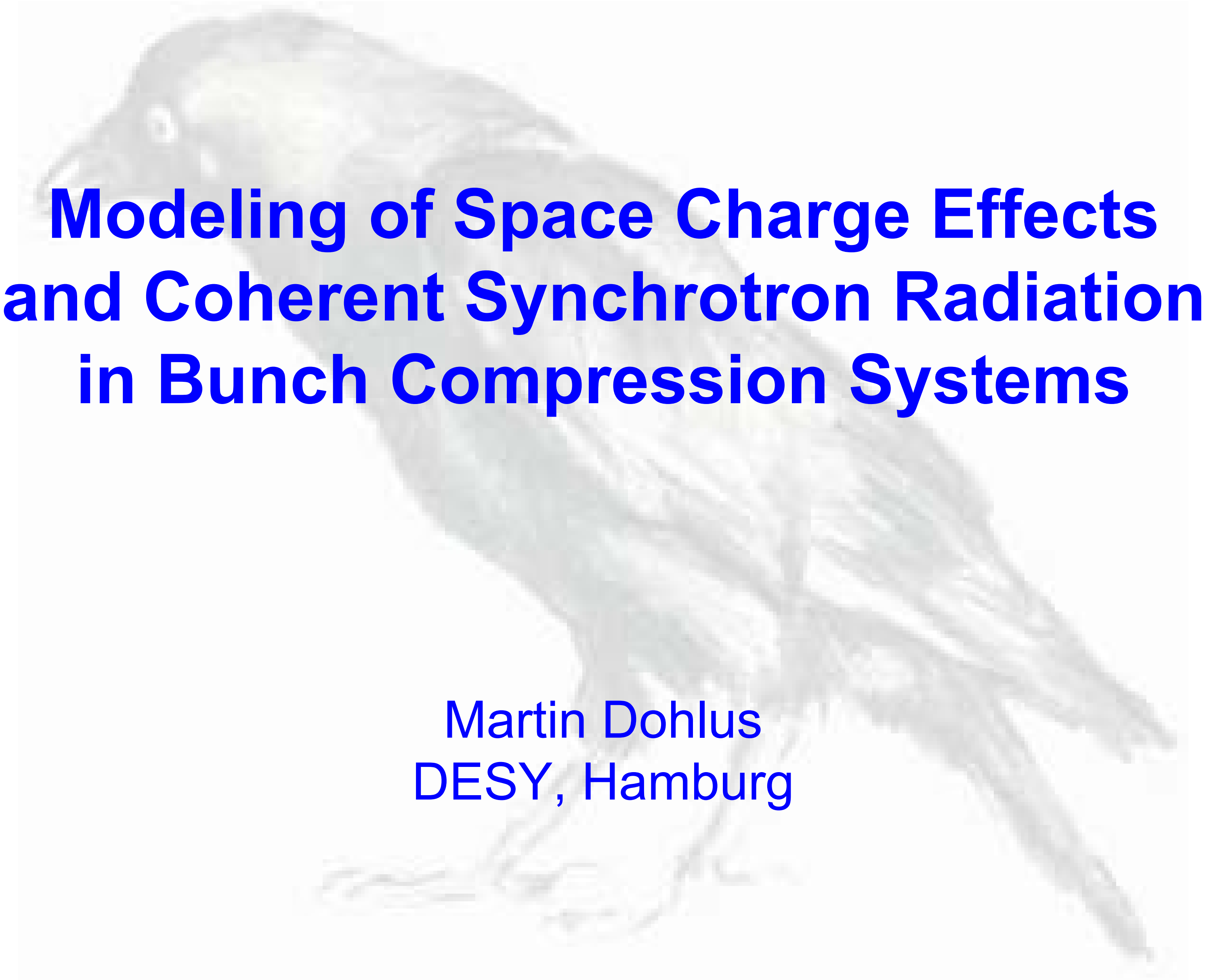


Modeling of Space Charge Effects and Coherent Synchrotron Radiation in Bunch Compression Systems

