MEASURING RING NONLINEAR COMPONENTS VIA INDUCED LINEAR "FEED-DOWN"

A. Parfenova*, G. Franchetti, I. Hofmann, GSI, Darmstadt, Germany

Abstract

The knowledge of the distribution in a ring of the nonlinear components is important for the resonance compensation. We suggest a method to measure the lattice nonlinear components based on the nonlinear tune response to a locally controlled closed orbit (CO) deformation. A test of this concept in the SIS18 synchrotron is presented.

INTRODUCTION

The nonlinear field errors in the magnets excite unwanted resonances, which cause beam loss and dynamic aperture reduction during the machine operation. For the 'new' SIS18 working point proposed in [1] ($Q_x = 4.2$, $Q_y = 3.6$) it may be necessary to compensate several of the existing resonances in order to avoid beam loss and improve machine performance. Therefore, a new technique to diagnose nonlinear field components based on the *tune response* to the deformed CO was developed. The approach used is similar to the orbit response matrix (ORM) method, where the CO response to the steering angle change provides information on the linear field errors. The method presented here extends the ORM analogy to the nonlinear errors with the difference that the tune response to the steering angle change is measured. The method is therefore referred to nonlinear tune response matrix (NTRM). The feed down effect of the nonlinear components at the level of linear tune due to the CO change is explored.

NTRM: THE LINEARIZED THEORY

The linear model of a circular accelerator is composed by a sequence of linear thick elements as drifts, quadrupoles and dipoles. The strengths of the linear focusing forces are defined by $k_x(s)$ and $k_y(s)$, where s is the longitudinal coordinate. It is assumed that the accelerator is equipped with N_t thin steerers. The longitudinal location of the tth steerer is s_t, and its steering angle is θ_{xt} in the horizontal plane, θ_{yt} in the vertical plane, respectively. N_l thin nonlinear elements are included in the ring. A nonlinear element can be a lattice sextupole or octupole as well as

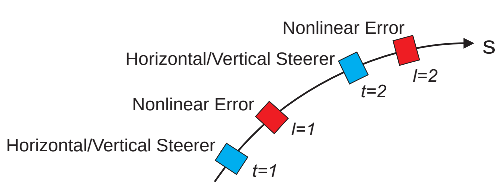

Figure 1: Nonlinear errors and steerer locations.

a dipole or quadrupole magnet nonlinear error. In general, the lth nonlinear error located at s_l is composed of several multipoles of integrated strength K_{nl}, $J_{nl}, n \geq 1$. Here the index n is used to indicate the order of the nonlinear component, and l the location. A schematic of the sequence of error-steerer is shown in Fig. 1. The CO is deformed by setting the N_t steering angles θ_{xt} and θ_{yt}, with $t = 1, ..., N_t$ to a value different from zero $(x_{CO}, x'_{CO}, y_{CO}, y'_{CO})$. In the *linear approximation*, if the CO deformation is not too large and the tunes are away from any resonance, the CO $x_{COl} = x_{CO}(s_l)$ and $y_{COl} = y_{CO}(s_l)$ at the location of the lth nonlinear element s_l is found as following

$$x_{CO\,l} = \sum_{t=1}^{N_t} M^x_{lt}\theta_{xt}\,, \qquad y_{CO\,l} = \sum_{t=1}^{N_t} M^y_{lt}\theta_{yt}\,, \tag{1}$$

where the matrices M^x_{lt} and M^y_{lt} are referred to the location of the nonlinear element and to the location of the steerer. The matrices M^x and M^y form the *orbit response matrix* $M = M^x_{lt} \oplus M^y_{lt}$ for the decoupled system. A test particle coordinate $(\delta x, \delta x', \delta y, \delta y')$ with respect to the deformed CO $(x_{CO}, x'_{CO}, y_{CO}, y'_{CO})$ is considered. If the coordinates of the test particle are small, then all the terms of higher order can be neglected as the tunes are far from any resonance [2]. The equations of motion reads

$$\begin{aligned} \delta x'' + (k_x + \tilde{k})\delta x &= \tilde{j}\delta y\,, \\ \delta y'' + (k_y - \tilde{k})\delta y &= \tilde{j}\delta x\,, \end{aligned} \tag{2}$$

where

$$\tilde{k} = \sum_{n\geq 1} \tilde{k}_n\,, \qquad \tilde{j} = \sum_{n\geq 1} \tilde{j}_n\,. \tag{3}$$

