

# COUPLER KICK FOR VERY SHORT BUNCHES AND ITS COMPENSATION

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## Abstract

In this contribution we estimate the effect of power- and higher order mode couplers on the beam properties in superconducting cavities of TESLA design. Two different effects are considered: the kick due to the asymmetry of the external radio-frequency field (coupler-rf-kick) and the kick due scattered electromagnetic field of the bunch (coupler-wake-kick). The rf field calculation needs a high precision continuous wave computation of a single fully three dimensional cavity while wake fields of short bunches are calculated in time domain in a short window that moves over a distance of many 3d-cavities to find the steady state solution of the periodic array of cavities. Coupler kicks can be compensated locally by a symmetric design, by cavity pairs with alternating orientation and by a skilful choice of their orientation. For kicks from the rf field even a global compensation along the linac is possible.

## INTRODUCTION

In ILC and European XFEL [1] projects the design of the main accelerator will be based on TESLA technology [2], and this or similar technology is used in many other new projects. Therefore it is of great importance to consider the effect of geometric asymmetries to the dynamics of charged particle beams.

Each TESLA cavity is supplied with one coaxial main coupler and two HOM couplers (see Fig. 1). The design and orientation of the couplers in the European XFEL project are shown in Fig. 2. The rotational symmetry is perturbed by power and HOM coupler so that even particles without transverse offset experience transverse kicks. These kicks are driven by external fields (rf) and by self fields (wakes). The bellows between cavities are neglected.

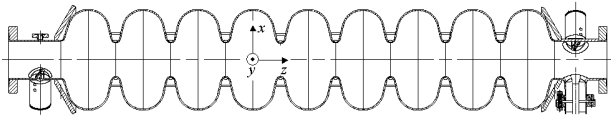


Figure 1: Overview of the TESLA cavity equipped with power- und HOM-couplers.

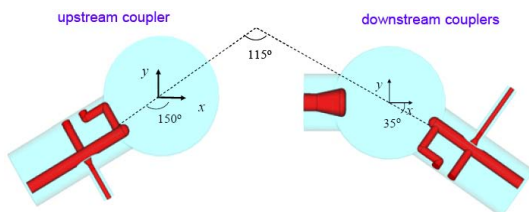


Figure 2: Geometry and orientation of TESLA couplers.

## FIELD CALCULATION

### RF Induced Coupler Kick

The time harmonic field in a perfect electric conducting cavity that is equipped with couplers and is traversed by a time harmonic beam can be written as

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \text{Re}\{\mathbf{E}(\mathbf{r})\exp(i\omega t)\} \\ \mathbf{B}(\mathbf{r}, t) &= \text{Re}\{\mathbf{B}(\mathbf{r})\exp(i\omega t)\} \\ \mathbf{E}(\mathbf{r}) &= a\mathbf{E}_0(\mathbf{r}) + b\bar{\mathbf{E}}_0(\mathbf{r}) \\ \mathbf{B}(\mathbf{r}) &= a\mathbf{B}_0(\mathbf{r}) - b\bar{\mathbf{B}}_0(\mathbf{r}) \end{aligned} \quad (1)$$

with  $a$  and  $b$  the forward and backward wave in the input coupler,  $\omega$  the frequency of excitation and  $\mathbf{E}_0$ ,  $\mathbf{B}_0$  the normalized forward solution. A standing wave field is stimulated if  $|a| = |b|$ . This condition is fulfilled in absence of a beam current and can be easily realized for EM field calculation either with port stimulation or by an eigenmode solver. To get a second and linear independent solution the field excitation by the beam current

$$\mathbf{J}(\mathbf{r}, t) = \text{Re}\{2I_b\delta(x)\delta(y)\exp(i\omega[t - z/c])\} \quad (2)$$