Martin Dohlus
DESY, Hamburg

SC and CSR effects are crucial for design & simulation of BC systems

CSR and related effects are challenging for EM field calculation

part 1: CSR codes

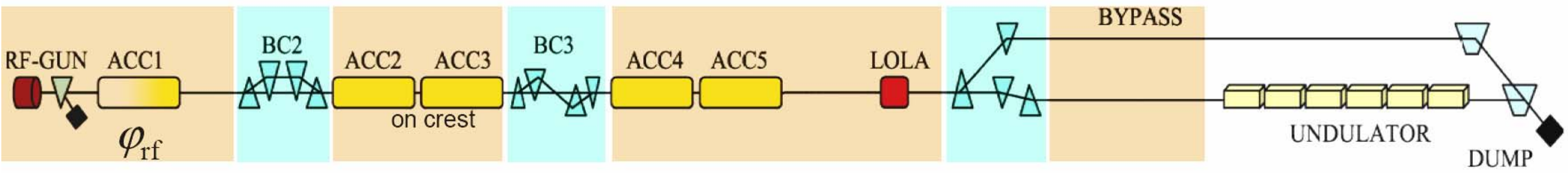
- effects
- approaches
  - Vlasov-Maxwell
  - paraxial approximation
  - 1d
  - sub-bunch
- Zeuthen benchmark example

part 2: simulation of BC systems

conclusions

effects

what is different in magnetic BC systems (compared to usual LINACS)?



r_{56} : there are dispersive sections with non-linear trajectories

chirp: there is a strong linear correlation between energy and longitudinal position

there is a variation of bunch shape

high I_{peak}/Energy after compression

linear trajectory (LT) most forces ~ $1/\gamma^2$	CSR & more energy independent effects

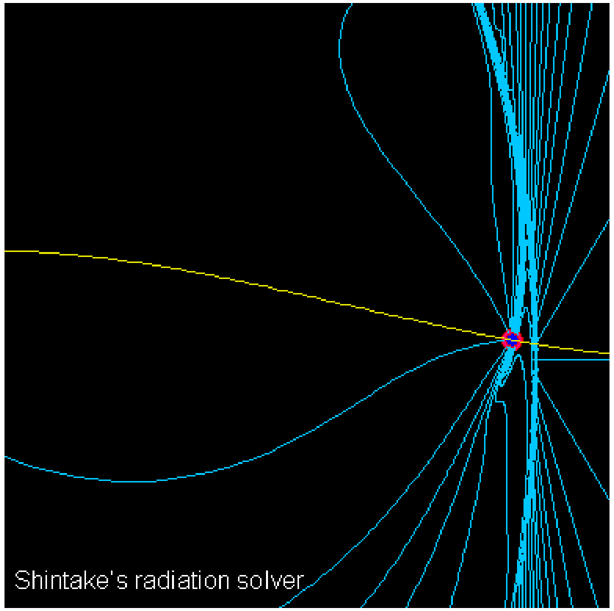
new types of tracking codes with more general electromagnetic field solvers

called "CSR codes"

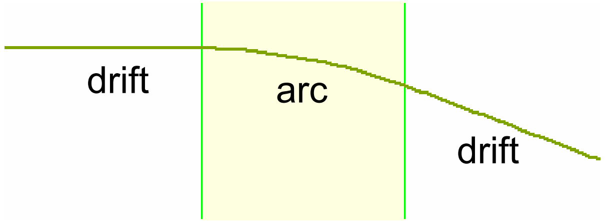
… effects

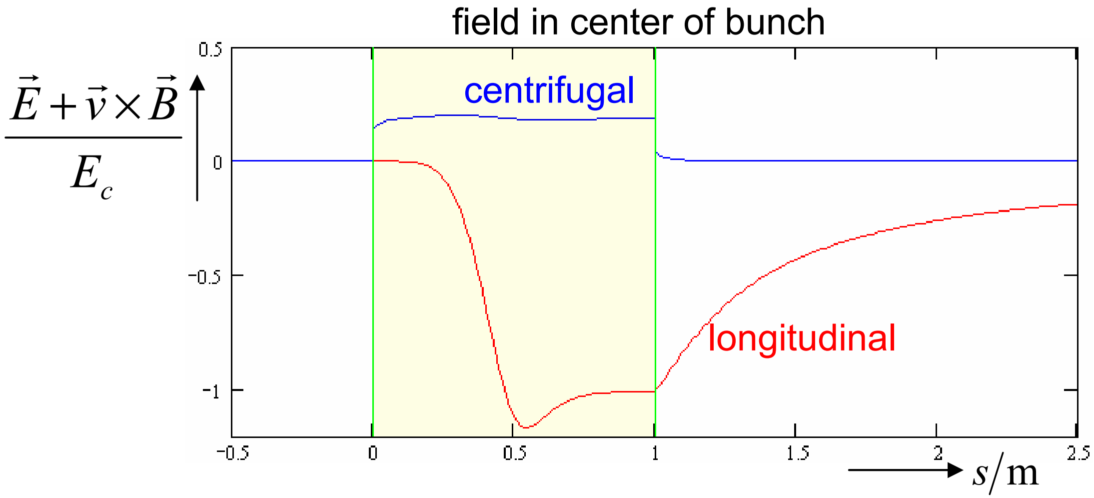
radiation effects

overtaking



long transients:



