PARAMETRIC STUDY OF A NOVEL COAXIAL BUNCHED BEAM SPACE-CHARGE LIMIT*

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Abstract

Recently, a non-trivial space-charge limit for off-axis bunched electron beams in a coaxial conducting structure was derived theoretically [1]. The space-charge limit describes the minimum strength of an external solenoidal focusing field which is needed to stabilize the beam's center-of-mass motion in the presence of induced surface charges on the coaxial structure. In this paper, we perform a parametric study of the space-charge limit to numerically determine its dependency on the conducting structure geometry, i.e., the ratio of the inner and outer conductor radii, as well as its' dependency on the transverse and longitudinal bunch distributions.

INTRODUCTION

When modeling the physics of high-current (100's A-10 kA) bunched electron beams which are often utilized in high-power microwave sources, such as klystrons, it is essential to understand the critical space-charge limits associated with these beams. Well-known space-charge limits, such as the Brillouin limit [2] for circularly symmetric unbunched beams, are regularly used in the design of klystrons [3] and ubitrons [4]. More recently, a new space-charge limit has emerged which predicts the onset of an instability for a slightly off-axis electron beam propagating in a circularly symmetric conducting structure. The essential physics of this instability is that the center-of-mass of the off-axis bunch "feels" an attractive force due to the image charges on the surface of the conductor. If an external magnetic solenoidal field is used for beam focusing there will be a minimum critical magnetic field, such that below this field, the beam's center-of-mass will move further off-axis. This instability was first computed for a circular conducting pipe [5], and then later computed for a coaxial structure [1].

In Ref. 1, the present author found a rather complicated expression for the total center-of-mass force and corresponding space-charge limit for a beam with an arbitrary charge distribution which was circularly symmetric about its' own axis. In addition, the author numerically calculated the space-charge limit for the special case of an annular ring (zero radial thickness) combined with a Gaussian longitudinal charge distribution.

*Work supported by DOE (DE-FG0292ER40747) and by the NSF (PHY-0552389).

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While this calculation was useful for understanding, the key physics of the instability it was not fully realistic since it did not include both a finite radial extent to the beam or a general search for how the space-charge limit depends on the ratio of the inner/outer radius of the beam.

In this paper, we extend the results of Ref. 1 by performing a full parametric study of the space-charge limit for annular electron beams in a coaxial conducting structure. We assume that the inner and outer radii of the structure are given by r_i and r_o. We will also assume that the electron bunch has a charge distribution which is uniform in the transverse direction and Gaussian in the longitudinal direction, i.e.

$$\rho(r,z)=\frac{[\theta(r-a)-\theta(r-b)]}{\pi\left(b^2-a^2\right)}\frac{e^{-z^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} \tag{1}$$

where a and b are the inner and outer radii of the electron beam and σ is the rms bunch length of the beam. The key parameters which we will be varying in this study are all normalized to the outer coaxial radius. In particular, we will be exploring the dependence of the space-charge limit on the normalized inner radius, r_i/r_o, the normalized beam radial thickness, $\delta r/r_o=(b-a)/r_o$, and the normalized bunch length, σ/r_o. In general, there is also a fourth parameter which is a measure of the average radius of the beam, i.e. $r_{ave}/r_o=(a+b)/2r_o$. However, in Ref. 1, we found that the space-charge limit was maximized when the beam was centered roughly midway between the two boundary surfaces. . Hence, in order to simplify this study, we will assume that $r_{ave}/r_o=(r_i+r_o)/2r_o$.

The space-charge limit for a beam with total number of particles N_b in the presence of a uniform magnetic field, $\vec{B}=B_o\hat{e}_z$, was found in Ref. 1 to be

$$\frac{16\pi N_b m_e}{B_0^2\varepsilon_0 r_o^3}\le\frac{1}{f} \tag{2}$$

where

$$f=r_o^3\int dzdrr\int dz'dr'r'\int_0^\infty dkk\cos[k(z-z')]$$
$$\times\frac{\rho(r,z)\rho(r',z')}{AB}\{C+D+(E-F)G\}$$

