Abstract

The clean environment of a lepton linear collider allows high-precision measurements for physics analyses. In order to exploit this potential, precise knowledge about the polarization state of the beams is also required. In this paper we concentrate on depolarization effects caused by the intense beam-beam interaction, which is expected to be the dominant source of depolarization. Higher-order effects, as well as critical analyses of the theoretical assumptions used in the past and theoretical improvements in the derivation of suitable equations, are given. Updates on existing simulation programs are reported. Numerical results for the design of the International Linear Collider (ILC) are discussed.

BASELINE ILC SOURCES

The full physics potential of the ILC can be realized only with polarized e− and e+ beams [1]. Polarized e− with a degree of polarization between 80% and 90% are foreseen for the baseline machine design. The electron source consists of a circularly polarized high-power laser beam and a high-voltage DC gun with a semiconductor photocathode [2], and a helical undulator based positron source [2] has been chosen as the most reliable solution for producing the required flux of order 10^14 positrons per second.

The positron source undulator operates on the main ILC e− beam generating circularly-polarized photons. An overview of a suitable full-scale superconducting undulator module is given in [3]. Polarized positrons are produced via an electromagnetic shower instigated by the circularly polarized synchrotron radiation striking a thin target. The production target has to be carefully designed to cope with the high heat load and strong thermal stresses, see [4]. Using the ILC baseline undulator a e+ polarization of about 30% to 40% should be achievable and could be used to enhance physics at the ILC interaction point (IP). The source can be readily upgraded by extending the undulator length to produce positrons with about 60% polarization. The undulator-based method has previously been experimentally verified by the E166 experiment [5].

DEPOLARIZATION EXPECTATIONS

The characteristics of the physics processes at the IP depend on the luminosity-weighted polarization of the beams. As the polarimeters can only measure the polarization either side of the IP, it is mandatory to model the depolarization processes expected during the beam interaction.

In the following sections we first list the possible depolarization effects in the interaction region and give numerical depolarization results obtained from the CAIN program for both the ILC and CLIC designs. Later in the paper we report on recent theoretical progress in the description of the depolarization effects and incoherent and coherent pair production during beam-beam interactions.

Effects during beam-beam interactions

In general, two effects influence the spin motion in electric and magnetic fields: a) spin precession governed by the Thomas–Bargmann–Michel–Telegdi (T-BMT) equation and b) the spin-flip Sokolov-Ternov (S-T) effect via synchrotron radiation emission. Usually the spin precession effect is dominant, but at higher energy the depolarization due to the S-T effect increases [6]. The analytically-based program CAIN [7] models both of these depolarization effects. In addition models of the two kinds of background processes that occur during beam-beam interactions, the production of incoherent and coherent e+/e− pairs, are also included with certain approximations that are discussed later.
The difference between the initial beam polarization and the luminosity-weighted beam polarization of the beams have been evaluated for the ILC baseline parameters [2] and the CLIC-G design [8], cf. Table 1, using CAIN and are shown in Table 2. In the ILC design, the predicted low values of the field strength parameter, $\Upsilon$, result in very small effects from the coherent first-order background pairs. At higher energy and also for the CLIC-G design with much higher beamstrahlung, significant contributions from the coherent pairs are expected.

Table 1: Parameters sets for the ILC and the CLIC design. More details can be found in [2, 8]

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>ILC baseline</th>
<th>CLIC-G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$/GeV</td>
<td>500</td>
<td>3000</td>
</tr>
<tr>
<td>$N / 10^{10}$</td>
<td>2</td>
<td>0.37</td>
</tr>
<tr>
<td>$n_B$</td>
<td>2625</td>
<td>312</td>
</tr>
<tr>
<td>$\beta_x/mm$</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>$\beta_y/mm$</td>
<td>0.4</td>
<td>0.09</td>
</tr>
<tr>
<td>$\sigma^x/nm$</td>
<td>640</td>
<td>40</td>
</tr>
<tr>
<td>$\sigma^y/nm$</td>
<td>5.7</td>
<td>$\sim$1</td>
</tr>
<tr>
<td>$\sigma_z/\mu$m</td>
<td>300</td>
<td>45</td>
</tr>
<tr>
<td>$D_x$</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>$\Upsilon$</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>$L/10^{34}cm^{-2}s^{-1}$</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2: Comparison of the luminosity-weighted depolarizing effects in beam-beam interactions for the ILC baseline with both 100% polarized and partially polarized beams, and for the CLIC-G parameter set with 100% polarized beams. T-BMT (S-T) denotes effects due to spin precession (synchrotron radiation)

