CRITICAL MAGNETIC FIELD DETERMINATION OF SUPERCONDUCTING MATERIALS*

A. Canabal[#], T. Tajima, LANL, Los Alamos, NM 87545, U.S.A.
S.G.. Tantawi, V. Dolgashev, SLAC, Menlo Park, CA, U.S.A.
T. Yamamoto, University of Tokyo, Tokyo, Japan

Abstract

Using a 11.4 GHz, 50-MW, <1 μs, pulsed power source and a TE013-like mode copper cavity, we have been measuring critical magnetic fields of superconductors for accelerator cavity applications. This device can eliminate both thermal and field emission effects due to a short pulse and no electric field at the sample surface. A model of the system is presented in this paper along with a discussion of preliminary experimental data.

INTRODUCTION

Superconducting RF technology is becoming more and more important. With some recent cavity test results showing close to or even higher than the critical magnetic field of 170-180 mT that had been considered a limit, it is very important to develop a way to correctly measure the critical magnetic field (H_c^{RF}) of superconductors in the RF regime. The system depicted in Fig. 1 allows for the determination of this critical field by measuring the cavity's quality factor at different power levels for different pulse lengths. Details of the system at the Stanford Linear Accelerator Center are given in [1] [2].

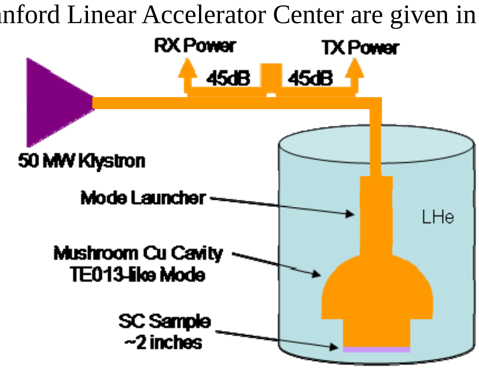

Figure 1: Simplified schematic showing a klystron, a mode launcher, a mushroom cavity and transmitted and reflected pickup ports.

SYSTEM OVERVIEW

The main component of the system [1][2], which operates at 11.424GHz, is the copper "mushroom" cavity, which does not present surface electric fields and concentrates most of the magnetic field on the bottom flange, i.e. where the superconducting sample is placed. Figure 2 shows the electromagnetic fields inside the cavity.

Preliminary test results are shown in Fig. 3, where Nb (RRR=250) was tested for pulse lengths of 0.5μs and 1μs at 1Hz and .1Hz of pulse repetition frequency (PRF). These results will be used to verify the model that will be presented in the following sections.

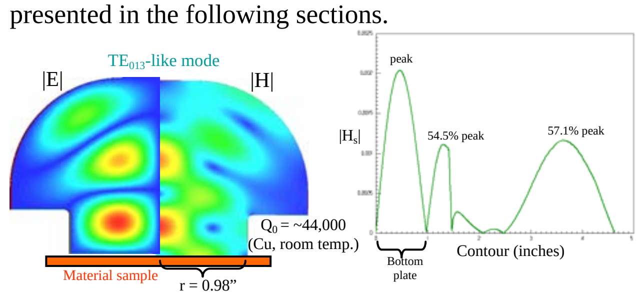

Figure 2: Electric and magnetic fields in the "mushroom" cavity (left) and magnetic field profile along the surface of the cavity (right).

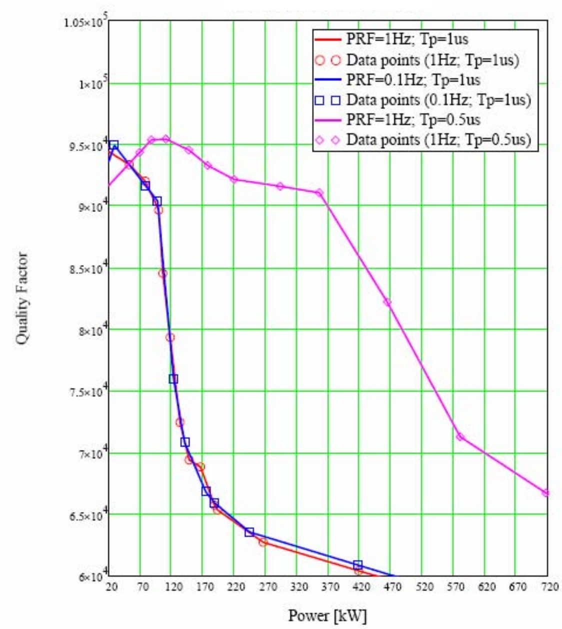

Figure 3: Nb (RRR=250) loaded Q as a function of incident power for different pulse lengths and pulse repetition frequency.