Equation (2) indicates that the nonlinear components around the ring produce an extra linear focusing component of strength $(k_x + \tilde{k}, k_y - \tilde{k})$ and a linear coupling term of strength $\tilde{j}$. The two first orders of the components $\tilde{k}_n$ and $\tilde{j}_n$ of the feed down due to deformed CO are: k_1 and j_1; $k_2 x_{CO} - j_2 y_{CO}$ and $k_2 y_{CO} + j_2 x_{CO}$. The first order contribution of the gradient error $\tilde{k}$ on the machine tunes with respect to the distorted CO is

$$\Delta Q_{x,y} = \frac{1}{4\pi}\int_0^C \beta_{x,y}(s)\tilde{k}(s)ds\,, \tag{4}$$

and the tunes with respect to the CO are given by $Q_x = Q_{x0} + \Delta Q_x$ and $Q_y = Q_{y0} + \Delta Q_y$. Here Q_{x0} and Q_{y0} are the tunes of the linear accelerator with the closed orbit corrected. Equation (4) can be written in matrix form using Eq. (1) for the CO at the location s_l and $\beta_{xl} = \beta_x(s_l)$, $\beta_{yl} = \beta_y(s_l)$

$$\Delta Q_x = {}_xQ + \sum_{t=1}^{N_t}({}_xQ^x_t\theta_{xt} + {}_xQ^y_t\theta_{yt})\,, \tag{5}$$

* a.parfenova@gsi.de

$$\Delta Q_y = {}_yQ + \sum_{t=1}^{N_t} \left({}_yQ_t^x \theta_{xt} + {}_yQ_t^y \theta_{yt} \right), \qquad (6)$$

where

$$\begin{aligned} {}_xQ &= \frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{xl} K_{1l}\,, \\ {}_xQ_t^x &= \frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{xl} K_{2l} M_{lt}^x\,, \\ {}_xQ_t^y &= -\frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{xl} J_{2l} M_{lt}^y\,; \end{aligned} \qquad (7)$$

$$\begin{aligned} {}_yQ &= -\frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{yl} K_{1l}\,, \\ {}_yQ_t^x &= -\frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{yl} K_{2l} M_{lt}^x\,, \\ {}_yQ_t^y &= \frac{1}{4\pi} \sum_{l=1}^{N_l} \beta_{yl} J_{2l} M_{lt}^y\,. \end{aligned} \qquad (8)$$

The linear components K_{1l}, J_{1l} contribute to the tune Q_x, Q_y independent on the CO deformation. Therefore the effective tune due to linear elements will be $Q_{x0,eff} = Q_{x0} + {}_xQ$ and $Q_{y0,eff} = Q_{y0} + {}_yQ$.

NUMERICAL RECONSTRUCTION

Validating the theoretical NTRM model a numerical reconstruction with MICROMAP of nonlinear components (strengths and polarity) for the SIS18 was performed. For this purpose 24 steerers (12 horizontal and 12 vertical) and 24 nonlinear errors (12 normal and 12 skew) given at the location of chromatic sextupoles were considered. The tune

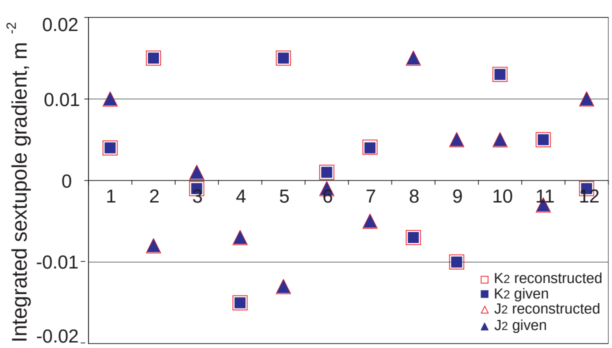

Figure 2: Comparison of 24 numerically reconstructed sextupolar errors (red) with the error set (blue).

of the machine was set to $Q_{x0} = 4.31$ and $Q_{y0} = 3.28$ away from the linear coupling resonance. The beam particle tune was computed using the X coordinate over 2048 turns. By the procedure described in the previous section the numerical coefficients ${}_xQ_t^x$ and ${}_xQ_t^y$ were retrieved and used to solve the linear system (7) for the unknown variables K_{2n} and J_{2n}. The results obtained by applying the NTRM model are shown in Fig. 2. The squares refer to normal components K_2, while the triangular markers refer to skew components J_2. The reconstructed values agree quite well with the given values.