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Depolarization $\Delta P_{tw}$</th>
<th>ILC 100/100</th>
<th>ILC 80/30</th>
<th>CLIC-G</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-BMT</td>
<td>0.17%</td>
<td>0.14%</td>
<td>0.10%</td>
<td></td>
</tr>
<tr>
<td>S-T</td>
<td>0.05%</td>
<td>0.03%</td>
<td>3.4%</td>
<td></td>
</tr>
<tr>
<td>incoherent</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.06%</td>
<td></td>
</tr>
<tr>
<td>coherent</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.3%</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>0.22%</td>
<td>0.17%</td>
<td>4.8%</td>
<td></td>
</tr>
</tbody>
</table>

Investigations of the sensitivity of the depolarization predictions on the form of the T-BMT model in CAIN have been carried out by scaling the field strength used in the model by a factor of 2, and similar investigations of the S-T effect have been carried out by scaling the ratio of initial and final state energies, $\zeta$, by a factor of 2. From these studies, the expected depolarization at the ILC IP is predicted to be $0.21\% \pm 0.01\% \pm 0.03\%$, where the first uncertainty is indicative of the sensitivity of the result to variations in the T-BMT model and the second uncertainty is indicative of the sensitivity to variations in the S-T model. For CLIC the analogous result is $4.8\% \pm 0.1\% \pm 1.5\%$, highlighting the dominance of the S-T effect at high energies.

**DESCRIPTION OF DEPOLARIZATION AND BACKGROUND EFFECTS**

*Spin precession*

Spin precession is described by the Thomas-Bargmann-Michel-Telegdi (T-BMT) equation:

$$\frac{d\vec{S}}{dt} = -\frac{e}{\gamma m^2}(\gamma \alpha + 1)\vec{B}_T + (a + 1)\vec{B}_L$$

$$-\gamma(a + \frac{1}{\gamma + 1})\vec{e}_v \times \frac{\vec{E}}{c} \times \vec{S},$$

(1)

where $a$ describes the anomalous magnetic moment of the electron given by the higher-order corrections to the $ee\gamma$ vertex. In the environment of strong colliding beams, however, the usual perturbation theory cannot be applied, and modified expressions for the anomalous magnetic moment in a medium have been derived in the literature [9]. The expressions have been evaluated in the no-scattering case and using the quasi-classical approximation that implies that the change in the momentum due to the strong fields has to be sufficiently slow. This condition is fulfilled if the Larmor radius of the particle motion due to the existing magnetic field in the bunches is much larger than the particle wavelength. Even in the strong field environment of the ILC such a quasi-classical approximation can be used and the T-BMT equation can be applied to describe the spin precession sufficiently accurately [7, 9]. However, the Furry representation (bound interaction picture), is also being studied [10], as it allows the contributions to be calculated separately. The dependence of the anomalous magnetic moment on the field parameter $\Upsilon$ previously derived using ‘by-consistency’ arguments [9] can thereby be re-derived and cross-checked with this technique [10].

*Spin-flip processes*

Usually the S-T spin flip process is expected to lead to a small depolarization of about 0.05%, cf. [11, 6], but this depolarization increases with energy as seen in Table 2. This effect is normally described quasi-classically, using the operator method of [12] for ultra-relativistic electrons in external fields. However real collisions may include less ideal bunches containing lower energy leptons. In addition, much higher field strengths at the IP, $\Upsilon$, are planned for CLIC, and a more general calculation of the S-T process allowing for quantised motion of the leptons is required. Such a calculation is provided by using the Furry representation in which the bunch field ($A^e_{\mu}$) is included in the Lagrangian level in the description of the fermion. For this calculation to proceed, the solution of the bound Dirac field equation is required.

