ACCURATE CALCULATION OF HIGHER ORDER MOMENTUM COMPACTION FACTOR IN A SMALL RING*

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Abstract

The key issues to obtain short electron beam bunch in storage ring is to reduce momentum compaction factor. When the linear momentum compaction factor is small enough, the higher order momentum compaction factor would play important roles in longitudinal beam dynamics. In a small ring, higher order momentum compaction factor is determined not only by the sextupoles and octupoles, but also by fringe field of main magnets. The contributions of different accelerator components to higher order momentum compaction factor were deduced by normal form theory and calculated by Lie algebra. As an example, the momentum compaction factor up to third order of HLS storage ring was calculated numerically.

INTRODUCTION

When the bunch length of electron beam is shorter than the concerned synchrotron radiation wavelength, the radiation would be enhanced coherently [1], which usually lies in the THz and FIR range. In a storage ring, this mechanism maybe essential to production of high power THz synchrotron radiation and become a new type of THz synchrotron radiation source [2]. In the low intensity regime disregarding collective beam effects, bunch length is proportional to square root of slippage factor, $\eta = \alpha_p - 1/\gamma^2$, where the α_p is momentum compaction factor and γ is relativistic energy factor. For ultra-relativistic electron beam, to obtain shorter bunch length the momentum compaction factor would be decreased significantly. In such storage ring, also named quasi-isochronous storage ring, the higher order momentum compaction factors would play important roles in the longitudinal beam dynamics and become concerned issues of a ring design. In the future plan of NSRL (National Synchrotron Radiation Laboratory), there is a possibility to convert the current HLS (Hefei Light Source) storage ring as a dedicated THz synchrotron radiation source [3] and it is necessary to calculate higher order momentum compaction factor accurately. Unlike large storage ring, except for multipole magnets, the fringe fields of main magnets would influence the higher momentum compaction factor considerably. In the following section, the effects of fringe field on higher order momentum compaction factor was discussed qualitatively and calculated by Lie algebra.

THEORY

In a storage ring, the path length of synchronous particle is C_0, due to chromatic effects, the path length of particles with a small momentum deviation is $C(\delta)$, which expresses as:

$$C(\delta) = C_0\left(1 + \alpha_0\delta + \alpha_1\delta^2 + \alpha_2\delta^3 + O\left(\delta^4\right)\right) \quad (1),$$

where the α_0, α_1 and α_2 are linear, second and third order momentum compaction factor and determined by lattice design.

Hamiltonian formalism is powerful tool to study particle dynamics in accelerators. The Hamiltonian of electron in external electro-magnetic field is as following:

$$H\left(x, p_x, y, p_y, z, \delta; s\right) = -a_s - (1+\delta)\left\{\left(1+\frac{x}{\rho}\right)\sqrt{1 - \frac{\left(p_x - a_x\right)^2 + \left(p_y - a_y\right)^2}{(1+\delta)^2}} - 1\right\} \quad (2),$$

where the ρ is curvature radius of reference orbit, the $a_{x,y,s} = \frac{A_{x,y,s}}{P_0}$ are momentum-normalized vector potential, which determined by electro-magnetic field, $\vec{B} = \nabla \times \vec{A}$, $\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$, the δ is relative momentum deviation. In normal case with piecewise constant approximation, the magnetic fields and vector potentials of main magnets are independent of longitudinal positions,

$$a_x = a_y = 0$$
$$a_s = -\frac{1}{2}\left(1+\frac{x}{\rho}\right)^2 + \frac{K}{2}\left(y^2 - x^2\right) + \frac{\lambda}{6}\left(3xy^2 - x^3\right) + \frac{O}{24}\left(x^4 - 6x^2y^2 + y^4\right) \quad (3),$$

where the K, λ and O are normalized quadrupole, sextupole and octupole gradients. According to K. Symon's results [4], in the fringe region of magnets, Maxwellian properties require that higher order components of magnetic field would exist. Including higher order components in fringe region up to order 4, the vector potential of rectangular dipole and normal quadrupole can be expressed as