has to be considered, with  $I_b = q/T_b$  the dc component of the beam current and  $T_b$  the bunch distance which is a multiple of the rf period. There are three other approaches to estimate a second linear independent solution without beam: 1) the power transfer to the beam is replaced by wall losses, 2) the standing wave solution at a different frequency  $\omega_2$  is used (with  $|\omega_2 - \omega| \ll \omega/Q_e$  and  $Q_e$  the external quality, see f.i. [3]) and 3) a decaying eigensolution is calculated. Few remarks:  $a$ ,  $b$ ,  $I_b$  and the cavity voltage are related by a constraint condition that involves  $Q_e$  and the longitudinal loss parameter [4];  $Q_e$  is usually rather high ( $10^6$  to  $10^7$ ) so that precise tuning (of the cavity or the frequency) is necessary to fulfil the resonance condition;  $Q_e$  is tuned by varying the penetration of the tip of the input coupler – this changes the field solutions  $\mathbf{E}_0$ ,  $\mathbf{B}_0$ ; the kick of the HOM couplers is related to the standing wave part (proportional to  $a+b$  for a certain choice of the reference plane); the kick of the power coupler depends individually on  $a$  and  $b$ ; for the TESLA cavity the kicks caused by the HOM couplers have the same magnitude as that from the power coupler; the coupler kick is about five orders of magnitude smaller than the longitudinal kick – therefore high precision field calculations are required.

The coupler kick is characterised by the normalized complex kick factor

$$\mathbf{v}(x, y) = \frac{\mathbf{V}(x, y)}{\mathbf{e}_z \cdot \mathbf{V}(0, 0)} \approx \begin{pmatrix} v_{x0} + v_{xx}x + v_{xy}y \\ v_{y0} + v_{yx}x + v_{yy}y \\ 1 + \dots \end{pmatrix} \quad (3)$$

with  $\mathbf{V}(x, y) = \int (\mathbf{E}(\mathbf{r}) + c\mathbf{e}_z \times \mathbf{B}(\mathbf{r})) \exp(i\alpha z/c) dz$  and complex coefficients  $v_{x0}, v_{y0}, v_{xx}, v_{xy}, v_{yx}, v_{yy}$ . The coefficients for the up- and down-stream couplers (TDR TESLA cavity adjusted to  $Q_{\text{ext}} = 2.5 \cdot 10^6$  and operated without reflections) have been calculated from a decaying eigen-solution [http://adweb.desy.de/~mpymax/mafia/HOM\_Coupler/index.html]. They are listed in Tab. 1.

Table 1: RF kick coefficients

	upstream	downstream
$v_{x0} \cdot 10^6$	-57+7i	-23+52i
$v_{xx} \cdot 10^6/\text{mm}$	1.0-0.7i	-3.7-2i
$v_{xy} \cdot 10^6/\text{mm}$	3.4+0.2i	3.0+0.4i
$v_{y0} \cdot 10^6$	-42-3i	30+5i
$v_{yx} \cdot 10^6/\text{mm}$	3.4+0.2i	3.0+0.4i
$v_{yy} \cdot 10^6/\text{mm}$	-1.1+0.6i	3.8+1.9i

A positron with the transverse coordinates  $x, y$  that flies in a distance  $s$  behind an on-crest-particle through a cavity with the accelerating voltage  $V_{\text{acc}}$  gets the momentum increment

$$\mathbf{p} = \text{Re} \left\{ \mathbf{v}(x, y) \exp(i\omega s/c) \right\} V_{\text{acc}} e/c. \quad (4)$$

### Self Induced Coupler Kick (Wake)

First estimations [5] considered the effect of couplers in an infinite beam pipe without cavities. The transverse wake was found to be capacitive and caused an averaged transverse wake of about 20 V per cavity for a gaussian bunch of 1nC and arbitrary length. Later calculations of coupler wakes have been presented and discussed on [6] and are summarized in [7]. The transverse wake is reduced by the presence of the cavity, and it changes its (previously capacitive) characteristic in a periodic environment of many cavities. In meantime a new set of calculations have been done at Fermilab [9], DESY and by TU Darmstadt to find out the steady-state periodic solution for bunch length 0.3 mm. The results of these calculations obtained by code ECHO [10] at DESY and by code PBCI [11] at TU Darmstadt are shown in Fig. 3 with the horizontal and vertical kick for different bunch lengths and different number of periods. The first value (point 0) is a result for one cavity with couplers between infinite beam pipes. The points  $n > 0$  are obtained by direct wake field integration algorithm through subtraction of on-axis kick after  $n-1$  cavities from the kick after  $n$  cavities. Tab. 2 gives the steady-state results for bunches of different length. Both codes use non-standard FDTD methods to suppress the numerical dispersion in bunch direction. The results of ECHO are obtained on a conventional PC with a single processor and coarse mesh resolution in the transverse plane. The estimated accuracy for the cases  $\sigma = 1$  mm and  $\sigma = 0.5$  mm is better than 5%. Here the longitudinal mesh was 5 steps on  $\sigma$ , the transverse mesh step was equal to  $\sigma$ . The case for  $\sigma = 0.3$  mm was calculated with 3 steps on  $\sigma$  longitudinally and without accuracy estimation. The PBCI results for  $\sigma = 0.3$  mm are obtained for an equidistant grid with

