TRANSVERSE MISMATCH OSCILLATIONS OF A BUNCHED BEAM IN PRESENCE OF SPACE CHARGE AND EXTERNAL NONLINEARITIES *

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Abstract

The damping of transverse mismatch oscillations depends on the combined effect of space charge as well as external nonlinearities. Previous studies of this problem for high intensity beams in a synchrotron have not included the combined effect of synchrotron oscillation and external nonlinearities on mismatch. In this paper we explore by $2\frac{1}{2}$D particle in cell simulations the effect on emittance growth, halo and beam loss caused by space charge, synchrotron oscillation and external nonlinearities.

INTRODUCTION

The interplay between space charge (SC) and nonlinearities (NL) in synchrotron has recently received much theoretical and experimental attention [1, 2, 3]. The beam losses prediction and control, which rely in the understanding of these interplay mechanisms, are the key issues for the new generation of high-intensity heavy ion rings, such as the SIS100 synchrotron of the FAIR project. Even if the basic mechanisms of beam loss can be investigated by means of simple particle-core models, a numerical approach is required in order to deal with the loss issue in real machines. Unfortunately a complete fully self consistent treatment which takes into account SC and NL for $10^5 \div 10^6$ turns is nowadays practically unfeasible even using a big cluster. In this context frozen or semi-frozen methods remain the only option[4]. A fully self consistent approach by using PIC code is however possible if we consider a limited number of turns ($\sim 10^2 \div 10^3$). In a recent paper [5] we presented the upgrade and some validation tests of the MICROMAP beam dynamics simulation library [6, 7] to include $2\,\frac{1}{2}$D space charge modeling (using a PIC technique) of a long and thin 3D bunch. In [5] was also presented a preliminary application of the library to the SIS100 (using a simplified lattice without nonlinearities) studying the damping of the mismatch oscillations induced by the mixing effect due to the synchrotron motion together with the space charge, and the formation of tails around the beam core. As $2\,\frac{1}{2}$D simulations are demanding in terms of CPU time requirements, in order to explore the coupling of SC and NL keeping short the simulation time, we take a SIS100 lattice equipped with squared magnets and stronger nonlinearities compared to the present ones [8]. With this lattice an important role will be necessarily played by the "sagitta" effect in the bending magnets. In this paper we extend the work done in [5] and examine the effects of this model for the SIS100 on an high intensity bunched beam off-centre at injection with a moderate transverse mismatch.

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WORKING POINT AND INITIAL CONDITION

The working point chosen for the simulation is the one with $Q_{x0} = 18.83$, $Q_{y0} = 18.73$ (WP1), the reasoning behind the choice of this working point are summarized in [9]. We consider a bunched beam (U^{28+}) with Gaussian transverse distribution injected ($E_{inj} = 200$ MeV/u) from the SIS18 into the bucket of the SIS100. The initial emittances (at 2σ) of the beam are $\epsilon_x = 35$ mm-mrad and $\epsilon_y = 15$ mm-mrad. We assume that during the injection the transverse distribution is truncated at 2.5σ in amplitude. All the particles with single particle emittances exceeding $(2.5)^2$ times the r.m.s. values are eliminated from the injected bunch. The edge emittances after the injection are then $\epsilon_x^{(edge)} = 54.7$ mm-mrad, $\epsilon_y^{(edge)} = 23.4$ mm-mrad and the r.m.s. emittances are $\epsilon_x^{(rms)} = 6.46$ mm-mrad, $\epsilon_y^{(rms)} = 2.77$ mm-mrad. The number of injected ions is $N_{bunch} = 7.5 \cdot 10^{10}$ and the longitudinal distribution of the bunch is Gaussian with $\sigma_z = 12$ m. The maximum tuneshifts obtained in the simulation are $\Delta Q_x = -0.30$, $\Delta Q_y = -0.45$. We also assume that the injection causes a moderate ($\sim 5\%$) transverse mismatch togheter with a displacement (~ 1 mm) of the beam centroid. Finally we consider also the synchrotron motion taking $T_{sync} = 233$ turns. In the simulation with MICROMAP we set $N_{particles} \sim 10^6$. For the SC calculations in the $2\frac{1}{2}$D approximation the number of slices along the bunch is $N_{slices} = 25$ and each transverse mesh is 128^2. In the tracking we take into account the complete nonlinear lattice (without collimators) of the SIS100 and we use 399 SC kicks per turn.

