INVESTIGATION OF HELICAL COOLING CHANNEL

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Abstract

A helical cooling channel (HCC) has been proposed to quickly reduce six-dimensional (6D) phase space of muon beams. It is composed of solenoidal, helical dipole, and helical quadrupole magnetic fields to provide focusing and dispersion needed for 6D cooling. A comprehensive investigation of the HCC is performed in present work including theoretical analysis, particle tracking and Monte Carlo simulation. Optimization of the HCC and estimation of its performances are presented. The practical design of the HCC is demonstrated.

INTRODUCTION

A helical cooling channel has been proposed by Y. Derbenev for 6D ionization cooling of muon beams [1]. The most comprehensive description of the HCC can be found in Ref. [2]. The main idea is to use a helical magnetic field dependent on the combination of cylindrical coordinates, $\theta - kz$ where $k = const$. Stable helical orbits $\theta = kz$ with momentum dependent radius are realized in such a field. This gives rise to transverse-longitudinal coupling, providing emittance exchange and 6D ionization cooling in the gas-filled channel. The idea was confirmed by later investigation and Monte Carlo simulation [3, 4, 5, 9]. At the same time, some constraints of the method was discovered including restrictions of the beam momentum, accelerating frequency, etc. Engineering problems are analyzed in papers [6, 7, 8]. Incorporation of high-gradient accelerating system in the channel is the current problem. A possible solution will be shown in this document.

BASIC RELATIONS

The helical magnetic field with period $\lambda = 2\pi/k$ can be described in terms of a scalar potential:

$$\Phi = B_s \left[\sum_{n=1}^{\infty} \frac{C_n}{kn} I_n(nkr)\, \sin n(\theta - kz) - z \right], \qquad (1)$$

where I_n are modified Bessel functions, C_n – arbitrary coefficients. Periodical helical solutions $\theta = kz$, $r = X/k = const$ are possible in this field, where normalized radius X as a function of the particle momentum p satisfies the equation:

$$\left(1 + F(X)\frac{1+X^2}{X^2}\right)\sqrt{1+X^2} = \frac{kpc}{eB_s}. \qquad (2)$$

The important function $F(X)$ is normalized z-component of the helical field:

$$F(X) = \sum_{n=1}^{\infty} C_n I_n(nX)\,. \qquad (3)$$

It determines dependence $X(p)$ which is dispersion. Here, we define the fundamental parameters: a normalized radius at reference momentum $X_0 = X(p_0)$, as well as the corresponding field and its derivative with respect to X: $F_0 = F(X_0)$ and $F'_0 = F'(X_0)$, respectively. They determine transverse tunes, coupling, cooling decrements, etc. Optimal parameters to reach equal decrements and fastest cooling are: $X_0 \simeq 1, F_0 \simeq F'_0 \simeq -0.2$.

COOLING SIMULATION

Cooling simulation is presented in Fig. 1 at following parameters [3]:

- Period length: $\lambda = 2\pi/k = 1$ m
- Field coefficients: $C_1 = -0.509,\ C_2 = 0.078$
- Field parameters: $X_0 = 1,\ F_0 = -0.234,\ F'_0 = -0.217$
- Transverse beta-functions: $\beta_1/\beta_2 = 23$ / 28 cm
- Central beam momentum $p_0 = 250$ MeV/c
- Solenoid field: B_s=6.97 T
- Accelerating frequency/gradient: 200 MHz / 30 MV/m
- Partial cooling length: $\Delta z = 20$ m.

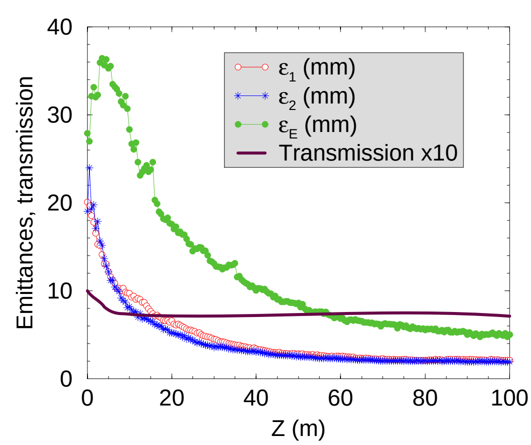

Figure 1: Normalized rms emittances and transmission vs longitudinal coordinate. 1-2 are transverse eigenmodes.

Equilibrium transverse emittance is about 2 mm in this example which is very close to the theoretical prediction. It is proportional to the helical period length: $\epsilon_\perp \simeq \lambda/500$. Required magnetic field depends on λ as well being proportional to p_0/λ. Unfortunately, lower momentum can cause additional particles loss, probably because of steeper dE/dx curve at lower momentum. In the example above, transmission falls from 73% to 21% (without decay) if the momentum decreases from 250 MeV/c to 183 MeV/c [5].

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RF REQUIREMENTS

As it is shown in Ref. [5], an increase of accelerating frequency can cause the cooling deterioration because of less transmission. For example, transmission of the cool channel considered in previous section falls from 73% at 200 MHz to 40% at 400 MHz at the same cooling factor. More growth of the frequency leads to total destruction of the beam. This is shown in Fig. 2 where this channel is considered at $f = 800$ MHz and zero initial emittances. It indicates the limitation of the RF frequency.