and

$$A=I_0(kr_o)K_0(kr_i)-I_0(kr_i)K_0(kr_o)$$

$$B = I_1(kr_o)K_1(kr_i) - I_1(kr_i)K_1(kr_o)$$
$$C = I_0(kr)I_0(kr')[K_0(kr_i)K_1(kr_i)/r_o - K_0(kr_o)K_1(kr_o)/r_i]$$
$$D = K_0(kr)K_0(kr')[I_0(kr_o)I_1(kr_o)/r_i - I_0(kr_i)I_1(kr_i)/r_o]$$
$$E = kI_0(kr_o)I_1(kr_i)K_0(kr_i)K_1(kr_o)$$
$$F = kI_0(kr_i)I_1(kr_o)K_0(kr_o)K_1(kr_i)$$
$$G = I_0(kr')K_0(kr) + I_0(kr)K_0(kr'),$$

and $I_\nu(x)$ and $K_\nu(x)$ are the modified Bessel functions of the first and second kind respectively. The limit shown in Eq. (2) was computed in Ref. 1 by calculating the total center-of-mass force on the bunch due to the induced conductor surface charge using a Green's function method. It was also shown in Ref. 1 that the center-of-mass motion can be derived from a Hamiltonian method which yields the space-charge limit in Eq. (2). The result in Eq. (2) can also be extended to periodic solenoidal magnetic fields. For this case, one replaces the magnetic field in Eq. (2) with its' root-mean-square value.

PARAMETRIC STUDY

The primary results of our parametric study of the off-axis space-charge limit are evaluations of the normalized function, $1/f(r_i/r_o, \delta r/r_o, \sigma/r_o)$, which is the right hand side of the space-charge limit in Eq. (2). For the charge distribution given by Eq. (1), it is possible to analytically compute the *r* and *z* integrals in Eq. (2). However, the remaining integral over *k* and hence, $1/f$, can only be evaluated numerically. Our study was performed by choosing a value of r_i/r_o, and then plotting $1/f$ as a function of $\delta r/r_o$ for different values of σ/r_o. We note that since the beam radii must be bounded by the conductor radii, i.e. $r_i \le a \le b \le r_o$, then the x-axes of the plots vary in the range $0 \le \delta r/r_o \le r_i/r_o$

Figures 1,2, and 3 show the numerical results of $1/f$ vs. $\delta r/r_o$ for the normalized inner conductor ratios of r_i/r_o = 0.25, 0.50, and 0.75, respectively. Within each figure, we use the normalized bunch lengths σ/r_o = 0.0 (solid curve), 0.5 (dashed curve), 1.0 (dotted curve), 1.5 (dashed with one dot), and 2.0 (dashed with two dots).

The figures contain a number of important features, which we will now explain. First, the value of $1/f$ for σ/r_o = 0.0 always goes to zero as the beam radii approaches the conductor radii, i.e. $\delta r/r_o \to r_i/r_o$. The reason for this is simple. For a zero longitudinal thickness beam, the induced conductor surface charges are highly localized and form a sharp ringlike distribution as the beam edges approach the conductor surfaces. At this point, the total center-of-mass force on the bunch will go to infinity, and the corresponding space-charge limit will be zero. Another obvious feature is that as the bunch length increases, $1/f$, also increases. Physically this occurs because a charge distribution which is less localized will induce a surface charge distribution which is also less localized. A third feature is that as the bunch length is increased (bunch becomes more two dimensional), the value of $1/f$, becomes less dependent on the value of $\delta r/r_o$. This is consistent with previous space-charge limit results for the case of a single conductor pipe [5] in which the space-charge limit for a two-dimensional bunch was shown to be independent of the choice in transverse charge distribution.

Finally, we can also see that as the inner conductor radius, r_i/r_o, is increased the space-charge limit is also increased. The physical reason for this is that as r_i/r_o is increased then the relative distance between the conductors is decreased. This gives rise to an overall increase in the beam-conductor surface interaction, and hence, a decrease in the space-charge limit.

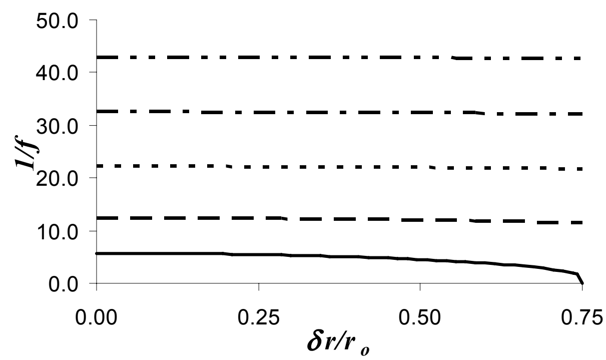

Figure 1: Plots of $1/f$ vs. $\delta r/r_o$ for the normalized inner conductor ratio of r_i/r_o = 0.25 and bunch lengths σ/r_o = 0.0 , 0.5, 1.0, 1.5, and 2.0.