CAVITY MODEL

In order to derive a working model of the system, we assume the response to be limited by the cavity, i.e. represented by the quality factors Q_L, Q_0, and Q_e, or respectively the loaded, unloaded and external quality factor. We start the modeling of the cavity with the power balance equation, according to Fig. 4

$$-\frac{dU_c}{dt} + P_i - P_r - P_L = 0$$

*Work supported by the US Department of Energy.
#acanabal@lanl.gov

where U_c is the energy stored in the cavity, P_i is the incident power, P_r the reflected power and P_L the power losses. We also define the relations $U_c = \alpha E_e^2$ and $P_L = \xi U_c$, where E_e is the electric field exiting the cavity, and α and ξ are constants that can be written as

$$\alpha = \frac{Q_0}{\omega_0 \beta}, \quad \xi = \frac{\omega_0}{Q_0}$$

with β being the coupling coefficient and ω_0 the angular frequency.

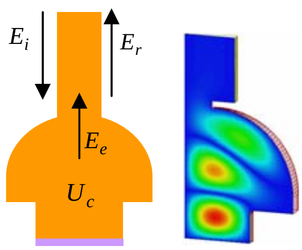

Figure 4: Simplified model of the mushroom cavity.

Given that $E_r = E_e + \Gamma E_i$ and $Q_0 = \beta Q_e$, along with some algebra manipulation we get to the following simple differential equation

$$\frac{2Q_L}{\omega_0}\frac{dE_e}{dt} = \frac{2Q_L}{Q_e}E_i - E_e$$

which has the following general solution

$$E_e = \frac{2Q_L}{Q_e}E_i\left(1 - \mathrm{e}^{-\frac{\omega_0 t}{2Q_L}}\right)$$

Incident Pulse Dependence

For a rectangular incident pulse of period T defined as

$$E_i = \begin{cases} \sqrt{P_i} & 0 \le t \le T \\ 0 & t > T \end{cases}$$

the solution can be particularized as

$$E_e = \frac{2Q_L}{Q_e}\sqrt{P_i}\left(1 - \mathrm{e}^{-\frac{\omega_0 t}{2Q_L}}\right) \qquad 0 \le t \le T$$

$$E_e = \frac{2Q_L}{Q_e}\sqrt{P_i}\left(1 - \mathrm{e}^{-\frac{\omega_0 T}{2Q_L}}\right)\mathrm{e}^{-\frac{\omega_0 (t-T)}{2Q_L}} \qquad t > T$$

from which all other fields can be derived. Of particular interest for the determination of H_c^{RF} is the value of $U_c = \frac{Q_e}{\omega_0}E_e^2$, which becomes

$$U_c = \frac{4Q_L^2}{\omega_0 Q_e}P_i\left(1 - \mathrm{e}^{-\frac{\omega_0 t}{2Q_L}}\right)^2 \qquad 0 \le t \le T$$

$$U_c = \frac{4Q_L^2}{\omega_0 Q_e}P_i\left(1 - \mathrm{e}^{-\frac{\omega_0 T}{2Q_L}}\right)^2 \mathrm{e}^{-\frac{\omega_0 (t-T)}{Q_L}} \qquad t > T$$

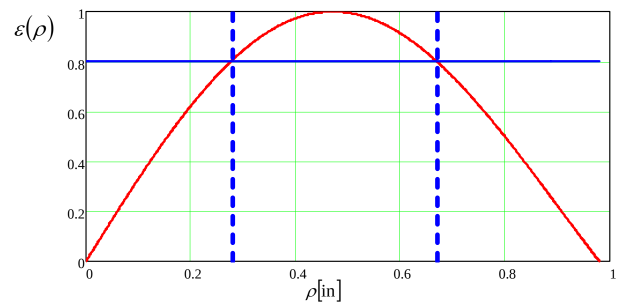

Figure 5: Normalized surface magnetic field.

