STRUCTURAL ANALYSIS FOR A HALF-REENTRANT SUPERCONDUCTING CAVITY

E. Zaplatin, FZJ, Jülich, Germany,
T.L. Grimm, W. Hartung, M. Johnson, M. Meidlinger, J. Popielarski, MSU, East Lansing, MI

Abstract
Structural analyses of half-reentrant mid-cell and multi-cell niobium cavities were performed, including the static and dynamic response. The effects of a helium vessel and stiffening rings were explored. Results were compared to other cell shapes. With the proper stiffening system, the structural properties of the various cavities are similar.

INTRODUCTION
A half-reentrant (HR) cavity (1300 MHz, $\beta = 1$) is being developed at Michigan State University [1] for use in a superconducting linear collider and other applications. The electromagnetic performance of a half-reentrant cell shape is similar to that of a fully reentrant cavity, but a multi-cell HR cavity can be cleaned using traditional techniques. This paper reports on structural analyses of the HR cavity for the mid-cell and multi-cell cases. The shift in the resonant RF frequency due to pressure differential was calculated as a function of wall thickness, with and without the helium vessel and stiffening rings. A modal analysis of microphonic excitations was done for the multi-cell case. The mid-cell results for the HR cavity were compared with simulations for the TeSLA [2], Cornell reentrant [3] and ICHIRO “low-loss” [4] cavities to better understand the behaviour and trends.

The ANSYS codes [5] were used for the simulations. We assumed the same mechanical properties for the cavity walls and connecting (stiffening) rings ($\text{Young modulus} = 105000 \text{ N/mm}^2$ and Poisson ratio $\nu = 0.38$).

PRESSURE RESPONSE OF MID-CELL
The response of the cavity to a pressure differential of 1 bar was simulated (vacuum inside the cavity, ambient pressure outside). The simulations were done with the cell-to-cell junction constrained by symmetry. The goal for these calculations was not to achieve the best accuracy, but rather to understand the behaviour and trends.

Figure 1 shows the Cornell reentrant, ICHIRO “low-loss” and HR middle cells in comparison with the TeSLA shape. The pressure differential changes the cavity shape and shifts the RF frequency of the accelerating mode. Inward deformation near the iris (the region of high electric field) increases the capacitance and hence reduces the frequency. Inward deformation near the equator (high magnetic field region) reduces the inductance and hence increases the frequency. Thus the effects tend to cancel one another. Although the equator region is generally more rigid than the iris region, the volume change near the equator is larger due to the larger radius. A stiffening ring can be used to change the frequency shift.

The deformation and frequency shift both depend on thickness of the cavity wall, so the wall thickness was varied in the simulations. We found that the frequency shift is approximately zero for an unstiffened cavity with a wall thickness of 3 mm. For the thicker walls, the frequency increases with pressure; for thinner walls, the frequency decreases with pressure (Fig. 2a). Note that, after the cavity is formed and etched, the wall thickness may be significantly different from the initial thickness of the sheet niobium.

The deformation as a function of distance along the cavity wall (Fig. 2c; the distance is measured from the equator) illustrates the effects discussed above. Even for the much smaller deformation with a 5 mm wall thickness, there is more change in volume near the equator, which produces a positive frequency shift.

The frequency shift as a function of wall thickness is different for the Cornell reentrant cavity due to the

\[ \text{Figure 1: Mid-cell geometries.} \]
\[ \text{Figure 2: Mid-cell response to 1 bar external pressure.} \]
difference in shape. In the reentrant case, the condition of zero frequency shifts is reached with a thinner wall and more displacement near the iris.

The addition of a stiffening ring affects the frequency shift with pressure, although the stiffening rings are often designed (as was the case for the TeSLA cavity) to minimise Lorentz force detuning.

Simulations with a stiffening ring located at half the equator radius \( \frac{R_{\text{ring}}}{R_{\text{eq}}} = 0.5 \) were done (Fig. 2d). The ring strengthens the iris region, so that the frequency shift is always positive.

Changing the ring position (Figs. 2f, 3 and 4), we see that there are two places for the ring where the frequency shift, but not the deformation, reaches zero. The smallest wall deformations correspond to nearly the largest frequency shifts, where there is minimal cancellation between effects near the equator and near the iris.

**9-CELL HALF-REENTRANT CAVITY**

The simulations for the middle cells give us only a basic understanding of cavity behaviour. A more realistic analysis is that of a multi-cell cavity with beam tubes, end cells (with compensation to ensure field flatness), and a helium vessel.