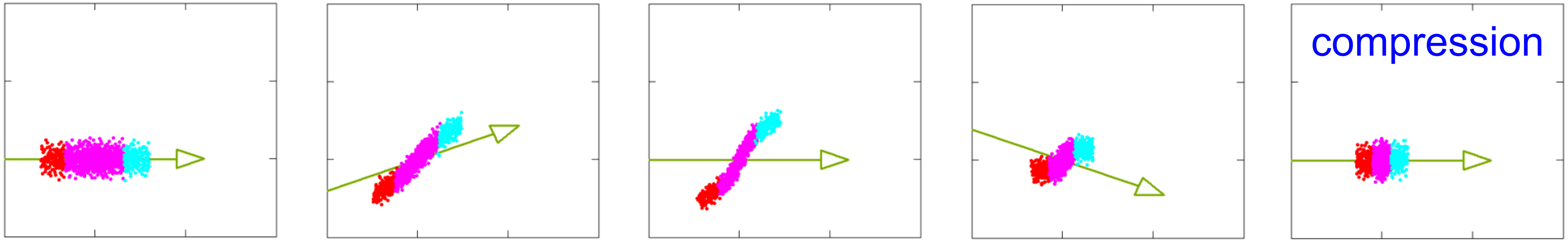


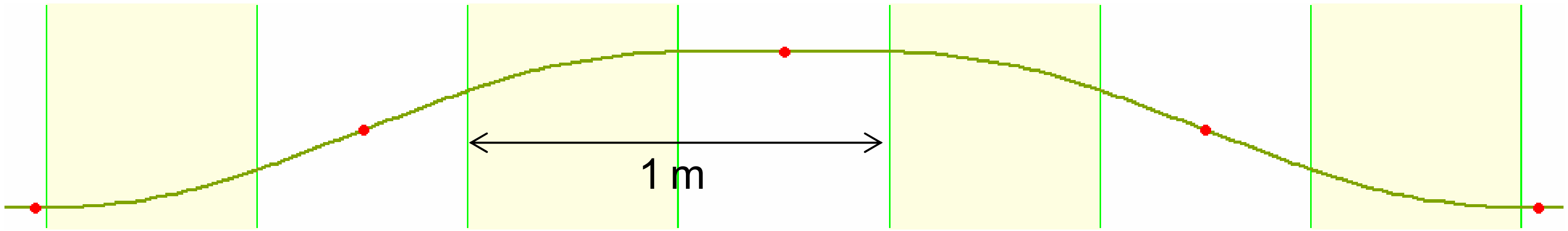
… effects

shape variation

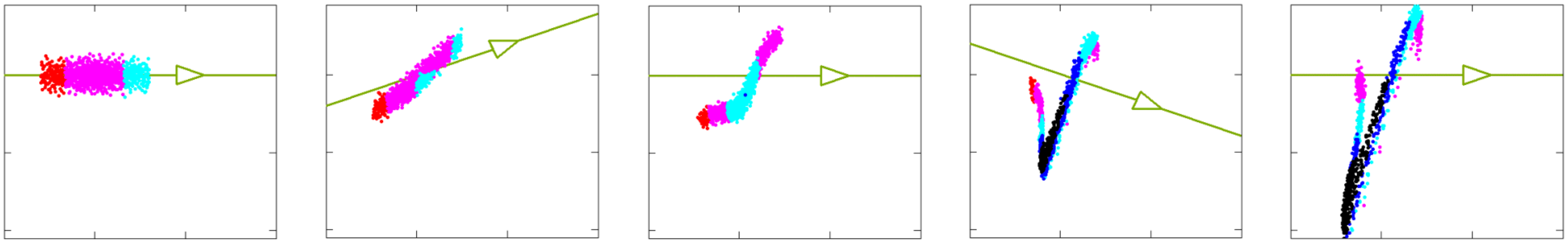
top view (horizontal plane), color = energy

without self-interaction





with self-interaction



emittance growth

approaches

1d (or projected) (1) sub-bunch approach (2) Maxwell-Vlasov (3)	retarded sources $\mathbf{E}(\mathbf{r},t)=\frac{1}{4\pi}\int\frac{\mathbf{Q}(\mathbf{r}',t')}{\lVert\mathbf{r}-\mathbf{r}'\rVert}dV' \qquad t'=t-\frac{1}{c}\lVert\mathbf{r}-\mathbf{r}'\rVert$ open boundary
paraxial approximation (4)	partial differential equation $\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=-\mathbf{Q}$ (closed) boundary condition