250·10<sup>6</sup> mesh points (6 steps per sigma) on a cluster with 408 processor cores and a simulation time of ~7 days. The curves obtained by PBCI agree reasonably with those obtained by coarse ECHO calculation.

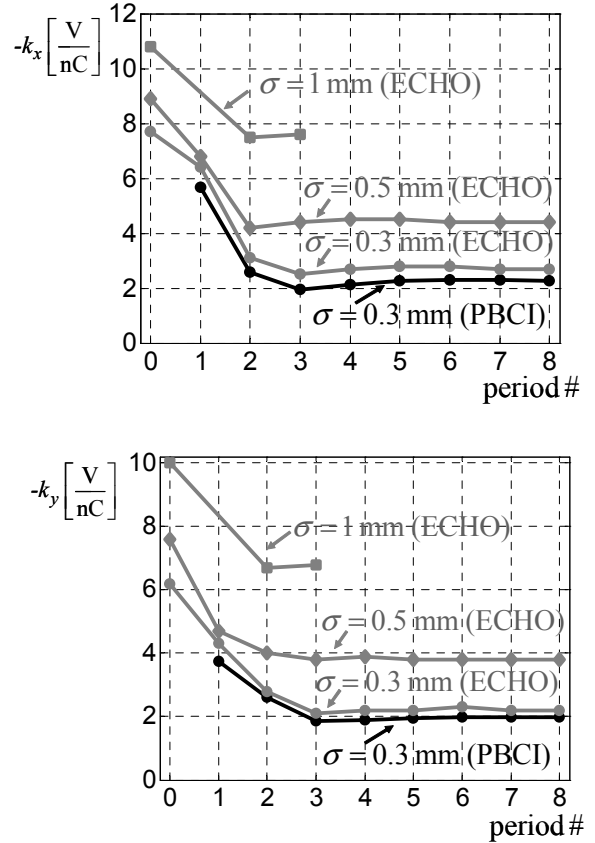


Figure 3: Horizontal and vertical kick vs. number of periods.

Table 2: Steady-state wake kick

Code	ECHO	ECHO	ECHO	PBCI
$\sigma$ [mm]	1	0.5	0.3	0.3
$k_x$ [V/nC]	-7.6	-4.4	-2.7	-2.3
$k_y$ [V/nC]	-6.8	-3.8	-2.3	-2

## DISCUSSION

### RF Induced Coupler Kick

The lowest order effect of rf induced coupling is a distortion of the beam trajectory. 1<sup>st</sup> order effects are related to the longitudinal and transverse position of individual particles. The longitudinal dependency causes shifts of slice centroids. The transverse dependency causes focusing or de-focusing including skew components. The integrated transverse field is

$$V_{x/y} = \text{const} + \frac{cs}{\omega} \text{Im} \left\{ v_{0x/y} \exp(i\varphi_{\text{acc}}) \right\} V_{\text{acc}} \quad (5)$$

with  $s$  the longitudinal coordinate,  $c$  the velocity of light and  $V_{\text{acc}}$ ,  $\varphi_{\text{acc}}$  the accelerating voltage and phase. The effect is reduced with the decreasing bunch length. For

typical XFEL parameters before the 1<sup>st</sup> bunch compressor ( $\sigma = 2.5\text{mm}$ ,  $V_{acc} = 20\text{MV}$ ,  $v_0 \sim 2E-4$ ) the rms spread of the integrated transverse field is about 250V.