$$([p - eA^e]_2 - m^2 - \frac{ie}{2} F_{\mu\nu}^e \sigma^{\mu\nu}) \psi(x, p) = 0$$

(2)

where

$$F_{\mu\nu}^e = k_\mu \frac{\partial A_\nu^e}{\partial x_\mu} - k_\nu \frac{\partial A^e_\mu}{\partial x_\nu}.$$  \[ \phi = k^\mu x_\mu \]

The solution is obtained [13] by writing it generally as a product of the Dirac bispinor $u_S(p)$ and a function $F(\phi)$ whose explicit form is to be determined by substitution into
the field Eq. (2)
\[ \psi_V(x,p) = u_S(p)F(\phi). \] (3)

Expansion of operator products in Eq. 2 and substitution of the general solution \( \psi_V(x,p) \) yields a first order differential equation in \( F(\phi) \). The solution is obtained by expanding in powers of photon momentum \( k \) and external field \( A^e \) and using the Lorentz gauge condition \( (k \cdot A^e) = 0 \)

\[ \psi_V(x,p) = \left[ 1 + \frac{e}{2(kp)} [k \cdot A^e] \right] e^{iF(k \cdot A^e)} e^{-ipx} u_S(p). \] (4)

The bunch field can be described by a constant crossed electromagnetic field. Work is in progress to calculate the S-T process using this description with general parameters. More comprehensive details can be found in [10].

**Incoherent pair production**

The production of background pairs is strongly dependent on the polarization state of the initial photons. These photons are either real (beamstrahlung) or virtual and their specific momenta and polarization depend on the interaction with the bunch electromagnetic fields. For the case of real initial photons the Breit-Wheeler cross-section for specific particle polarization states is required in order to calculate the contribution. In CAIN this cross-section was written down only for the product of circular polarizations of initial photons. The full cross-section is a sum over specific functions of 4-momenta products and the polarization vector components [14]. Since the Breit-Wheeler process contributes only a small portion of the background pairs the effect of including full polarizations is likewise small. Initial simulations predict a slight increase in overall pair numbers. Final results will require inclusion of the full polarization in the virtual photon processes.

The other incoherent processes, Landau-Lifshitz, Bethe-Heitler and the Bremsstrahlung process are calculated using the equivalent photon approximation (EPA). In general this approximation is only valid [15] if the polarization of the virtual photons has been included in the calculations. The corresponding updates for CAIN are in progress.

**Coherent pair production**

The coherent production of pairs via the first order interaction between a beamstrahlung photon and beam field is included already in CAIN. However the second order stimulated Breit-Wheeler process also takes place in the presence of the bunch fields. The cross-section calculation for this process involves products of four Volkov solutions.

Naively, in comparison to the first order coherent process, the second-order cross-section is diminished by an order of the fine structure constant. However the bunch field has the effect of allowing the bound virtual lepton to reach the mass shell. The resulting resonances are rendered finite by inclusion of the electron self-energy in the external field and the stimulated Breit-Wheeler cross-section can exceed the first order coherent process. A detailed investigation is required to gauge the effect on produced pairs [10].

**CONCLUSIONS**

An analysis of depolarization and higher-order processes during the beam-beam interaction at a linear collider has been carried out. In line with previous studies, the ILC is predicted to display a depolarization of about 0.2% during each bunch crossing. For the current CLIC-G parameters, a much higher depolarization of around 4.8% is expected due to the strong beamstrahlung and higher \( \Upsilon \) parameter.

The current CAIN implementation now includes updates allowing for full polarization of coherent background pairs as well as for the incoherent pairs for the case of real initial photons. The inclusion of additional processes is ongoing.

New theoretical studies are currently underway for the description of the T-BMT and S-T effect in strong field environments during the beam-beam interaction. Both the anomalous magnetic moment as well as the spin-flip effects are being evaluated using the Volkov solution for the particle wave function in an external field. These studies become particularly relevant at higher energies and for a high beamstrahlung parameter.

**REFERENCES**

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[4] I.R. Bailey et al., A prototype target wheel for the ILC positron source, these proceedings; L.R. Jenner et al., “A study of mechanical and magnetic issues for a prototype positron source, these proceedings.