

# COLLECTIVE EFFECTS IN THE CLIC DAMPING RINGS\*

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## Abstract

Possible performance limitations coming from collective effects in the CLIC damping rings are the subject of this paper. In particular, the consequences of space charge, due to the very high beam brilliance, and of the resistive wall impedance, due to the locally very small beam pipe, are considered potentially dangerous in spite of the high beam energy. Space charge has been studied in detail with the HEADTAIL code, which was modified in order to take into account a finer lattice structure. This study also includes requirements on the broad band impedance of the damping rings and ion effects in the electron ring (electron cloud in the positron ring is treated in a companion paper). Its goal is to identify all the potential design constraints determined by these phenomena.

## INTRODUCTION

Collective effects can be the bottle-neck in the performances of any accelerator and usually pose limitations to its ultimate performances. In the design stage of an accelerator, it is necessary to budget a total impedance that is consistent with the beam intensity requirements. Besides, the intensity requirement must be checked not to clash with other possible multi-particle limitations coming from space charge, electron/ion effects, Intra-Beam Scattering.

The impedance of an accelerator is usually thought of as made of three main contributions: resistive wall, several narrow band resonators modeling cavity-like objects, and one broad-band resonator that models the rest of the ring. The broad-band resonator, which models the global effect of all discontinuities of the beam pipe and several non-resonating objects (like pick-ups, kickers, etc.), is short range and mostly affects the particle dynamics within one single bunch. Both resistive wall and narrow band resonators usually produce severe multi-bunch effects, since they are associated to slowly decaying wake fields causing mainly the motion of one bunch to affect subsequent bunches.

The aim of this paper is to review the impact of collective effects for the CLIC Damping Rings, whose parameters are summarized in Table 1.

Table 1: Damping Ring parameters used in our study

Energy	$p_0$ (GeV)	2.424
Norm. transv. emitt.	$\epsilon_{xn,yn}$ (nm)	381, 4.1
Bunch length	$\sigma_z$ (mm)	1.53
Momentum spread	$\sigma_\delta$	$1.43 \times 10^{-3}$
Bunch spacing	$\Delta T_b$ (ns)	0.5
Bunch population	$N_b$	$4.1 \times 10^9$
Circumference	$C$ (m)	365.2
Coupling	(%)	0.13
Mom. compact.	$\alpha$	$8 \times 10^{-5}$
Number of bunches	$n_b$	312
Tunes	$Q_{x,y,s}$ (m)	69.82, 33.80
Store time/train	$T_{st}$ (ms)	20
Energy loss	$\Delta E$ (MeV/turn)	3.857
Damping times	$\tau_{x,y,z}$ (ms)	1.5, 1.5, 0.74
RF frequency	$f_{rf}$ (GHz)	2
RF voltage	$V_{rf}$ (MV)	4.115
Bend length	$L_{bend}$ (m)	0.545
Bend chamber rad.	$R_{bend}$ (cm)	2
Number of bends	$N_{bend}$ (m)	96
Wiggler length	$L_w$ (m)	2
Wiggler field	$B_w$ (T)	2.5
Number of wigglers	$N_w$ (m)	76
Wiggler radius	$r_w$ (mm)	9

## SINGLE BUNCH EFFECTS

### Space Charge

The incoherent (direct) space charge tune spread is given by:

$$\Delta Q_y = \frac{N_b r_e C}{(2\pi)^3 / 2 \gamma^3 \sigma_z} \sqrt{\frac{1}{\epsilon_y}}$$

$$\left\langle \frac{\sqrt{\beta_y}}{\sqrt{\beta_x \epsilon_x + D_x^2 \sigma_\delta^2 + \sqrt{\beta_y} \epsilon_y}} \right\rangle \simeq 0.188 \quad (1)$$

In this formula,  $\epsilon_{x,y}$  are the beam geometric emittances, which can be obtained from the normalized ones dividing by  $\gamma$ . The incoherent tune shift is certainly more critical in the vertical plane. The horizontal one, due to the larger emittance, is in the order of 0.01. Furthermore, the coherent tune shift coming from image charges is much below this value (in both planes), because it is inversely proportional to the square of the mean chamber radius, rather than to the product of the beam rms sizes.

