

SPIN FLIP OF DEUTERONS IN COSY - SPINK TRACKING *

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Abstract

Results of measurements of deuteron vector and tensor polarization in the storage ring COSY have been published by the SPIN@COSY collaboration[1]. In this experiment a RF dipole was used that produced spin flip. The strength of the RF-induced spin resonance was calculated from the amount of spin flipping using the Froissart-Stora formula.. In this note we present the simulation of the experimental data (vector polarization) with the spin tracking code *Spink*.

SPINK

The code *Spink* simulates the spin motion through the lattice of a synchrotron[2][3]. In this code each machine element, or an element “slice” if appropriate, is treated as “thin” for spin motion. The spin angle rotation μ is then calculated by the Thomas-BMT spin equation[4]

$$\frac{d\mathbf{S}}{dt} = \frac{q}{\gamma m} \mathbf{S} \times \mathbf{F} \quad (1)$$

with

$$\mathbf{F} = (1 + G\gamma)\mathbf{B}_\perp + (1 + G)\mathbf{B}_\parallel \quad (2)$$

containing the magnetic field components transverse and longitudinal with respect to the velocity \mathbf{v} , where G is the particle gyromagnetic ratio ($=-0.142987$ for deuterons) and γ the Lorentz energy factor. The spin rotation is calculated along the particle orbit, while the magnetic field that appears in the equations are calculated in the laboratory reference frame.

The vector spin is treated as a 3-dimension real vector. *Spink* uses a coordinates system that defines as: \hat{x} , radial, \hat{y} , vertical, and \hat{z} , longitudinal.

Some details of the algorithms used in *Spink* are presented in a companion paper (by A.Luccio and F.Lin) to this Conference.

RF DIPOLE SIMULATION

The COSY experiment was performed using an RF dipole that produced an horizontal oscillating magnetic field at a location on the beam trajectory. The RF field gives a vertical periodic kick to the orbit and produces a proportional kick to the deuteron spin. The orbit kick modulates the vertical betatron motion of the beam. When the frequency of modulation of the orbit f_{RF} , the frequency of

spin precession (spin tune) $G\gamma$ and the frequency of circulation of the beam in the ring f_c match through the condition

$$f_{RF} = f_c(k \pm G\gamma), \quad (3)$$

with k an integer, spin flips are excited deflecting the spin from its direction (originally vertical) to a direction closer to the horizontal.

To obtain this flip, the frequency of the dipole was linearly varied in time, while the momentum of the deuteron beam was kept constant, until the condition (3) was met, with $k = 1$.

In *Spink* the expression for the kick imparted to the vertical component of the momentum p_y is

$$\begin{cases} K = \frac{1}{B\rho} \left(\int B_{RF} d\ell \right) \cos \phi, \\ \phi = 2\pi \int f_{RF}(t) dt, \end{cases} \quad \frac{\delta p_y}{p} = K, \quad (4)$$

with f_{RF} a linear function of time. $B\rho = pc/c$ is the momentum of the beam, $\int B_{RF} d\ell$ the strength of the RF dipole, and p_y the vertical momentum of a deuteron.

The spin precession kick by the RF dipole is simply expressed in *Spink* as

$$\mu = G\gamma K. \quad (5)$$

The spin matrix for the rotation of the vector spin in the RF dipole is (rotation around an horizontal axis, to first order in the components of the particle velocity)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \mu & \sin \mu \\ 0 & -\sin \mu & \cos \mu \end{pmatrix}. \quad (6)$$

The result that leads to Eq.(5) to first order is obtained from the Thomas-BMT equation integrated through the RF dipole, when the vertical oscillations of the beam are taking into account. This results is in agreement with a recent note by E.Courant[5].

With *Spink* we simulated the results of the cited SPINK@COSY article, in particular the curves of Fig.2, measured vector deuteron polarization for different speed of crossing the resonance, and Fig.4, resonance strength for different vertical betatron tune of COSY.