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$$\begin{cases} a_x = -\dfrac{B_1\cos(\alpha)}{2}y^2 + \dfrac{B_2\sin(\alpha)\cos(\alpha)}{2}xy^2 \\ -\dfrac{B_3\sin^2(\alpha)\cos(\alpha)}{4}x^2y^2 + \dfrac{B_3\cos(\alpha)}{24}y^4 \\ a_y = 0 \\ a_s = -Bx + \left(\dfrac{B_1\sin(\alpha)}{2} - \dfrac{B}{2\rho}\right)x^2 - \\ \dfrac{B_1\sin(\alpha)}{2}y^2 + \left(\dfrac{B_1\sin(\alpha)}{3\rho} - \dfrac{B_2\sin^2(\alpha)}{6}\right)x^3 \\ +\left(\dfrac{B_2\sin^2(\alpha)}{2} - \dfrac{B_1\sin(\alpha)}{2\rho}\right)xy^2 \\ +\left(\dfrac{B_3\sin^3(\alpha)}{24} - \dfrac{B_2\sin^2(\alpha)}{8\rho}\right)x^4 \\ +\left(\dfrac{B_2\sin^2(\alpha)}{2\rho} - \dfrac{B_3\sin^3(\alpha)}{4}\right)x^2y^2 + \dfrac{B_3\sin(\alpha)}{24}y^4 \end{cases} \quad (4),$$

$$\begin{cases} B_1 = \dfrac{B'(s)}{\cos(\alpha)} \\ B_2 = \dfrac{B''(s)}{\cos^2(\alpha)} - \dfrac{B'(s)\sin(\alpha)}{\rho\cos^3(\alpha)} \\ B_3 = \dfrac{B'''(s)}{\cos^3(\alpha)} - \dfrac{3B''(s)\sin(\alpha)}{\rho\cos^4(\alpha)} + \\ \dfrac{B'(s)\left(1+2\sin^2(\alpha)\right)}{\rho^2\cos^5(\alpha)} - \dfrac{B'^2(s)\sin(\alpha)}{(B\rho)\cos^4(\alpha)} \end{cases}$$

$$\rho(s) = \frac{(B\rho)}{B(s)} \qquad \frac{d\alpha(s)}{ds} = -\frac{1}{\rho(s)}$$

$$\begin{cases} a_x = -\dfrac{K'(s)}{2}xy \\ a_y = 0 \\ a_s = \dfrac{K(s)}{2}\left(y^2 - x^2\right) + \dfrac{K''(s)\left(x^4 - 6x^2y^2 + y^4\right)}{48} \end{cases}$$

where the ‘ indicates the derivative respect to longitudinal position. A differentiable function is needed to describe the longitudinal distribution of magnetic field in fringe field, and 4 parameters Enge function was used in this paper [5]. To study the effect of various magnets to path length, the normal form transformation is a convenient mathematical tool [6]. For far away from resonances case, through normal form transformation, ignoring the resonant terms, the effective Hamiltonian was obtained,

$$\begin{cases} H = f_2 + h_3 + h_4 \\ f_2 = \mu_x J_x + \mu_y J_y + \dfrac{1}{2}\alpha_c\delta^2 \\ h_3 = C_{x1}J_x\delta + C_{y1}J_y\delta + C_3\delta^3 \\ h_4 = C_{xx}J_x^2 + C_{xy}J_xJ_y + C_{yy}J_y^2 + C_{x2}J_x\delta^2 + C_{y2}J_y\delta^2 + C_4\delta^4 \end{cases} \quad (5),$$

where the $J_{x,y}$ are horizontal and vertical action. In the effective Hamiltonian, the second order terms give the transverse eigen mode tunes and linear momentum compaction factor, and the third order terms give the transverse chromaticities and second momentum compaction factor, and the third order terms give the second order chromaticities, tune shifts with amplitude and third order momentum compaction factor. The effects on transverse dynamics, such as chromaticities and tune shifts with amplitude were discussed in the earlier papers [7]. The attention of this paper was focused on momentum compaction factors and the effect of Betatron oscillation on path length also ignored. Setting $J_{x,y} = 0$, the momentum compaction factors then obtained directly. The contribution of sextupole component to second order momentum compaction is as following:

$$\alpha_{2,sext} = \sum_{sext}\lambda_i\eta_{xi}^3\delta^2 \quad (6),$$

where η_{xi} is horizontal dispersion at sextupole i. Also, the sextupole-like components of dipole fringe field also contribute to second order momentum compaction,

$$\alpha_{2,sext-like} = \frac{\eta_x^3}{12}\left\{B_2 - B_2\cos(2\alpha) + 4B_0B_1\sin(\alpha)\right\} \quad (7),$$

where meanings of symbols are referred to (4). For a constructed storage ring, the contribution of dipole fringe field is fixed and the sextupole strength should be tuned according to (6) to control second order momentum compaction. Except for second order effects of sextupole and sextupole-like components, the octupole-like components of dipole fringe field and quadrupole fringe field are main sources of third order momentum compaction. If necessary, the octupole magnet can be used to control third order momentum compaction factor. The contribution of octupoles magnet and octupole-like components of fringe field to third order momentum compaction factor is

$$\alpha_{3,oct,oct-like} = \frac{\eta_x^4}{48}\begin{Bmatrix} 3B_0B_2 - K\cdot K'' + 2O \\ -3B_0B_2\cos(2\alpha) - 2B_3\sin^3(\alpha) \end{Bmatrix} (8),$$

where meanings of symbol are referred to (3) and (4). As expectation, the octupole-like components in fringe field of dipole and quadrupole magnets contribute to third order momentum compaction factor directly. In the formula (8), the second order effect of sextupole and sextupole-like components on third order compaction factors were not included and are dependent on Beta functions at sextupoles and Betatron phase advance between sextupoles.