A possible emittance growth due to space charge has been studied with macroparticle simulation by tracking the beam in the DR over 1000 turns with a variable number of space charge kick locations chosen randomly in the lattice (and

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averaging over different seeds). It is clear that, with a high enough number of kicks the growth levels at about 10% (see Fig. 1), which could be still intolerable if extrapolated to the 16000 turns of residence time of the beams in the storage rings. However, this might still be a numerical effect and besides, it could quickly saturate as the emittance grows and the space charge effect is reduced. Simulation campaigns with higher numbers of space charge kicks and scans in the parameter space are foreseen in the future. It is also planned to track the beams with space charge and at least one nonlinearity in the machine to study whether possible phenomena of periodic resonance crossing could be important for the CLIC-DRs.

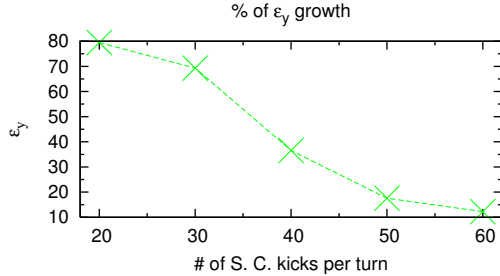


Figure 1: Space charge induced emittance growth as a function of the number of space charge kicks given to the bunch around the ring.

### Longitudinal and Transverse Broad-band Impedance

The broad-band resonator model gives a description of the impedance at frequencies  $\sim \omega_r = c/b$ , where  $b$  is the beam pipe half-height. In the higher frequency range, the more detailed diffraction model gives better results, but it needs to be applied only on a scale  $|z| \ll b$  (where  $z$  is a longitudinal coordinate along the bunch). In the lower frequency range, the broad band model does not take into account possible resonant responses to sharply defined frequencies (cavity modes). These cavity modes occur at frequencies below the cut-off frequency  $c/b$  (since they have to be trapped) and give rise to wake functions which ring for long periods of time. Neglecting these long-range contributions, we certainly do not get accurate results from the model in the range  $|z| \gg b$ . Taking into account that the CLIC-DR vacuum chamber radius ranges between 8 and 20 mm, the frequency  $\omega_r$  lies between 2.4 and 6 GHz. A sufficiently smooth vacuum chamber design, as is in modern accelerators, can easily allow for an impedance of about  $1 \Omega$ , or even lower. The transverse impedance is roughly linked to the longitudinal one by:

$$Z_1^\perp(\omega) = \frac{C}{\pi b^2} \frac{Z_0^\parallel(\omega)}{n}, \quad (2)$$

where  $C$  is the ring circumference. Assuming the longitudinal impedance to be  $1 \Omega$ , we then easily get to a transverse impedance of about  $0.3 - 2 M\Omega/m$ .

A broad band impedance is expected to cause microwave instability above a certain threshold, which we can roughly

evaluate using the Boussard criterion (coasting beam criterion extended to bunched beams). However, even below threshold, there could be enough potential well distortion as to cause bunch lengthening, which can be associated to bunch shape deformation or coherent undamped dipole motion of the bunch within the bucket. The Boussard criterion for longitudinal microwave instability (or turbulent bunch lengthening) is [1]:

$$\left| \frac{Z_0^\parallel}{n} \right| < 1.7 \ln(2) Z_0 \frac{|\eta| \gamma}{N_b r_0} \sigma_\delta^2 \sigma_z \quad (3)$$

Plugging the CLIC-DR values into the above formula, we obtain the expected threshold of microwave instability to sit around  $44 m\Omega$ . This value is very low, but it becomes reasonably high ( $1-7 \Omega$ ) if we include in the above formula the suppression factor  $(b/\sigma_z)^2$  [2], which for the DRs ranges between 30 and 170.