During each experiment at a given input pulse width T, different values of Q_L are collected for different values of P_i. These Q_L values are calculated by fitting a "Q circle" to the corrected S_{11} data in the complex plane. Therefore, the previous expressions can then be used to calculate the stored energy in the cavity along with the other fields and parameters.

Critical Magnetic Field

To determine the value of H_c^{RF}, an understanding of the sample quenching from the superconducting state is necessary. We start by decomposing the loaded quality factor Q_L

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_c} + \frac{1}{Q_s},$$

where Q_c represents the quality factor due to the copper cavity and Q_s the quality factor due to the sample. Since the external quality factor of the coupling aperture is 106000, we can solve for the two absolute states based upon the experimental data in Fig. 3:

- Fully superconducting sample: $Q_L \approx 95000$

$$Q_s = \infty \Rightarrow \frac{1}{Q_c} = \frac{1}{Q_L} - \frac{1}{Q_e} \Rightarrow Q_c = 915455$$

- Fully normal conducting sample: $Q_L \approx 60000$

$$\frac{1}{Q_s} = \frac{1}{Q_L} - \frac{1}{Q_e} - \frac{1}{Q_c} \Rightarrow Q_s^{nc} = 162857$$

According to Fig. 2, the surface magnetic field on the sample can be expressed as a Bessel function, namely we can write

$$H_s(\rho) \propto J_1\left(\frac{3.8317\rho}{a}\right)$$

where a is the sample radius, which is 0.98 inches. Moreover, we can write the normalize field as

$$\varepsilon(\rho) = \frac{H_s(\rho)}{\max[H_s(\rho)]}$$

which is represented in Fig. 5. Therefore, as the surface magnetic field is increased beyond the sample's critical value, part of the sample becomes normal conducting while the rest is still superconducting. This is depicted in Fig. 6. It is important to mention that since the incident pulse is so short, there are no pulsed heating effects and consequently no thermal runaway can occur. Also note that the fully normal conducting state could only be achieved by an infinite surface magnetic field since the

Bessel function has zeros at the center and at the edges of the sample. However, the asymptotic value of $Q_L \approx 60000$ previously used for this calculation is a good approximation.

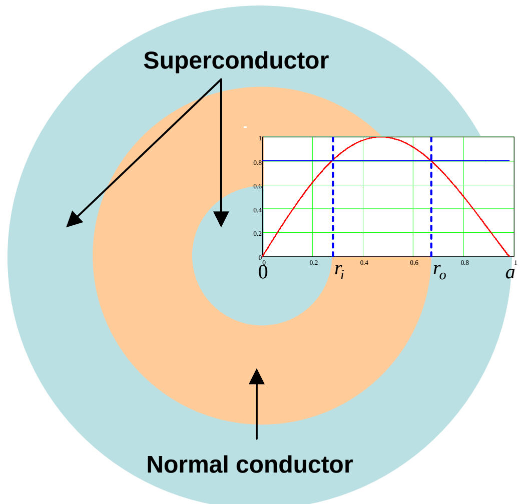

Figure 6: Quenching process of the superconducting sample as the surface magnetic field (Bessel function) increases.