An end cell was designed for the HR cavity using ANSYS. The iris radius was increased by 15% on the non-reentrant side and small adjustments were made on the re-entrant side (Fig. 5a). The accelerating field profile for a 9-cell HR cavity is shown in Fig. 5b. The calculated field varies by up to 10% depending on the mesh density.

**Pressure Response**

When both beam tubes are fixed, the calculated response of the 9-cell HR cavity to 1 bar pressure differential is very similar to previous result for the mid-cell case, whether the stiffening rings are omitted (Fig 6b) or included (Fig. 6c). As seen in Fig. 6, when one of the beam tubes is free, the cavity response is significantly different.

Let us consider a 9-cell HR cavity inside a cryostat similar to the cryostat designed for 6-cell 805 MHz elliptical cavities [6], see Fig. 7. Bellows are included in the helium vessel to accommodate the motion produced by the tuner. When the cavity and cryostat are under vacuum and the helium vessel is at 1 bar, the pressure differential is exerted not only on the cavity walls, but also on the inside surfaces of the helium vessel, including end dishes. One of the dishes is firmly attached to the wall = 1.8 mm \( R_{\text{ring}}/R_{\text{eq}} = 0.4 \)

wall = 5 mm \( R_{\text{ring}}/R_{\text{eq}} = 0.55 \)

\( R_{\text{ring}}/R_{\text{eq}} = 0.65 \)

Figure 3: ICHIRO cavity deformation for 1 bar pressure differential with different ring positions.

\( R_{\text{ring}}/R_{\text{eq}} = 0.4 \)

\( R_{\text{ring}}/R_{\text{eq}} = 0.55 \)

\( R_{\text{ring}}/R_{\text{eq}} = 0.65 \)

Figure 5: End-cell corrections and electric field profile along the cavity axis for a 9-cell HR cavity.

\( R_{\text{ring}}/R_{\text{eq}} = 0.55 \)

\( R_{\text{ring}}/R_{\text{eq}} = 0.65 \)

\( R_{\text{ring}}/R_{\text{eq}} = 0.7 \)

Figure 6: 9-cell HR cavity with 1 bar pressure differential.
outer cylinder of the helium vessel; the other is connected via the bellows. Depending on the radius of the bellows ($r_{xx}$, Fig. 7), the inward force on the cavity walls will be either larger or smaller than the outward force on the end dish. Thus, the cavity will be either shortened or stretched. There is a particular bellows radius for which the shift in frequency with pressure is zero (Fig.7).

**Microphonic Analysis**

As discussed above, it is possible to make the frequency shift equal to zero with the proper choice of bellows radius, even without stiffening rings. However, the benefit of a stiffening ring is clearly shown from the cavity modal analysis. The simulation model and results for the first few mechanical eigen-modes of the 9-cell HR cavity are shown in Fig. 8 and Table 1. The lowest mechanical mode’s frequency increases by nearly a factor of 3 with the stiffening ring present. One of beam pipes and the helium vessel dish were fixed for these simulations. The position of the stiffening ring was not optimised: the ring was placed at half the equator radius.

**CONCLUSION**

Modal analysis of a 9-cell half-reentrant cavity indicates that stiffening rings increase the rigidity of the structure significantly. The helium vessel can be designed to minimise the shift in frequency due to pressure differential. Lorentz force detuning must also be considered in the structural analysis; Lorentz force detuning simulation results will be presented later [7].

**REFERENCES**


<table>
<thead>
<tr>
<th>mode</th>
<th>with ring Freq. / Hz</th>
<th>no ring Freq. / Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>163.858</td>
<td>58.448</td>
</tr>
<tr>
<td>2</td>
<td>163.888</td>
<td>58.454</td>
</tr>
<tr>
<td>3</td>
<td>263.978</td>
<td>95.500</td>
</tr>
<tr>
<td>4</td>
<td>285.463</td>
<td>95.506</td>
</tr>
<tr>
<td>5</td>
<td>285.531</td>
<td>105.262</td>
</tr>
<tr>
<td>6</td>
<td>447.085</td>
<td>151.471</td>
</tr>
<tr>
<td>7</td>
<td>447.181</td>
<td>151.489</td>
</tr>
<tr>
<td>8</td>
<td>525.695</td>
<td>205.930</td>
</tr>
</tbody>
</table>

Table 1: Modal analysis results for the 9-cell HR cavity.