(1) Schneidmiller, Stupakov, Emma, Borland, Dohlus, …; ELEGANT, CSRtrack, …
(2) R.Li, Kabel, Dohlus, Limberg, Giannessi, Quattromini; ???, TrafiC^4, CSRtrack, TREDI
(3) Warnock, Bassi, Ellison
(4) Agoh, Yokoya

$\left.\begin{array}{l}(3)\\(4)\end{array}\right\}$ autor's code

Vlasov-Maxwell

Warnock, Bassi, Ellison: Progress on Vlasov Treatment …, PAC2005

4d phase space density in beam frame: $f = f(\mathbf{r},\mathbf{p},s)$

2d projections
history (@ retarded time)

$$\rho(\mathbf{r},t) = \int f(\mathbf{r},\mathbf{p},\beta ct)\,d\mathbf{p}$$
$$\tau(\mathbf{r},t) = \int p_x f(\mathbf{r},\mathbf{p},\beta ct)\,d\mathbf{p}$$

3d charge and current density distributions:
(in lab frame)

$$\rho_L(\mathbf{R},Y,t) = Q\cdot\boxed{H(Y)}\cdot\rho(\mathbf{r},t)$$

vertical profile

$$\mathbf{J}_L(\mathbf{R},Y,t) = \ldots$$

3d fields (by retarded source integration) $\mathbf{E}(\mathbf{R},Y,t)$

2d projections (weighted averaged) $\mathbf{E}(\mathbf{R},t) = \langle \mathbf{E}(\mathbf{R},Y,t)H(Y)\rangle_{Y\in\mathrm{gap}}$

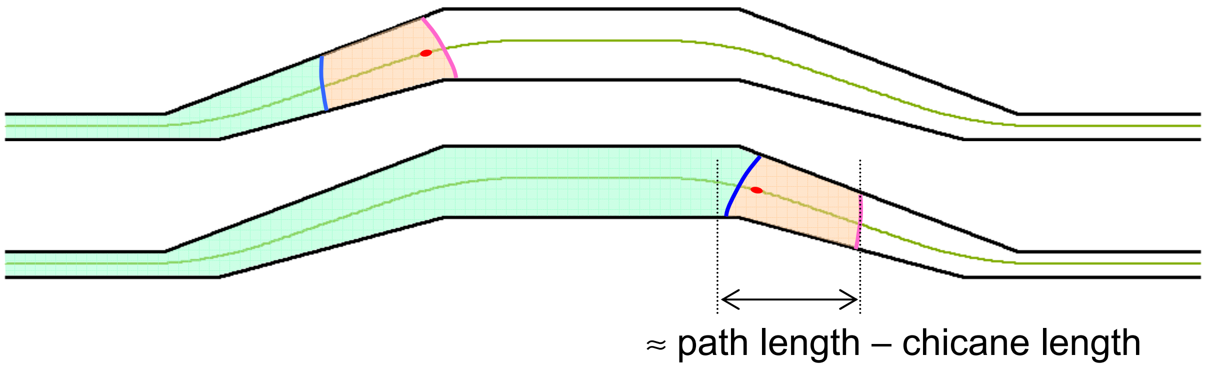
Lorentz force
EoM in beam frame, integration of Vlasov Equation

em-field calculation with PDE?