In the transverse plane, head-tail interaction through a wake field can drive the bunch unstable. Regular head-tail instability can be avoided by running the ring at some slightly positive value of chromaticity. Strong head-tail instability, also known as Transverse Mode Coupling Instability (TMCI), can nevertheless affect the DR bunch and cause rapid beam loss if the transverse impedance is strong enough. For a round beam in a round chamber, a criterion to find the threshold of TMCI, obtained from the application of the kinetic theory and the mode coupling condition, applicable in the two limits of short or very long bunch, reads [3]:

$$\xi < \frac{Q_s}{\omega_r \sigma_t} \quad \text{if } \omega_r \sigma_t \leq 1 \quad (4)$$

$$\xi < \sqrt{2} Q Q_s (\omega_r \sigma_t)^2 \quad \text{if } \omega_r \sigma_t \gg 1 \quad (5)$$

where

$$\xi = \frac{\omega_r / 2\pi < \beta_y > R_T N_b e}{3.75 Q E / e}$$

If we consider  $\omega_r$  ranging between  $2\pi \times 2.4$  and 6 GHz, the discriminating number  $\omega_r \sigma_t$  remains below 1. Consequently, the DR bunch is in the regime of short bunch and Eq. (4) should be applied to predict the threshold for the TMCI. In terms of broad-band resonator parameters and in convenient units, we can simply recast this equation in the following form:

$$\frac{R_T [\text{k}\Omega/\text{m}] f_r^2 [\text{GHz}]}{Q} \leq 0.6 \frac{E [\text{GeV}] Q_s}{< \beta_y > [\text{m}] Q_b [C] \sigma_t [\text{ps}]}, \quad (6)$$

where  $Q_b = N_b e$  represents the bunch charge in Coulomb and  $f_r = \omega_r / 2\pi$ .

Using the DR parameters from Table 1, Eq. (6) yields

$$\frac{R_T [\text{M}\Omega/\text{m}] f_r^2 [\text{GHz}]}{Q} \leq 5.5 \times 10^2,$$

which translates into a threshold of  $15 M\Omega/m$  for the  $R_T$  parameter of the transverse broad band resonator in the frequency range of interest (6 GHz) and for a quality factor  $Q = 1$ . As said above, the present design of storage rings should allow for an impedance well below this value.

## MULTI-BUNCH EFFECTS

### Coupled Bunch Instabilities

The rise time of the coupled bunch modes caused by resistive wall are calculated from the imaginary part of the formula:

$$\Delta\omega_{\mu,m}^{x,y} = -\frac{i}{2} \frac{\Gamma(m+1/2)}{2^m m!} \frac{N_b r_0 c^2 \langle \beta_{x,y} \rangle}{\gamma C \sigma_z} \cdot \frac{\sum_{p=-\infty}^{\infty} Z_1^{x,y}(\omega_p) h_m(\omega_p - \omega_{\xi_{x,y}})}{\sum_{p=-\infty}^{\infty} h_m(\omega_p - \omega_{\xi_{x,y}})}, \quad (7)$$

with

$$h_m(\omega) = \left( \frac{\omega \sigma_z}{c} \right)^{2m} \exp\left(-\frac{\omega^2 \sigma_z^2}{c^2}\right)$$

$$\omega_p = (pM + \mu + Q_{x,y} + m\nu_s)\omega_0$$

$$\mu = 0, 1, \dots, M-1$$

$$m = 0, \pm 1, \pm 2, \dots$$

We consider the resistive wall impedance (inversely proportional to the square root of the conductivity and to the third power of the beam radius) and use the pipe vertical size from the wigglers (flat structure).  $M$  is assumed to be the harmonic number of the radiofrequency. The numbers obtained are pessimistic because 1) the wigglers only cover half of the ring, therefore the worst growth time would still be about twice larger, and 2) they assume a ring fully filled with bunches, which would approximately scale by another  $n_b/M$  factor the real growth rate of the instability. The growth/damping rates for  $m = 0$  are plotted in Fig. 2. The minimum growth time of 1 ms corresponds to about 1000 turns (and is also in the order of the vertical damping time) and can be easily damped with a transverse feedback.