For the variation of Q_L as the sample quenches we write the sample quality factor as

$$Q_s = \frac{\omega_0 U_c}{R_s P_L^s} = \frac{\omega_0 \mu_0 \iiint_V |\varepsilon(\rho)|^2 dv}{R_s \iint_S |\varepsilon(\rho)|^2 ds} = \frac{\omega_0 \mu_0 \iiint_V |\varepsilon(\rho)|^2 dv}{R_s \int_{r_i}^{r_o} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}$$

that can be rewritten as

$$Q_s = \frac{\omega_0 \mu_0 \iiint_V |\varepsilon(\rho)|^2 dv}{R_s \int_{r_i}^{r_o} 2\pi\rho |\varepsilon(\rho)|^2 d\rho} \cdot \frac{\int_0^{\rho=a} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}{\int_0^{\rho=a} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}.$$

This yields to

$$Q_s = Q_s^{nc} \cdot \frac{\int_0^{\rho=a} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}{\int_{r_i}^{r_o} 2\pi\rho |\varepsilon(\rho)|^2 d\rho} = Q_s^{nc} \cdot \frac{9.322 \cdot 10^{-4}}{\int_{r_i}^{r_o} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}$$

By combining all the quality factors together we can write

$$Q_L(\varepsilon) = \left[\frac{1}{Q_e} + \frac{1}{Q_c} + \frac{\int_{r_i(\varepsilon)}^{r_o(\varepsilon)} 2\pi\rho |\varepsilon(\rho)|^2 d\rho}{Q_s^{nc} \cdot 9.322 \cdot 10^{-4}} \right]^{-1}$$

where $r_i(\varepsilon)$ and $r_0(\varepsilon)$ are the numerically calculated half inverse of the normalized magnetic field ε of Fig. 5.

RESULTS

Fitting of the experimental data to this model is shown in Fig. 7. The agreement is very good except in the Q-slope region, which was not considered in the model.

For the calculation of the critical magnetic field we assume that the critical field squared and the stored energy are related by a constant

$$|H_{max}|^2 = \zeta U_c$$

Using a commercial electromagnetic solver package such as Ansoft HFFS or CTS Microwave Studio, we can determine the value of the constant to be

$$\zeta = 4.7059 \times 10^{10} \frac{A^2}{m^2 J}$$

Also, since we now know how to calculate the value of U_c for different pulse lengths, we can solve for the critical magnetic field. From our experiments with both 0.5 μs and 1 μs, the sample should quench at the same field regardless of the incident pulse. The stored energy can then be found to be $U_c = 0.05J$, and this value yields a critical magnetic field of a little over 60mT.

CONCLUSIONS

This value of 60mT is derived from both 0.5 μs and 1 μs incident pulses. However, it is known that Nb should have a H_c^{RF} of at least 180 mT. Possible causes of this discrepancy are:

- Temperature in the RF surface is not exactly 4.2K. A value of 60mT assuming 180mT as the critical field yields to a temperature of about 7.5K on the sample's surface. Further tests will enforce temperature of the sample to be that of the liquid helium by improving the design of the sample holder
- Pulsed heating is not taking into account. Next experiments will test smaller pulse lengths to guarantee that only magnetic quench takes place.

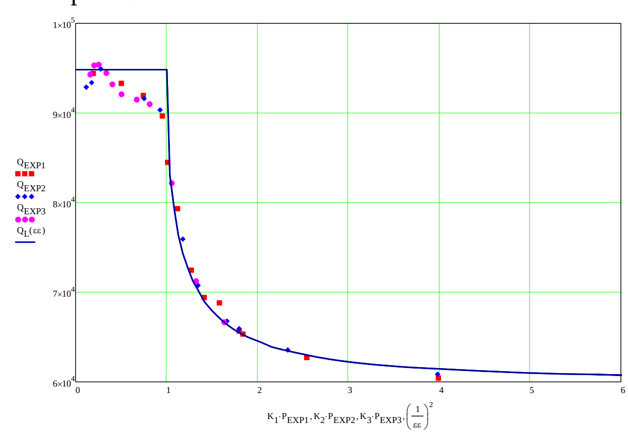

Figure 7: Experimental data fit to derived model.

REFERENCES

[1] C. Nantista et al., "Test Bed for Superconducting Materials," *Proceedings of the 21st Particle Accelerator Conference (PAC05)*, Knoxville, TN, May 16-20, 2005, p. 4227.

[2] S. Tantawi et al., "Superconducting Materials Testing With a High-Q Copper RF Cavity," *Proceedings of the 22nd Particle Accelerator Conference (PAC07)*, Albuquerque, NM, June 22-26, 2007.