We tracked the particles in the same condition as in the experiment, machine and beam parameters, at constant momentum of the deuterons, namely

$$pc = 1.85 \text{ GeV},$$

and with an RF Dipole strenght

$$B_{RF} d\ell = 6.10^{-4} \text{ T.m.}$$

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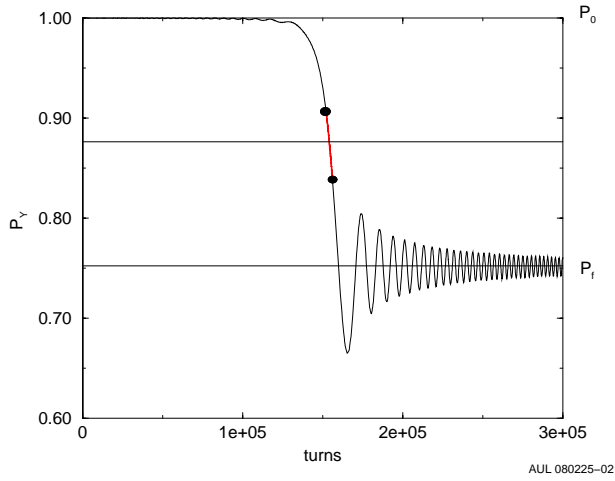


Figure 1: Example of spin motion. Vertical component of the polarization vs. turn number of particles in COSY. The vertical betatron tune for this example was $\nu_y = 3.783$. The horizontal lines mark the final polarization P_f and the value of the polarization in the center of the resonance.

The speed of crossing is defined as

$$\alpha = \frac{\Delta f_{RF}}{\Delta t} \text{ Hz/sec}, \quad (7)$$

where the experiment used a fixed $\Delta f_{RF} = 300$ Hz and Δt was varied. The spin showed a flipping at the required frequency, Eq.(3). The vertical vector spin component behaves as shown in Fig.1 Since this is a well isolated spin resonance, we can calculate its strength ϵ_{RF} using the Froissart-Stora formula[6]

$$P_f = P_i \left\{ 2 \exp \left[-\frac{(\pi \epsilon_{RF})^2}{\alpha^2} \right] - 1 \right\}, \quad (8)$$

where P_f and P_i are the final and initial polarization, respectively. Note that Spink does NOT use any pre-calculated formula to predict the resonance strength ϵ_{RF} , which is only calculated from the ratio P_f/P_0 and Eq.(8).

RESULTS

We first simulated the experimental results summarized in Fig.2 of the COSY paper by *Spink* tracking, *i.e.* resonance strength vs. speed of crossing, using Eq.(5) for the spin kick. The results are shown in our Fig.2. The circles are the measured values and the squares the results of *Spink*. Another set of data is shown with diamonds in the figure, obtained from a simulation where the spin kick was made (incorrectly) proportional to $1 + G\gamma$. Since for the deuteron momentum used in the experiment the coefficient for the spin kick is either the spin tune $G\gamma = -0.20084$ or $(1 + G\gamma) = 0.79916$, the results would differ by a factor 4 (hence the title of the SPIN@COSY paper ‘‘Unexpected Reduction...’’). Comparison with the experiment shows that the first value for the spin kick coefficient in the RF Dipole

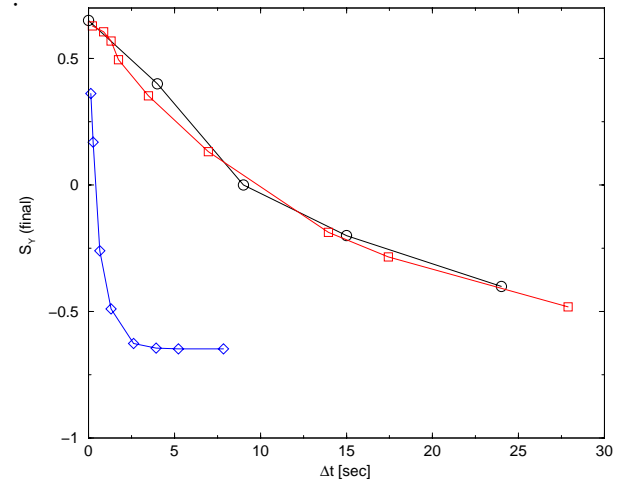


Figure 2: Comparison between *Spink* simulation and measured values for polarized deuterons resonance induced by a RF Dipole. Circles indicate measured values for vector polarization, squares are values simulated with a $G\gamma$ coefficient in the RF Dipole, diamonds with $(1 + G\gamma)$. The vertical axis is the vertical component of the spin, the horizontal axis is the duration of the RFD action, inversely proportional to the speed of resonance crossing.