INTERPLAY BETWEEN SPACE-CHARGE AND NONLINEARITIES IN SIS100

In order to investigate the interplay between SC and NL in SIS100 we consider the four simulations listed below:

- case I: without SC – without NL,
- case II: with SC – without NL,
- case III: without SC – with NL,

- case IV: with SC – with NL.

Case I: without SC – without NL

This case is trivial. The mismatch oscillations are initially washed out on a time scale of $20 \div 30$ turns due to the mixing effect of the synchrotron motion but since the dynamics is completely linear the single particle emittances (SPE) are invariant of the motion. As a consequence the mismatch oscillations periodically reappear following twice the periodicity of the synchrotron motion. Obviously neither losses nor emittance growth are observed.

Case II: with SC – without NL

This case is similar to the one presented in [5]. Again we observe the damping of the mismatch oscillations on a time scale of $\sim$ 10 turns (shorter than in the previous case) but in this case, due to collective effects, the process is irreversible. The SPEs spread in the emittance plane forming a small tail around the beam core. In Fig. 1 we plot the distribution of the SPEs as a function of the parameter $t = \epsilon_x/\epsilon_x^{rms} + \epsilon_y/\epsilon_y^{rms}$, where (ϵ_x, ϵ_y) are the SPEs and $(\epsilon_x^{rms}/\epsilon_y^{rms})$ the r.m.s. emittances of the beam. The quantity t "measures" the distance of a particle from the beam core in the emittance plane. The black plot is the distribution at the beginning of the simulation, the red one the distribution after 100 turns when the diffusion process has saturated. In Fig. 2 we plot the percentage of particles of the beam in the tail (*i.e.* outside the beam "edge" represented by the blue line in Fig. 1) as a function of the time. We see that $\sim$ 0.1 % of the particles is in the tail. We have also observed that this number is a function of T_{sync}, the longer is the synchrotron period the higher is the number of particles in the tail. In the limit $T_{sync} \to \infty$ we find that approximately $\sim 0.3\%$ of the particles is in the tail. Since the lattice is linear losses are not an issue. The change in the r.m.s. emittances is negligible.

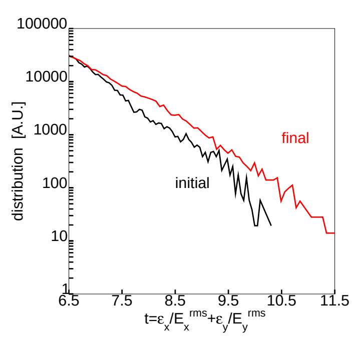

Figure 1: Case II. Distribution of the SPEs as a function of $t = \epsilon_x/\epsilon_x^{rms} + \epsilon_y/\epsilon_y^{rms}$ at the beginning of the simulation (black) and after 100 turns (red).

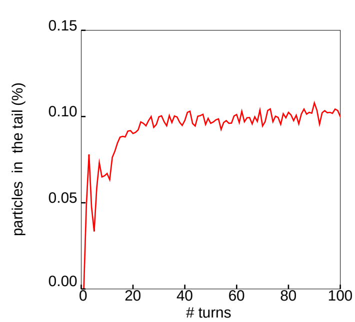

Figure 2: Case II. Percentage of particles in the tail.

Case III: without SC – with NL

In this case nonlinearities induce a periodic rippling in the r.m.s. emittances as shown in Fig. 3. This rippling can be clearly seen also in the emittance plane (see Fig. 4) where nonlinearities pile up a cluster of particles that moves parallel to the beam edge, with a cycle of $\sim$ 25 turns, between the positions A (see Fig. 4-left panel) and B (Fig. 4-right panel). We also observe that the cluster size increases while moving from A towards B. In both figures the black plot are the SPEs at the beginning of the simulation, the blue dashed line is the "unperturbed edge". When the cluster is in A (B) then ϵ_x^{rms} has a local maximum (minimum) and ϵ_y^{rms} a local minimum (maximum) as shown in Fig. 3. Even if the lattice is nonlinear, losses are not present.