### Self Induced Coupler Kick (Wake)

It can be seen that the kick factor at both coordinate plans for  $\sigma = 0.3\text{mm}$  is about 2 V/nC, that is an order of magnitude lower than a preliminary estimation of Ref. [5]. This is a consequence of a shadowing effect of the cavity and of a linear decrease of the steady-state wake with the decrease of the bunch length [6, 9].

## COMPENSATION

Coupler kicks can be compensated locally by a symmetric design and by cavity pairs with alternating orientation [3]. They can be reduced by symmetrising shunt elements and by a skilful choice of the orientation of up- and downstream couplers. Symmetrising measures do not reduce all 1<sup>st</sup> order (offset dependent) terms. F.i. the normalized kick factor of a pair of TESLA cavities is:

$$v_x \approx -2d_x x, \quad v_y \approx 2f_0 + 2f_y y,$$

if the geometry of the second cavity is mirror symmetric to the first with respect to the  $y$ -plane. For the case of a second cavity that is rotated by 180 degrees around the longitudinal axis, the offset independent part cancels completely but there is still  $xy$ -coupling:

$$v_x \approx 2d_x x + 2d_y y, \quad v_y \approx 2f_x x + 2f_y y.$$

To compensate 0<sup>th</sup> order effects **and**  $xy$ -coupling, quartets of mirrored and rotated cavities have to be considered or quartets with two perpendicular mirror planes.

The compensation of kicks is local if it takes place on a length short compared to the betatron wavelength. While local compensation is optic independent, a global compensation needs the balance of effects on a much longer scale and depends on optical functions and rf settings.

Fig. 4 shows an example of global compensation in the European XFEL. In the first module a bunch of 1nC with a peak current of about 50A is accelerated to 130MeV (a similar setup was investigated in [8]). Due to time dependent effects and  $xy$ -coupling the horizontal projected emittance grows by about 10%. The section between 22m and 65m is needed for diagnostics, a LASER heater and a dogleg. It was simulated with two different lattices that result in the same Twiss parameters at the end, but with different betatron phase advances ( $\varphi_1$ ,  $\varphi_2$ ). In the bad case ( $\varphi_1$ ) the horizontal emittance is increased by about 40% while the optimal case ( $\varphi_2$ ) reduces growth to few percent which is less than after the 1<sup>st</sup> module.

## CONCLUSION

RF coupler kicks cause a larger variation of the transverse fields along a bunch than wakes, even for a bunch of few hundred micron length. Compensation mechanisms have to be considered (for rf induced kicks) to avoid projected emittance growth in the European XFEL.

A global compensation scheme with optimized phase advance between rf modules is proposed. The impact of wakes is negligible, according to newer calculations that consider short bunches in periodic structures.

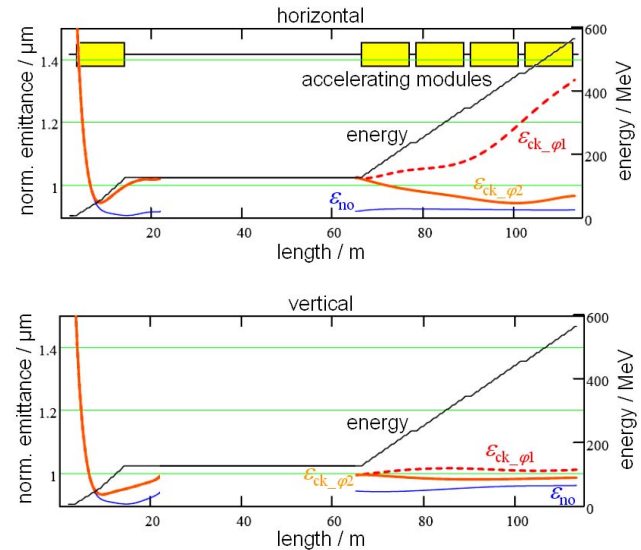


Figure 4: Global compensation of emittance growth before the first bunch compressor in the European XFEL.  $\epsilon_{no}$  is normalized projected emittance without couplers.  $\epsilon_{ck\_φ1}$  and  $\epsilon_{ck\_φ2}$  are emittances with rf-coupler fields and different phase advance between the first two modules.

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