STUDY OF THE VALIDITY OF K. BANE'S FORMULAE FOR THE CLIC ACCELERATING STRUCTURE*

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Abstract

The comprehension of short range wakefields is essential for the design of CLIC. Useful tools are the Karl Bane's formulae which provide geometrical parameterization of the short range wake for periodic rotational-symmetric structures. The comparison of 2D computations based on ABCI with predicted results and the study of the range of validity of these formulae are the subjects of this paper. An extended model for rounded iris structures is also proposed.

INTRODUCTION

The short range wakefield is one of the main limitations to be taken into account in the design of the CLIC accelerating structures.

The short range wake, that can not be damped, determines the maximum charge per bunch and the minimum aperture of the structures, and has thus a strong impact on the luminosity of the collider and on the efficiency of the accelerator.

The main tools for the study of the short range wake have been provided by Karl Bane in terms of analytical formulae that correlate the wake to the geometrical parameters of a periodic array of cells (1,2). K. Bane work has been done for the NLC accelerating structures. The validation of these formulae for a different geometrical range "CLIC range" is the first target of the present work.

A study of the effect of rounded irises instead of rectangular ones is also presented.

HIGH FREQUENCY LONGITUDINAL IMPEDANCE AND SHORT RANGE WAKE

For a periodic array of cavities according to Yokoya and Bane [1,2] the high frequency impedance (so for large wave number k) is approximately given by:

$$Z_L(k) = \frac{iZ_0}{\pi k a^2}\left[1 + (1+i)\frac{\left(1 - 0.465\sqrt{g/p} - 0.07\, g/p\right)p}{a}\sqrt{\frac{\pi}{kg}}\right]^{-1}$$

with $Z_0 = 377\Omega$, "p" the periodicity, "a" the iris aperture and "g" the cavity length (see Fig. 1).

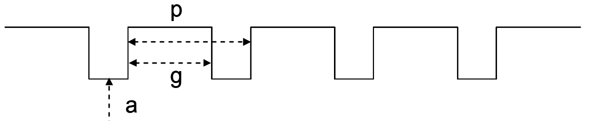

Figure 1: Basic shape of 2D periodic structure.

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The short-range wake is obtained by Inverse Fourier transforming:

$$W_L(s) = \frac{Z_0 c}{\pi}\exp\left(\frac{\pi s}{4 s_{00}}\right) erfc\left(\sqrt{\frac{\pi s}{4 s_{00}}}\right) \tag{1}$$

with

$$s_{00} = \frac{g}{8}\left(\frac{a}{\left(1 - 0.465\sqrt{g/p} - 0.07\, g/p\right)p}\right)^2$$

for $s>0$ and $W_L(s) = 0$ for $s<0$ and s_{00} determined by fitting of numerical results [1].

For small s (1) can be rewritten in the following simpler way:

$$W_L(s) \approx \frac{Z_0 c}{\pi a^2}\exp\left(-\sqrt{\frac{s}{s_{00}}}\right) \tag{2}$$

The relation between the longitudinal wake W_L and transverse wake W_x for small s is the following [2,3]:

$$W_x(s) = \frac{2}{a^2}\int_0^s W_l(s')ds' \tag{3}$$

Combining (2) and (3) we get:

$$W_x(s) = \frac{4 Z_0 c s_{00}}{\pi a^4}\left[1 - \left(1 + \sqrt{\frac{s}{s_{00}}}\right)\exp\left(-\sqrt{\frac{s}{s_{00}}}\right)\right] \tag{4}$$

The (4) is the K. Bane formula for transverse short-range wakefield; the value of S_{00} and its dependence on *a,g,p* has been modified by K. Bane by fitting with numerical results and it is represented by the following expression [2]:

$$S_{00} = 0.169\frac{a^{1.79} g^{0.38}}{L^{1.17}} \tag{5}$$

NUMERICAL COMPUTATIONS

The short range wakefield has been computed with the 2D code ABCI [4] which solves the Maxwell equations in the time domain for axis-symmetric structures. ABCI makes use of a moving mesh which drastically reduces the number of mesh points to be stored. The moving mesh option allows thus the computation of very short bunches.

In order to evaluate the limit of the computation, several Gaussian beams have been computed with the same geometry (a/p=0.41; g/p=0.81; p=4.861 mm). The ratio between the sigma and the mesh density has been kept constant for the different computations. The results show very good agreement for sigmas larger than 70 microns but also not tolerable discrepancies for short beam bunches (sigma of the order of 30-50 microns); see fig. 2. In the rest of this note all the computations have been done for sigmas of 100 microns to have reasonably correct results.

