSLS VUV ring is presented.

INTRODUCTION
A real-time orbit feedback utilizes beam position monitors (BPM) data to observe motion of a circulating beam. The deviation of orbit from the nominal is multiplied by an inverse matrix to find the required trim strength to suppress noise. Usually the response matrix inversion is done using singular value decomposition (SVD) method [1-3].

By increasing the number of retained eigenvectors one improves accuracy of inversion and generally improves suppression of beam motion [1, 3]. However, the real machine has errors in measuring of the orbit caused by the various reasons such as noise in BPM electronics, vacuum chamber motion, etc. The orbit feedback system can translate these errors into the unwanted beam motion.

The optimal number of retained eigenvectors depends on the relation of measurement noise and real beam motion, induced by external factors such as quadrupoles vibration. To illustrate this we can consider to ultimate cases: for ideal BPMs full set of eigenvectors will be utilized, but for ideal machine without beam motion there is no need for orbit feedback (i.e. number of eigenvectors is zero no matter how small BPM noise is).

THEORETICAL CONSIDERATIONS
The VUV storage ring at NSLS has 24 BPMs and 16 correctors for each plane [4]. All 16 correctors in the vertical plane can be used for orbit feedback but for the horizontal plane only 8 can be utilized. Therefore, we considered only the vertical plane. It was assumed that beam motion is excited by uncorrelated motion of the quadrupoles. Because mechanical frequencies are substantially smaller than betatron frequency the beam reacts instantaneously to change in the positions of the quadrupoles. The beam orbit is measured with some errors and correction is applied.

The noise in the BPM receivers was measured using a BPM calibrator [5]. CW sine wave was applied to the receiver inputs and oscilloscope traces were acquired at the output of the receiver. The spectrum of noise was obtained by Fourier transformation and is shown in Fig. 1. The spectrum is flat up to 2 kHz in accordance with the design of the receiver [6]. R.m.s. error in the BPM measurement is about 12 microns. Orbit feedback has 100 Hz bandwidth and therefore in simulation lower value of BPM error was used.

Figure 1: Frequency spectrum of electronic noise of BPM receiver.

The motion of the quadrupoles is about 1 micron in the 100 Hz bandwidth. Based on the noise data we wrote a simulation script in MATLAB. The quadrupoles were randomly moved and closed orbit was found. The measurement noise was added to the BPM receivers. Trim strength was calculated and correction was applied. R.m.s orbit deviation from the reference was calculated and normalized on the beam size. The results of simulations with 128 seeds are shown in Fig. 2.

Figure 2: Modeled dependence of beam motion vs. number of eigenvectors in orbit feedback.

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As one can see the noise goes down and reaches the minimum when 6 eigenvectors are used. Presently VUV ring utilizes 8 eigenvectors and therefore it was interesting to compare simulations with real machine.

**EXPERIMENTAL SET-UP**

In order to receive independent from BPM measurements of beam motion we utilized position sensitive diode (PSD) installed on the U5A beamline. This diagnostic beamline is used for monitoring beam size with camera with visible synchrotron radiation from a bending magnet. The radiation was focused on the PSD with long focal length lens. The vibration experienced by the lens and beamline mirrors prevented absolute measurements of beam motion, but relative noise figure was available. For this purpose signal PSD was sampled with digital oscilloscope and processed in order to see beam motion in 1-30 Hz range. The results are shown in Fig. 3.

![Figure 3: Measured dependence of beam motion of vs. number of eigenvectors utilized by the orbit feedback system.](image1.png)

The good agreement between theoretical predictions and experimental results was observed. Unfortunately, we were unable to explore larger number of eigenvectors, because orbit feedback became unstable due to the errors in the measured response matrix.

**ANALYSIS FOR NSLS-II**

We did similar modeling for the NSLS-II storage ring using the expected quadrupole motion and noise in beam measurement. The results are shown in Fig.4.

![Figure 4: Calculated beam motion for the NSLS-II storage ring.](image2.png)

As in case for NSLS VUV storage ring amplitude of beam motion greatly diminishes with increase of number of eigenvectors. The broad minimum is reached when number of eigenvectors is about 60. That means that amount of calculations for orbit feedback algorithm can be significantly reduced by retaining small number of eigenvectors.

**CONCLUSION**

We considered both theoretically and experimentally storage ring orbit feedback system based on SVD algorithm. It was shown that the optimal number for retained eigenvectors exists in order to provide minimal beam motion. Good agreement between theoretical approach and experimental results was found.

**REFERENCES**