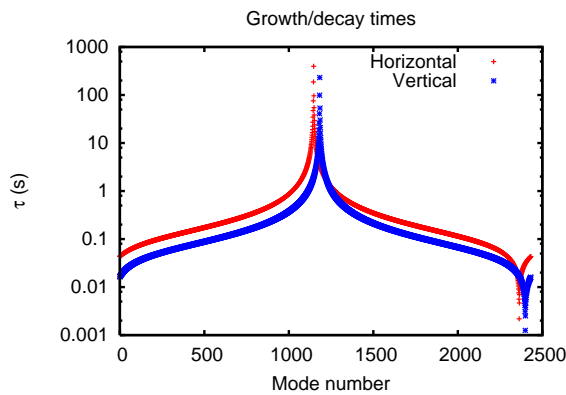


Figure 2: Growth/damping times of the  $m = 0$  coherent modes as a function of the  $\mu$  mode number for a coupled bunch resistive wall instability in the CLIC-DRs.

### Ion Effects

In the electron DR, the ion oscillation frequency inside the bunch train during the store is in the range of 300 MHz (horizontal plane) to about 1 GHz (vertical plane), to be divided by the the square root of the mass number of the ion  $\sqrt{A}$ . However, not all ion types will be trapped in the bunch train, and the critical mass for trapping of a singly charged ion is:

$$A_{crit} = \frac{N_b \Delta T_b c r_p}{2 \sigma_y (\sigma_x + \sigma_y)} \simeq 13 \quad (8)$$

This means that ions like  $\text{CO}^+$ ,  $\text{N}_2^+$  or  $\text{H}_2\text{O}^+$  will be trapped and will accumulate around the electron beam, potentially becoming a source of fast ion instability. The induced tune shift introduced by the ion cloud at the end of the train is given by

$$\Delta Q_{ion} \simeq \frac{N_b n_b r_e C}{\pi \gamma \sqrt{\epsilon_x \epsilon_y}} \left( \frac{\sigma_{ion} p}{k_B T} \right) \simeq 0.05 \quad (9)$$

where a standard pressure  $p=1$  nTorr and 2 MBarn of ionization cross section were assumed. The exponential rise time of the fast ion instability is:

$$\tau_{inst} \simeq \frac{0.1 \cdot \gamma \sigma_x \sigma_y}{N_b n_b c r_e \beta_y \sigma_{ion}} \left( \frac{k_B T}{p} \right) \sqrt{\frac{8}{\pi}} \quad (10)$$

and it amounts to about 1.1  $\mu\text{s}$ , which is about one revolution time and would therefore require a very demanding multi-bunch feedback system to be controlled. Therefore, a lower vacuum would be desirable.

## CONCLUSIONS

In conclusion, the requirements in terms of longitudinal and transverse broad band impedance (low frequency part of the total machine impedance) are not too stringent because a few  $\Omega$  in longitudinal and apparently a few  $\text{M}\Omega/\text{m}$  in the transverse plane would still be acceptable to guarantee the beam stability against single bunch effects. Space charge causes a very large incoherent tune spread, dangerous in terms of resonance crossing, and can be potentially a source of emittance growth. However, this specific study needs further investigation and cleaning of possible numerical effects.

Resistive wall multi bunch effects do not seem to be critical because of the large rise time of the associated instabilities (in the order of the damping times). The fast ion instability poses a constraint on the acceptable vacuum level, which must be much smaller than 1 nTorr.

## REFERENCES

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