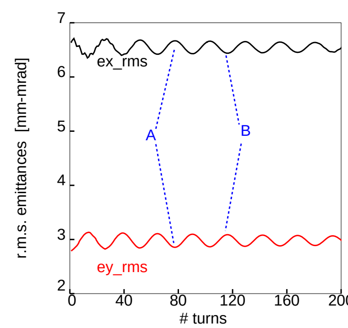

Figure 3: Case III. R.M.S. emittances of the beam as a function of the time.

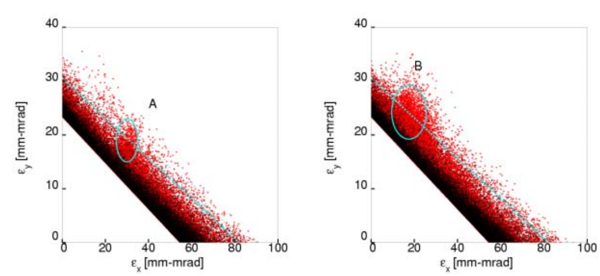

Figure 4: Case III. SPEs (in red) for two different times during the simulation. In black are the values at the beginning.

Case IV: with SC – with NL

The complete case is the most interesting one. In Fig. 5 we show the behaviour of the r.m.s. emittances. We observe an initial partial emittance exchange which saturates after ~ 20 turns. This is the characteristic feature of the Montague resonance [10, 11]. Studying the dynamics in the emittance plane we observe the formation in ~ 10 turns of a large particle cluster, due to both the SC and NL, as shown in Fig. 6. The cluster is responsible for the sudden emittance exchange. The number of particles in the cluster is approximately $2 \div 3\%$ of the total. Compared to

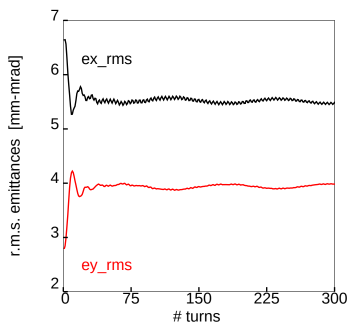

Figure 5: Case IV. R.M.S. emittances of the beam as a function of the time.

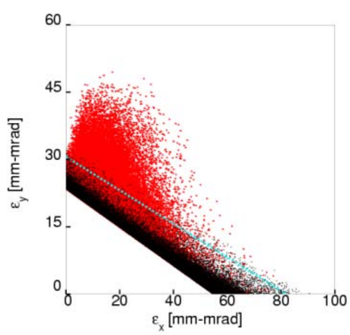

Figure 6: Case IV. SPEs at turn #8 (in red) and at the beginning of the simulation (in black).

the cases II an III, now the SPEs can reach a large distance from the "edge" of the distribution, as a consequence particles get lost due to the finite size of the dynamic aperture. In Fig. 7 we plot the losses as a function of the time: we see that in the first 10^3 turns we are loosing approximately 1 % of the particles.

CONLUSION

In this work we found that magnet nonlinearities induce relevant effects in the dynamics of a high intensity bunched beam in a SIS100 lattice version with rectangular bending magnet. The interplay of the closed orbit affected by the dipole "sagitta" with the magnet nonlinearities (stronger

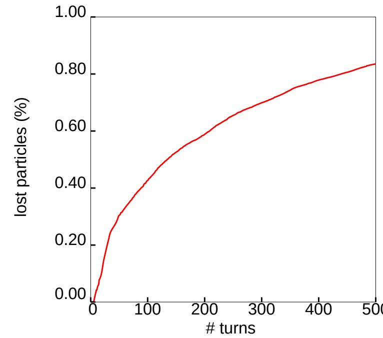

Figure 7: Case IV. Percentage of the beam loss as a function of the time.

here than in the present design) is possibly the major responsible for a feed down tuneshift which brings the effective accelerator tunes close to the Montague resonance (see Fig. 5). This effect is absent in actual SIS100 magnets. We conclude that the actual SIS100 working point WP1 should be benchmarked, for a fine tuning, with full $2\frac{1}{2}$ D simulations for the present version of the SIS100 nonlinear lattice.

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