To calculate the momentum compaction factors, instead of using above formula, the Lie algebra was used to include second order effect of sextupole and sextupole-like components. Knowing the Hamiltonian of individual component in storage ring, BCH formula was used in the concatenation procedure. Combining the BCH theorem and normal form transformation, the effective Hamiltonian of whole ring was obtained, including various phase-independent terms, such as tunes, chromaticities, tunes shift with amplitude and momentum compaction factors. For a large ring, fringe fields maybe have little effect on effective Hamiltonian, and using of (3) is accurate enough. For a small ring, the soft fringe field effects should be investigated and the (4) should be used. In real calculation, each magnet with soft fringe was divided into many pieces with infinite length, where the gradient and its derivative were approximated as constant. In this approximation, the higher order components up to order 4 were considered, so the momentum compaction factors up to order 3 is accurate. In calculation, the gradient information came from the fitting results of magnet design data.

HLS EXAMPLES

In the upgrade proposal of NSRL, the current HLS storage ring will be converted as a coherent THz synchrotron radiation source. In the upgrade, the main focusing structure of HLS storage ring was keep and their focusing parameters were changed to obtain low momentum compaction factors. Taking the quasi-isochronous lattice parameters of HLS storage ring as an example, various order momentum compaction parameters were calculated under the piecewise approximation and soft fringe field model of main magnets. The figure 1 and figure 2 displayed the longitudinal distributions of dipole and quadrupole strength obtained by Enge function fitting to measured data. The main results are listed in the table 1. It is not suppressing that the momentum compaction factors considering soft fringe field of main magnets are different with the results of hard edge model. The tunes and linear momentum compaction factor are different due to using hard and soft quadrupole model. The difference in higher order momentum compaction is due to higher order magnetic field components in soft model. Adjusting Q3 and Q4 little, the linear momentum compaction factor of soft model is 2.5e-4.

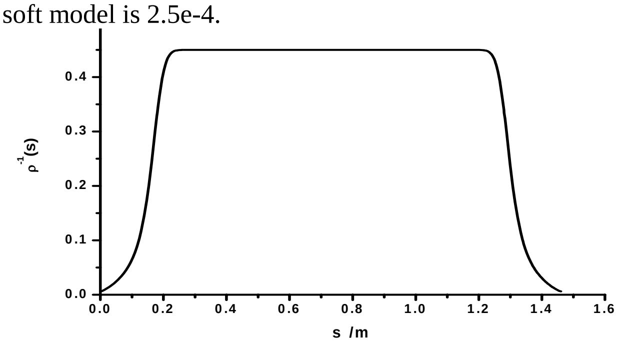

Figure 1: Longitudinal distribution of curvature in dipole

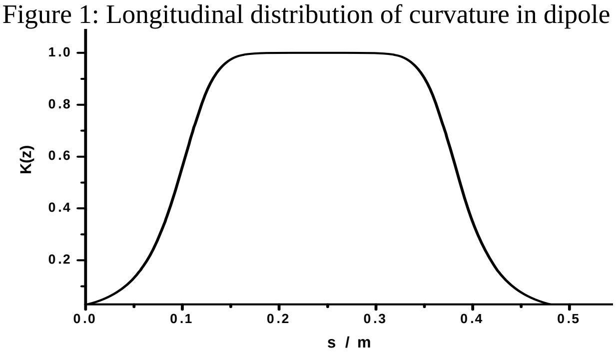

Figure 2: Longitudinal distribution of $K(s)$ in quad

Table 1: Calculated momentum compaction factors

	Hard edge model	Soft edge model
tunes	3.3738/2.71	3.3496/2.704
Corrected chromaticity with same sextupoles	~1/1	~1.2/0.5
α_1	0.0001278	0.00102
α_2	1.49	0.32
α_3	-2.32	-2.80

CONCLUSION

In this paper, the correction of soft fringe field of main magnets to the momentum compaction factors was discussed and calculated. For a small ring, like HLS storage ring, this correction is can not be ignored. For quasi-isochronous operation mode, accurate calculation of momentum compaction factor is important to discussion of nonlinear longitudinal dynamics. If necessary, additional family of sextupoles and octupoles should be added to control second order and third order momentum compaction factors.

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