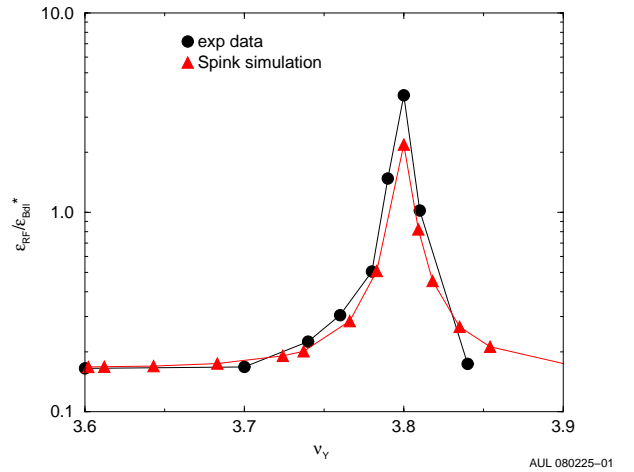


Figure 3: Comparison with Fig.4 of the quoted article. Circles: experimental data, normalized. Triangles: *Spink* simulation.

is correct and the second is wrong, in agreement with the cited Courant note.

The measurements shown in Fig.4 of the COSY paper, resonance strength vs. COSY betatron tune, were also simulated with the results reported in our Fig.3. This figure shows the measured resonance strength, normalized to some theoretical value ϵ_{RF}^* of the resonance strength, assumed proportional to the RF Dipole integrated magnetic field with a coefficient $1 + G\gamma$. The results of our simulation, for sake of comparison, have been normalized to the same value of the resonance strength, however containing a coefficient $G\gamma$. At variance with the previous Fig.2, this

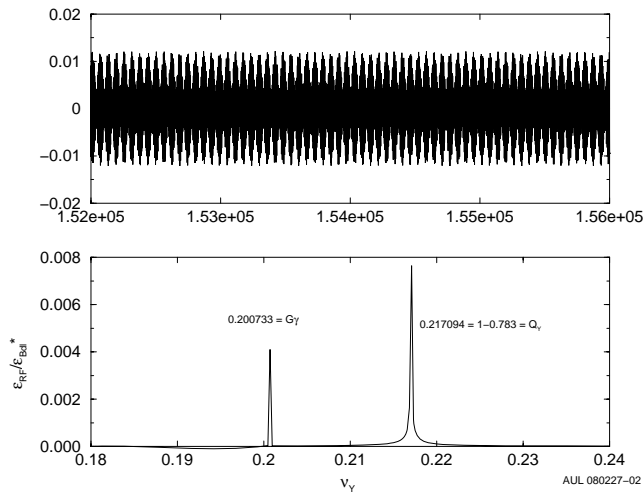


Figure 4: Vertical orbit vs. turn number (upper curve), and its FFT spectrum using $2^{12} = 4096$ points (lower diagram)

curve cannot be used to establish which coefficient should be used in the formula for the spin kick in the RF Dipole, and its cognitive value lays in the description of the behavior of the resonance strength as a function of the vertical betatron tune of COSY, which is well reproduced when we replace the experiment with simulation.

The vertical kick on the orbit by the RF dipole induces a vertical modulation of the vertical betatron oscillation. An FFT spectrum of the vertical beam orbit shows both the betatron tune line and the frequency of the modulation. The position of the modulation spectral line shifts during the RF ramp and coincides with the resonant frequency of Eq.(3) when the deuteron spin actually flips. This is reported in Fig.4 that shows the vertical orbit in the turn interval highlighted in Fig.1 and its spectrum.

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