EXPERIMENTAL PROOF OF PRINCIPLE

The presented NTRM model was experimentally validated in reconstruction of sextpolar errors in the SIS18. The coherent betatron oscillations of a coasting beam were excited by a fast kick at extraction energy of about 416 MeV/u and a medium intensity level of approximately $10^8 - 10^9$ particles. The kick of about 0.15 mrad was given on $45°$ in both x- and y-planes. The chromaticity was corrected and 2048 turns were measured. The fractional part of tunes were retrieved using FFT with data filtering [3]. A one turn injection was optimized to create a 'pencil' like beam to exclude finite beam size effects on the tune. Transverse rms-emittances obtained from the measured beam profiles are $(\epsilon_x, \epsilon_y) \approx (1.4, 1.4)$ mm mrad. The tune error estimated in simulations due to the finite beam size is of the order of 10^{-5} in the horizontal and vertical planes for the applied kick amplitude. The co-

Table 1: Additional strengths applied in the sextupoles.

Normal errors	l	ΔK_2	Cal.	Exp.	Rel. Err.,
		$\times 10^{-2}$, $[m^{-2}]$			%
S1	1	-2	-1.999	-1.797	10.5
	2	1	1.001	1.018	1.8
S2	1	-4	-3.998	-4.133	3.3
	2	2	2.002	1.546	22.7
S3	1	-8	-7.995	-7.609	4.9
	2	4	4.007	3.902	2.5
S4	1	5	5.008	4.971	0.6
	2	-3	-2.997	-2.739	8.7
Skew errors	l	ΔJ_2	Cal.	Exp.	Rel. Err.,
		$\times 10^{-3}$, $[m^{-2}]$			%
S1	1	8.32	8.35	7.13	14.6
	2	8.32	8.35	7.29	12.7
S2	1	8.32	8.33	8.76	5.2
	2	-8.32	-8.31	-4.52	45.6

herent oscillations are measured for each steering setting of the changed CO. The tune response with chromatic sextupoles powered on (referred to the setting $S0$) is measured. Then two sextupoles for chromatic correction get a small extra probing strength error, and the tune response is remeasured for the same CO deformation. By subtracting the two tune response curves, the resulting differential tune response depends solely from the extra probing error added on the sextupoles. As the probing errors are folded linearly into the terms ${}_xQ_t^x$, the experimental task is to measure the differential tune response and obtaining ${}_xQ_t^x$. For completeness, the measurement was repeated for several probing error strengths ΔK_2, signs and locations. When the normal probing errors are excited, only horizontal deformation of the CO can reveal them (the terms ${}_xQ_t^y$ and ${}_yQ_t^y$ are absent), see Fig. 3. The same procedure was repeated in the vertical plane with ${}_yQ_t^y$, where additional two skew sextupoles were powered, see Fig. 4. The tune response in the vertical plane to the vertical CO change was measured. The results obtained from the simulations and experiments are summarized in Table 1.