The CLIC bunch length, which is around 40-50 microns, is at the limit; for this reason it is very important to have also a non-numerical approach to study the short range wakefield especially to describe non Gaussian distributions.

K.B. formulae are obviously independent from the bunch length and distribution and are thus the preferred tool to describe short range wakefields.

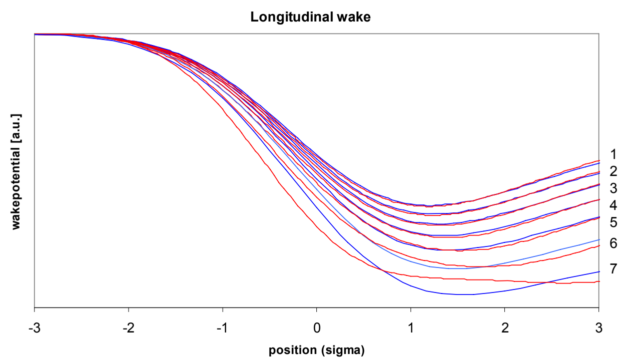

Figure 2: ABCI (red lines) and convolutions of K. Bane (blue lines) longitudinal wake for different sigmas (1=0.15 mm, 2=0.13mm, 3=0.11 mm, 4=0.09mm, 5=0.07mm, 6=0.05mm, 7=0.03mm; for a Gaussian distribution).

K. Bane gives for transverse wakefield (4) (5) the following geometrical range of validity: $0.34 \leq a/p \leq$ and $0.54 \leq g/p \leq 0.89$ [2].

The present CLIC structure studies cover a much larger area in the *a/p*, *g/p* plane, for this region a check of validity of eq. 4 and 5 has been done; see fig. 3.

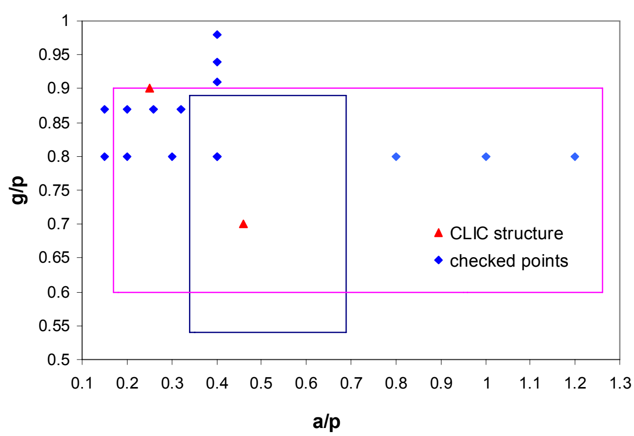

Figure 3: The range of validity gave by K.B. is in blue, the one required by the CLIC structures is in red. The first and last cell of the present CLIC reference structure are shown in red.

The numerical results show that the K. Bane formula range of validity is much larger for the longitudinal wakefield and it covers the full CLIC region in the space *g/p*, *a/p*. For the transverse wakefield, numerical results show good agreement with K. Bane in the full *g/p* range but also not negligible discrepancies for low value of *a/p*; see fig. 4.

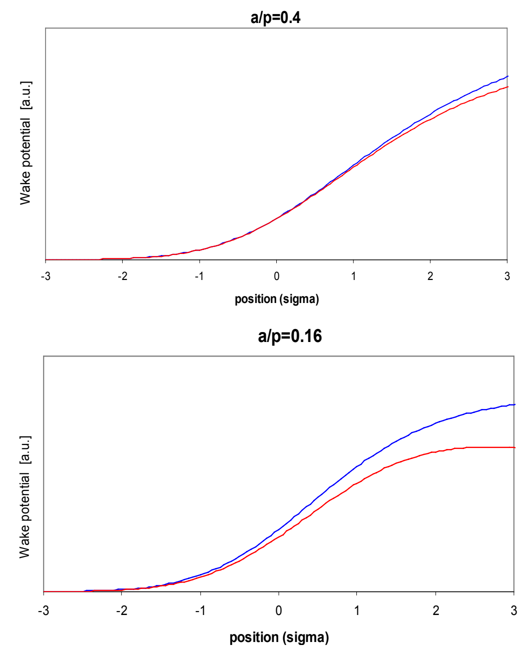

Figure 4: The range of validity of (4) and (5) does not cover the full CLIC range. For small aperture (a/p<0.3) the predicted values (blue) are significantly different of the ones computed with ABCI (red).