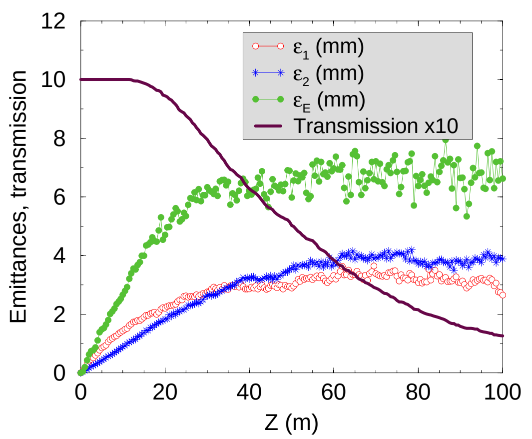

Figure 2: Normalized rms emittances and transmission vs longitudinal coordinate at 800 MHz RF.

Above consideration is clarified by Fig. 3 where phase space time-energy is shown. Phase trajectories in the left-hand plot are rather similar to the "usual" ones, and only small difference appears because of a modulation of the flying time by betatron oscillations (cooling is detectable as well). At 200 MHz, the modulation is small in comparison with the separatrix, which retains more or less standard form through this. However, 4 times shorter 800 MHz separatrix is comparable with amplitude of the modulation, which produces strong perturbation of the phase trajectories (right-hand plot). From above analysis, we found that the optimum helical period (λ) is related with the RF wavelength (λ_{RF}):

$$\lambda_{RF} >\sim \lambda \tag{4}$$

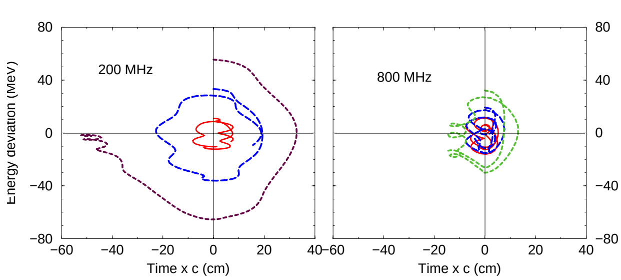

Figure 3: Phase space time-energy at different frequencies.

PRACTICAL HCC DESIGN

Helical Solenoid

Prospective engineering design of the HCC proposed in Ref. [6] is schematically sketched in Fig. 4. This helical solenoid (HS) can be presented as a set of parallel ring coils shifted from each other to form a helical-shape channel. Required field is reached by taking of appropriate ratio [coil diameter] / [period length] and additional homogeneous field. For this, HS should be placed inside a usual solenoid. Corresponding example is considered below.

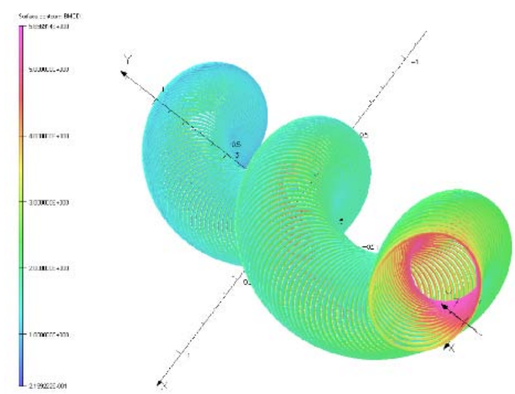

Figure 4: Helical solenoid.

- Period length: $\lambda = 2\pi/k = 1$ m
- Helical solenoid diameter 0.547 m
- Field parameters: $X_0 = 1,\ F_0 = -0.235,\ F_0' = -0.215$
- Transverse beta-functions: $\beta_1/\beta_2 =$ 24 / 28 cm
- Beam momentum $p_0 = 250$ MeV/c
- Longitudinal field on the channel axis:

...Helical solenoid 9.75 T
...Additional solenoid -2.76 T

- Accelerating frequency/gradient: 200 MHz / 15 MV/m
- Partial cooling length: $\Delta z = 41$ m.

Function $F(X)$ is almost linear in this case being composed from a lot of Bessel functions. Results of cooling simulation are presented in Fig. 5:

A drawback of this scheme is small diameter of the helical solenoid coil. This does not allow to install there pill-box cavities satisfying Eq. (4). Development of RF cavities compatible with this channel is a challenging problem.

