WAKE FIELD SIMULATIONS FOR STRUCTURES OF THE PITZ RF PHOTOINJECTOR: EMITTANCE GROWTH ESTIMATIONS [*]

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Abstract

One of the main concerns in the design of electron guns is the generation of low-emittance beams. One source of emittance growth is the beam-surrounding effect, which can be estimated from the wake potentials along the beam path. For the calculation of these potentials an accurate knowledge of the short range wake fields induced in the different parts of the gun with geometrical discontinuities is necessary. The computation of these wake fields is a challenging problem, as an accurate resolution for both the small bunch and the large model geometry is needed. Here with the help of numerical wake-potential calculations we analytically estimate the emittance growth for the RF electron gun of the Photoninjector Test Facility at DESY Zeuthen (PITZ).

INTRODUCTION

The X-FEL project requires high quality beams with ultra-short electron bunches. In order to predict the energy spread and emittance growth of such bunches, an accurate knowledge of the short range wake fields is necessary. The effect of the wake fields on the bunch of particles can be estimated by calculating the so called wake potentials [1].

Simulations of wake fields from short bunches of particles in accelerators are difficult to perform, since high computational resolution is required due to the high frequency fields excited by the bunches. So, the abilities of codes such as MAFIA are limited due to the huge amount of memory needed. For that reason, recently, the especialized code Parallel Beam Cavity Interaction (PBCI) [2] was developed. The PBCI code was designed for massively parallel wake field simulations in arbitrary three-dimensional geometry. The algorithms used include a purely explicit and dispersion free split-operator scheme as well as a domain decomposition approach for highly balanced parallel computations [2]. The Finite Integration Technique (FIT) [3, 4] is used for the spatial discretization of the wake fields. In order to reduce the amount of memory needed, and since the PBCI is a dispersion free code in the longitudinal direction, a moving window technique has been successfully implemented.

In our numerical simulations we calculate the longitudinal wake potentials and by applying the Panofsky-Wenzel Theorem [1] we obtain the transverse wake potentials.

It is alto worth mentioning that we have successfully validated our code [5] against results from the commercial software CST PARTICLE STUDIO™ [6].

In the following, we present results from wake field simulations of the PITZ diagnostics double cross section with the metallic mirror included. Afterwards, we use these numerical results to analytically estimate the emittance growth in the horizontal direction where an effect from the metallic-mirror structure is expected.

THE PITZ DIAGNOSTICS DOUBLE CROSS SECTION: NUMERICAL RESULTS

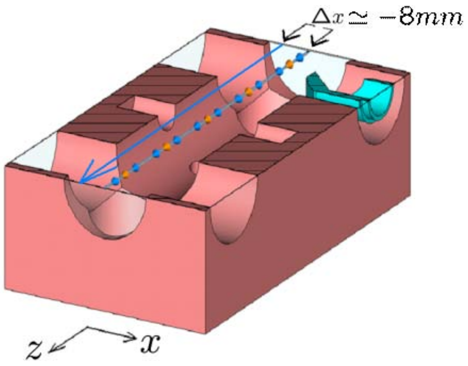

Figure 1: Diagnostics double cross section of the PITZ injector with the mirror. Half of the symmetric structure is depicted, where a horizontal cutting-plane view ($y = 0$) can be seen. The beam path (blue line) is shifted Δx with respect to geometrical center ($\Delta x = 0$ corresponds to the main axis of the system).

In Fig. 1 an schematic representation of the PITZ double cross section with the mirror included is presented. This is ten-port structure, which is not rotationally symmetric. In the figure is also schematically plotted a beam path shifted some horizontal distance Δx from the geometrical center. Different Δx values are considered here to estimate and minimize the possible negative effects of the mirror structure on the particle bunch. We note that positive (negative) Δx values correspond to the case of the beam being shifted to (away from) the mirror.

Notice that wake potentials describe the momentum change of a test charge particle when interacting with the wake fields. This interaction depends on a distance s to the bunch head [1, 5, 2]. Here the particle bunch is represented by a Gaussian distribution in both the longitudinal and the

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transverse directions, which moves with ultra-relativistic velocity.

In Fig. 2 we present results of the numerical computed horizontal wake potentials W_x vs. the distance s for different Δx values. It is worth mentioning that in the case of the mirror being absent W_x would vanish if $\Delta x = 0$. We observe that the Δx effect on W_x is important, i.e. positive and negative W_x amplitudes are observed and well distinguishable from each other. In order to estimate the overall effect of W_x we calculate the horizontal kick factor κ_x [1], whose behavior is shown in Fig. 3. We observe that κ_x monotonically increases from negative values at $\Delta x = -11$mm to positive values for $\Delta x > -8$mm. A minimum of the absolute value of κ_x is observed at $\Delta x \simeq -8$mm. So, in this region the wake-potential effect on the bunch center are small (In Fig. 1 the beam path is schematically plotted for this case).