The results show how the transverse wake is largely overestimated in the present case (a=2mm, a/p=0.33, g/p=0.8333 and σ= 0.1 mm). A better comprehension of the effect of rounded irises and the validation of eq. 4 over a large range of *a/p* and *g/p* could provide a useful tool for optimising the design of RF structures.

EXTENSION OF THE K.B. FORMULAE TO ROUNDED IRIS STRUCTURES

One of the main limitations of the K. Bane formulae concerns the shapes of the irises; the reference geometry is a simple 2D periodic array of squared irises. Real structures have obviously rounded or elliptical irises for evident reasons of peak electric field and machining.

Both longitudinal wake and transverse wake depend largely on the aperture *a* ($w_l \propto a^{-2}; W_x \propto a^{-3}$); the rounding of the iris introduces thus a significant approximation especially for the transverse wake. The approximation is negligible only for very short bunches ($\sigma_z/a \ll 1$). An extension of the model based on a fitting of numerical results for geometries with rounded irises or a simple combination of different *a, g*, structures are two possible options to improve the estimation of the wake. The second approach is proposed; see fig. 5.

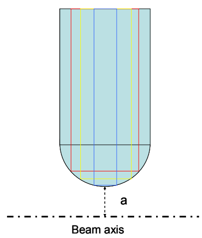

Figure 5: Example of the model; a rounded iris as combination of 4 rectangular ones.

A simple geometry (a/p=0.33, g/p=0.83) has been computed; in this case a is smallest aperture (a=2) and the radius of curvature is (p-g)/2.

The computed ABCI results have been compared with the composition of the convolutions of K. B. formulae of seven different *a, g* structures with the Gaussian beam.

The proposed model to represent rounded irises is thus the following:

$$Wx(s) = \frac{4Z_0 c}{M\pi}\sum_{m=1}^{M}\frac{S_{0m}}{a_m^4}\left[1-\left(1+\sqrt{\frac{s}{S_{0m}}}\right)\exp\left(-\sqrt{\frac{s}{S_{0m}}}\right)\right] \quad (6)$$

where $S_{0m} = 0.169\frac{a_m^{1.79}\, g_m^{0.38}}{p^{1.17}}$ $a_m = a_0 + \frac{(p_0 - g_0)}{2}(1-\sin(\alpha_m))$; $g_m = g_0\cos(\alpha_m)$ and $\alpha_m = 90^\circ\left(1-\frac{m}{M-1}\right)$ and M is a positive integer large enough (M=7 in the present case) to provide a good representation of the problem.

The results are in good agreement with the computations especially for the transverse wake; see fig. 6.

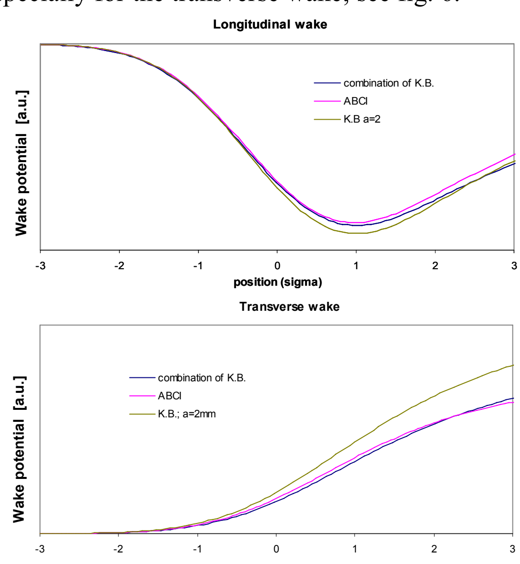

Figure 6: Longitudinal wake (top) and dipole wake (bottom).

The results show how the transverse wake is largely overestimated in the present case (a=2mm, a/p=0.33, g/p=0.8333 and σ= 0.25 mm). A better comprehension of the effect of rounded irises and the validation of Eq. 6 over a large range of *a/p* and *g/p* could provide a useful tool for optimising the design of RF structures.

Eq. 6 should also be optimised in order to fit different sigma values; the use of weight coefficients for the different terms *(a,g)* seems to be the best solution.

CONCLUSION

The range of validity of K. Bane formulae covers almost all CLIC requirements with the exception of the dipole wake for low value of *a/p* where the wake seems to be overestimated. A further approximation of K.B. formulae is due the rounding of the irises; also in this case the analytical formulae tend to overestimate the wake. Eq.6 seems to be a promising tool to describe the effect of the rounded iris. A new fitting of Eq. 6 for the CLIC geometrical range is under way.

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