HS and HS Correction Coil

Another possible way to incorporate the pill-box type RF cell in the HS coil is promoted by adding an HS correction coil as shown in Fig. 6. A red solid circle is the RF pill-box cavity, a blue ring is the HS coil, a small green ring is the HS correction coil, and a big green ring is the pure solenoid coil, respectively. Optimal parameters are achievable in this case at more diameter of the helical solenoid coil. As shown in Fig. 6, there is an additional gap between

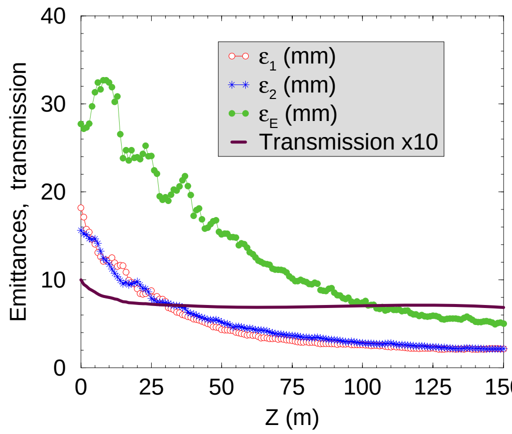

Figure 5: Cooling simulation at helical solenoid.

the coil and the RF cell. It is required for the realistic design of the HCC, i.e. thermal insulation, mechanical support, and so on. We put an 8 cm gap in this design study.

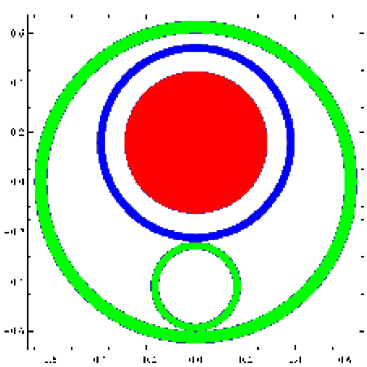

Figure 6: Schematic of HS coil with HS correction coil. Origin is the center of the HCC magnet.

An example is given below at following parameters [3]:

- Period length: $\lambda = 2\pi/k = 1$ m
- Helical solenoid diameter 0.734 m
- Helical solenoid current -129 A/mm^2
- Correction coil diameter 0.3 m
- Correction coil current 387 A/mm^2
- Solenoid coil diameter 1.2 m
- Solenoid coil current -31 A/mm^2
- Beam momentum $p_0 = 250$ MeV/c
- Longitudinal field on the channel axis: HS coil -4.5 T

...Correction HS coil -1.0 T

...Additional solenoid coil -0.3 T

- Accelerating frequency/gradient: 400 MHz / 16 MV/m
- Partial cooling length: $\Delta z = 40$ m.

A brief study of the RF structure in the HCC has been done. The average power dissipation in one cavity is $\sim$3 kW. The number of cavity per helical period is 20. Therefore, the total power dissipation is 60 kW/helical period. A detailed discussion for the practical design of the HCC can be found in Ref.[9].

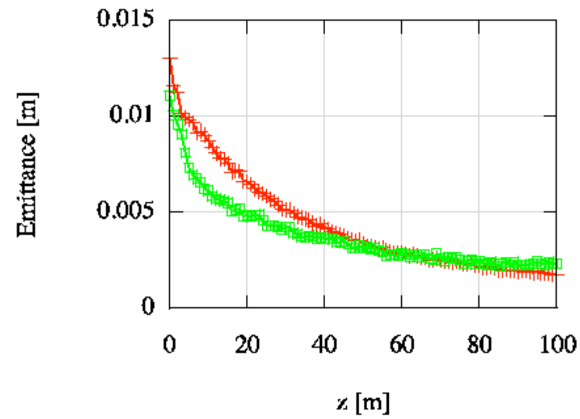

Figure 7: Transverse (red +) and longitudinal (blue box) emittances in the new HCC.

CONCLUSION

We investigated the characteristic of the HCC to optimize its cooling performance. We discover rule of thumb as below.

- Period length λ is the most important parameter to design the helical cooling channel.
- Achievable transverse emittance $\epsilon \simeq \lambda/500$.
- Required solenoid field B_s (T) $\sim 7/\lambda$ (m).
- Admissible RF wavelength $\lambda_{RF} >\sim \lambda$.

The practical HCC has been demonstrated. The most challenging problem is an embedding of RF cavities into the helical solenoid magnet. A development other then pillbox cavities is first possibility Another way is adding the HS correction coil. However, the current correction coil is not very effective to apply the required field strength on the helical reference orbit. The amplitude is about one fifth of strength of the HS coil. Probably, the elliptical shape of the HS coil is another solutions.

REFERENCES

[1] Y. Derbenev, http://www-mucool.fnal.gov/mcnotes/public/ps/muc0108/muc0108.ps.gz (2000).

[2] Y. Derbenev and R. Johnson, Phys. Rev. ST-AB, 8, 041002 (2005).

[3] K. Yonehara *et al.*, PAC'05, TPPP052.

[4] V. Balbekov, Fermilab-TM-2400-APC (2007).

[5] V. Balbekov, http://muonsinc.com/lemc2008/presentations/balbekov_LEMC_08.PPT .

[6] V.S. Kashikhin *et al.*, Proc. of Applied Superconductivity Conf., p. 1055 (2006).

[7] V.S. Kashikhin *et al.*, EPAC'08, WEPD014.

[8] S. Kahn *et al.*, EPAC'08, MOPP090.

[9] K. Yonehara, http://www.muonsinc.com/lemc2008/presentations/hcc_ky_042308.ppt