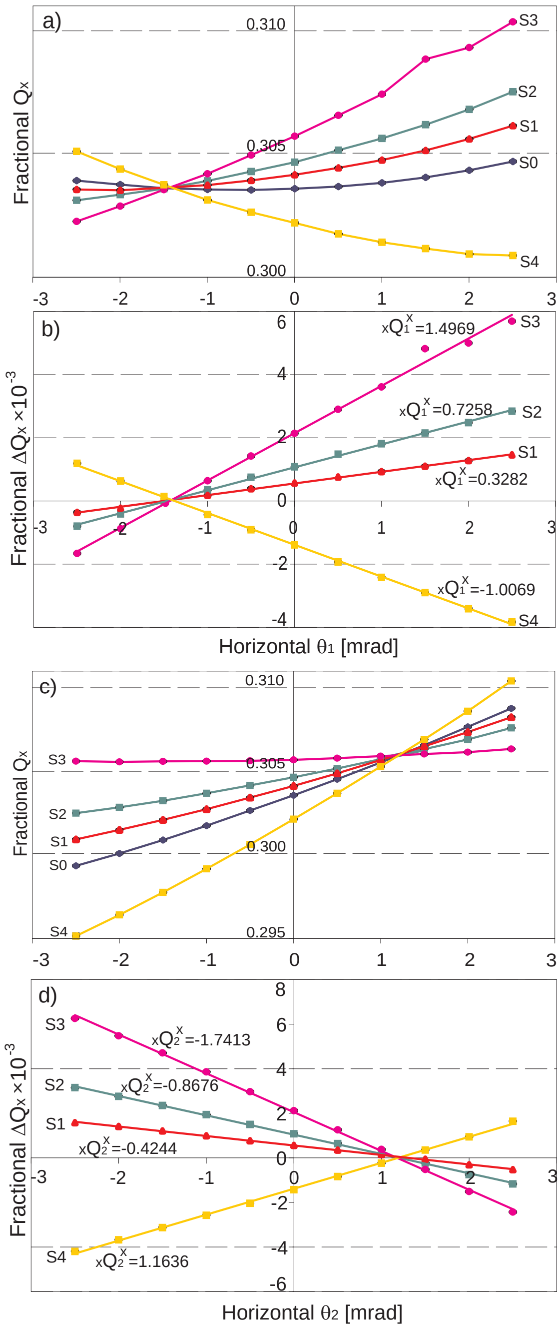

Figure 3: Measured a), c) fractional part of the horizontal tune vs. horizontal steering angles θ_1 and θ_2 for different strengths of the excited two normal sextupoles. The corresponding differential tune response b) and d).

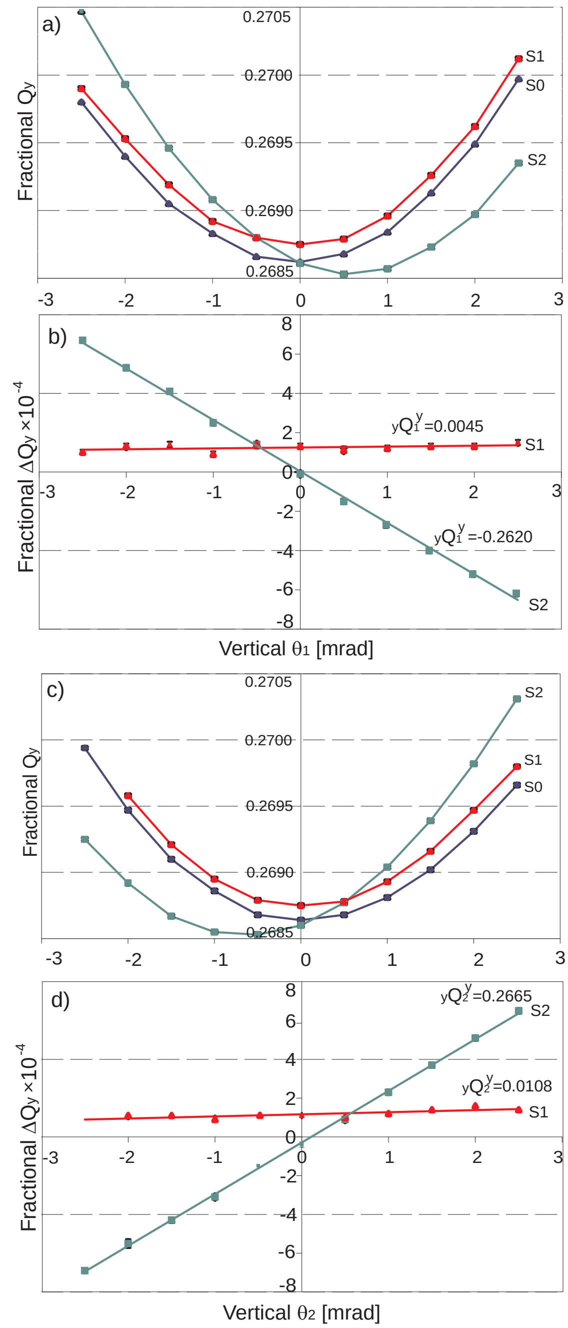

Figure 4: Measured a), c) fractional part of the vertical tune vs. vertical steering angles θ_1 and θ_2 for different strengths of the excited two skew sextupoles. The corresponding differential tune response b) and d).

CONCLUSION

The theoretical basis of the NTRM model was presented. Two normal and two skew sextupolar errors of the order of natural errors ($K_2 \approx 0.01\ m^{-2}$) were reconstructed in the SIS18. In general, the accuracy in reconstruction of sextupolar errors is better than 10 % for sufficiently large errors out of the chosen range. The practical side of the reconstruction technique is still under development. It can be also applied to reconstruct octupolar errors, finally, to reconstruct sextupolar and octupolar field errors in the complete machine in order to compensate them.

REFERENCES

[1] R. Bär and et., “SIS Operations at High Beam Intensities”.

[2] S.Y. Lee, “Accelerator Physics”, 2004.

[3] R. Bartolini and et., “Tune evaluation in simulations and experiments”, Phys Ref. Special Topics, vol.5, 1996, p. 147.