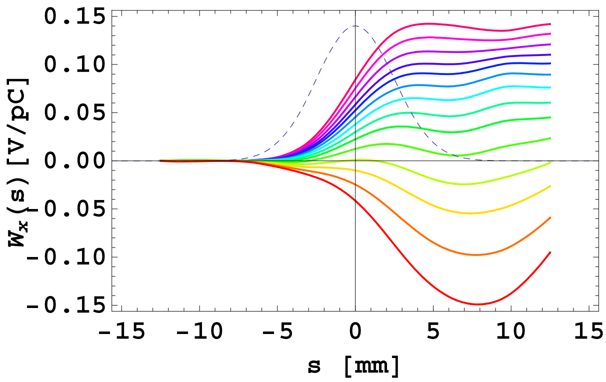

Figure 2: W_x vs. s for different Δx values (curves with different colors). $\Delta x \in$ [-11mm,2mm] (Δx values considered in simulations can be seen in the horizontal axis of Figs. 3). Bottom-up curves correspond to increasing Δx values starting from the minimum $\Delta x = -11$mm to the maximum $\Delta x = 2$mm. For comparison the dotted line shows the Gaussian-profile with $\sigma_z = 2$mm.

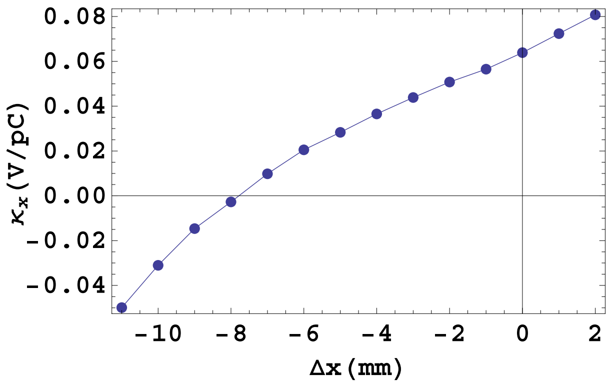

Figure 3: κ_x vs. Δx. The parameters are the same of Fig. 2.

ANALYTICAL ESTIMATION OF THE EMITTANCE GROWTH

In this section we estimate the effect of the horizontal wake potential on the eimittance growth of a Gaussian bunch. For that we follow the method proposed in Ref. [7].

This method takes into account that the random nature of the electron motion in the the bunch yields a Maxwell-Boltzman distribution in transverse phase space given by

$$\rho_0(x,x',s) = \aleph_0 e^{-(\gamma x^2+2\alpha x x'+\beta x'^2)/(2\varepsilon_{x,0})} f(s), \qquad (1)$$

where $x' = dx/ds$ is the slope of the particle trajectory, $\varepsilon_{x,0}$ can be interpreted as an unperturbed emittance, and $f(s) = \exp(-s^2/(2\sigma_z^2))/\sqrt{2\pi\sigma_z^2}$ is the longitudinal particle distribution with a width σ_z. In Eq. (1) $\aleph_0$ is a normalization constant, and the parameters α, β, γ and $\varepsilon_{x,0}$ can be estimated from a tracking code [8].

In the present case we assume that the horizontal kick due to the W_x changes the slope of the particle trajectory in a $\Delta x'$ value, i.e. $x' \to x' + \Delta x'$. This change on the phase space value $\Delta x'$ can be estimated from the change in the horizontal momentum Δp_x, namely $\Delta x'(s) = \Delta p_x/p_z$, where p_z is the longitudinal momentum E/c, E is the total energy and c is the light velocity. By using first order perturbation theory on the energy function of the particle bunch it is straightforward to obtain Δp_x in terms of a change of energy ΔE_x, i.e. $\Delta p_x = \Delta E_x/(\beta_z c)$ with β_z as the normalized velocity in the longitudinal direction. Finally, the change of energy can be estimated via horizontal wake potential by defining $\Delta E_x = eQW_x$ [8], where e is the electron-test charge and Q is the bunch charge.