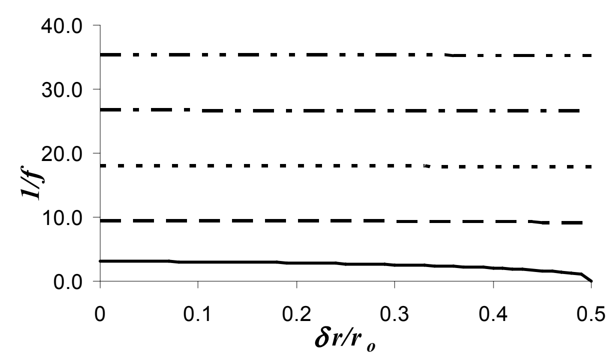

Figure 2: Plots of $1/f$ vs. $\delta r/r_o$ for the normalized inner conductor ratio of r_i/r_o = 0.50 and bunch lengths σ/r_o = 0.0 , 0.5, 1.0, 1.5, and 2.0.

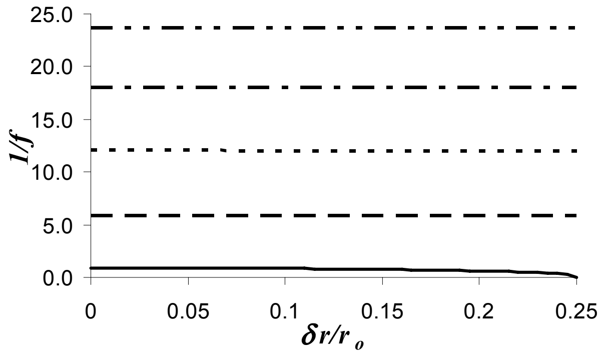

Figure 3: Plots of $1/f$ vs. $\delta r/r_o$ for the normalized inner conductor ratio of r_i/r_o = 0.75 and bunch lengths σ/r_o = 0.0 , 0.5, 1.0, 1.5, and 2.0.

DISCUSSION

We will now address a few key points regarding our parametric study of the coaxial space-charge limit. The first point deals with the behavior of the zero bunch length σ/r_o = 0.0 curve near its' zero point shown in Figures 1,2, and 3. This zero point can be readily shown using the asymptotic behavior of the Bessel functions found in Eq. (2). In particular, one can show that the space-charge limit near its zero point scales as,

$$\frac{1}{f} \propto -\frac{1}{\ln\left[1 - r_i/r_o - \delta r/r_o\right]}. \tag{3}$$

In essence, the zero point of the space-charge limit for a zero bunch length charge distribution occurs at the point $\delta r/r_o = r_i/r_o$, but the rate of decrease of the space-charge limit near this point is exponentially slow. The exponentially slow decrease in the space-charge limit is an artifact of the extended charge distribution, and how it couples to the induced surface charge on the wall. That is, even though the 2-D, pancake-like, electron bunch has a center-of-mass force acting on it which is approaching infinity as the beam edges approach the conductor pipe surfaces, this force is infitesimally small compared to the force on a 1-D ringlike charge distribution with its' beam edge approaching the conductor surface.

Another key point of the space-charge limit which is not immediately apparent from this study, is that the limit can be drastically lowered when the transverse bunch distribution is not centered near the midpoint of the two conductor surfaces. This effect is most noticeable when the bunch length is sufficiently short which is in agreement with the results of this study in that long bunch lengths yield space-charge limits that are independent of the transverse distribution. For a zero bunch length charge distribution which is a 1-D ringlike distribution, the space-charge limit approaches zero as the beam edge approaches either conductor surface.

SUMMARY

In this paper, we have shown the results of a parametric study for a bunched beam space-charge limit in coaxial conducting structures. The space-charge limit derived in Ref. 1 is an off-axis center-of-mass instability in which the beam experiences a force from induced surface charges while an external magnetic field is acting to focus the beam. In the case of a coaxial structure, which has two conducting surfaces, both surfaces yield a non-negligible contribution to the center-of-mass force. The parameters which were explored in this study were the normalized bunch length, the annular beam thickness, and the inner to outer conductor radius ratio. Qualitatively, we have found that a higher inner to outer conductor radii ratio will yield a lower space-charge limit. In addition, we have found that larger bunch lengths yield a higher space-charge limit with little dependence on the transverse density distribution. Finally, we have found that a zero bunch length beam has a zero space-charge limit when its' edges approach the conductor radii.

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