From the formulation above we are able to estimate the $\Delta x'$ value from the horizontal wake potential W_x. This in turn allows to estimate the emittance change by first defining a distribution expression $\rho = \rho_0(x, x' + \Delta x', s)$, then calculating the final emittance,

$$\varepsilon_x = \int\int\int \frac{\rho}{2}(\gamma x^2 + 2\alpha x x' + \beta x'^2) ds\, dx\, dx'. \qquad (2)$$

Notice that in order to integrate Eq. (2) an analytical expression for the horizontal wake potentials in Fig. 2 is needed. Here we concern with the effect of the wake potential in the center of the bunch $s = 0$. So, we take a linear fitting of W_x in this region, i.e. $W_x \simeq w_0 + w_1 s$ for $|s| \simeq 0$. By using this approximation in Eq. (2) we finally obtain

$$\varepsilon_x = \varepsilon_{x,0} + \frac{\beta}{2}\left(\frac{eQ}{\beta_z^2 E}\right)^2 \left({w_0}^2 + {w_1}^2\sigma_z^2\right). \qquad (3)$$

The relative emittance growth can be defined as

$$\%\varepsilon_x = 100\frac{\varepsilon_x - \varepsilon_{x,0}}{\varepsilon_{x,0}}$$

An estimation of $\%\varepsilon_x$ is presented in Fig. 4. We observe that the overall $\%\varepsilon_x$ values are small. Notice, for instance,

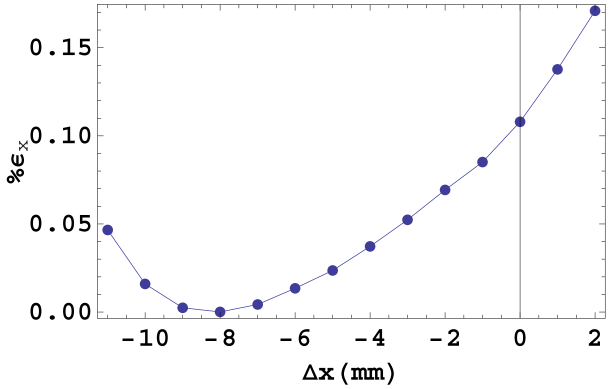

Figure 4: $\%\varepsilon_x$ vs. Δx. The parameters are the same of Fig. 2. $E = 4.2$MeV, $\beta =$8m, $\beta_z \simeq 1$, and $\varepsilon_{x,0} = 4.62$ mm-mrad.

that a bunch located at the geometrical center of the structure experiment an emittance growth of $\simeq 0.1\%$. We observe also that, as in the case of the horizontal kick factor κ_x (Fig. 3), a minimun is present at $\Delta x \simeq -8mm$. This result shows the direct correlation between the horizontal wake potentials and the emittance growth.

Notice that the emittance-growth values in Fig. 4 for large negative Δx are due to geometric effect of the port located at the opposite side of the mirror. In the region of positive Δx values the emittance growth is mainly due to the mirror structure. The present results, though calculated for the transverse center of the bunch, suggest that the horizontal bunch tail close to the mirror would suffer larger distortions than that located on the opposite side. On the other hand, it also suggests that a displacement of the bunch path $\Delta x \simeq -8$mm, as shown in Fig. 1, would reduce the overall effect of the mirror structure on the beam.

CONCLUSIONS

In the present paper we have studied the emittance growth of the PITZ diagnostic double cross section due to the effect of a horizontal wake potential.

The asymmetry introduced by the mirror in the structure of the double cross section is responsible for the horizontal wake potential, which have been numerically calculated with the recently developed code PBCI. We have performed simulations for particle beams horizontal displaced from the geometrical center of the structure, showing that the horizontal wake potential is strongly affected by the horizontal beam position.

In particular we have observed that when the beam is displaced in the direction of the mirror the maximum of the wake potentials tends to be positive and increases in value. On the other hand, if the beam path is shifted such that the distance to the mirror structure increases the wake-potential maximum reduces its value and eventually becomes negative. Calculations of the horizontal kick factor show that a minimum appears for an approximated 8mm displacement from the geometrical center and opposite to the the mirror.

In order to study the emittance growth of the system we have followed a theory where a small change of the slope particle trajectory is estimated from the wake potential results. Afterwards, an effective emittance is calculated with the help of a modified Maxwell-Boltzman distribution of the transverse phase space. This analytical theory predicts small values of the emittance growth. An emittance-growth minimum is found at same place where the absolute value of the horizontal-kick-factor also presents a minimum. The overall results suggest that a particle beam horizontal displaced to the region of mininum emittance may reduce the effects of to the mirror structure in the PITZ double cross section.

Further studies are needed to verify and estimate the emittance effects on the bunch shape.

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