(on a grid)

the problems (direct time domain calculation):

calculation window >> bunch volume



mesh: $V/\sigma^3 \propto 10^8$

number of time steps $\propto$ chicane length / $\sigma \propto 10^6$

numerical dispersion

no way with explicit schemes (my personal opinion)

but: strong shielding: calculation window can be reduced

neglect backward waves; Field(x,y,s,t) is a slowly function of u=s-ct

paraxial approximation: large steps in u

frequency domain

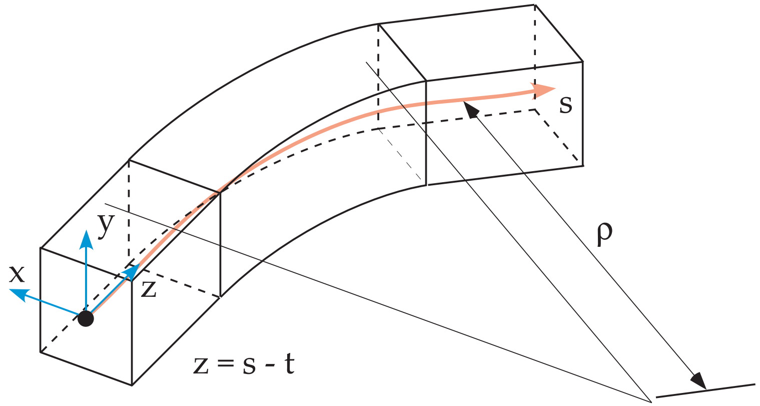
paraxial approximation

T. Agoh: PhD Thesis, Dec. 2004

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\breve{\mathbf{E}} = \mu_0\left(\nabla \breve{J}_0 + \frac{\partial \breve{\mathbf{J}}}{\partial t}\right)$$

wave equation in time domain

accelerator coordinates x,y,s



Fourier transformation $e^{ikz} = e^{ik(s-ct)}$

weak s-dependence (forward propagation)

pipe size small compared to bend radius

relativistic particles $\gamma >> 1$

paraxial approximation for transverse em-fields

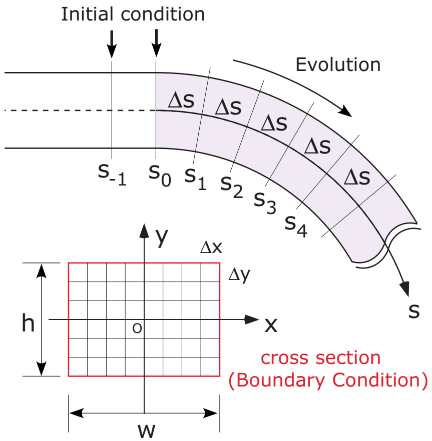
$$\frac{\partial E_\perp}{\partial s} = \frac{i}{2k}\left[\left(\nabla_\perp^2 + \frac{2k^2 x}{\rho}\right)E_\perp - \mu_0 \nabla_\perp J_0\right] \quad \rightarrow \quad E_s = \frac{i}{k}\left[\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \mu_0 J_0\right]$$

… paraxial approximation

T. Agoh: PhD Thesis, Dec. 2004

advantages:

- (curved) rectangular beam-pipes defined by coordinate planes
- bending radius needs not to be constant
- mesh based computation (explicit, frequency by frequency)
- resistive wall effects



- generalization to arbitrary transverse cross-sections and smooth variation of longitudinal profile

problems:

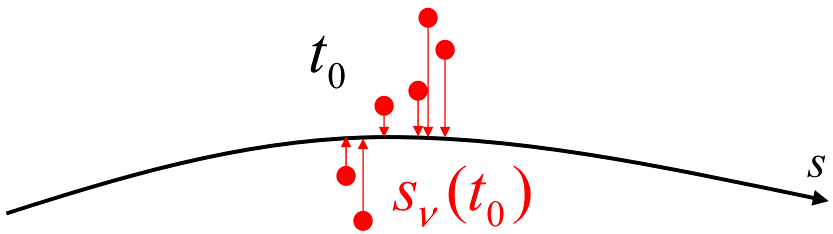
- free space or large chamber
- non smooth variations $\rightarrow$ stimulation of backward waves
- distributions with fine structure

special care:

- singularity of 1d beams
- transverse beam dimensions & SC effects
- variation of bunch shape

“CSR” codes: 1d

$$\dot{\mathbf{p}}_\nu = q\left(\mathbf{e}_{\nu\|} E^{(\lambda)}(s_\nu, t) + \mathbf{v}_\nu \times \mathbf{B}^{(\mathrm{ext})}\right)$$



some physics is missing

no transverse self-forces

no transverse dimensions,
rigid 1d charge distribution:

$$\lambda^{(\delta)}(s - t_0 c) = \sum q_\nu \delta((s - t_0 c) - (s_\nu - s))$$

$$\lambda(s - t_0 c) = \lambda^{(\delta)}(s - t_0 c) \otimes \left(g\left(s/\sigma\right)/\sigma\right)$$

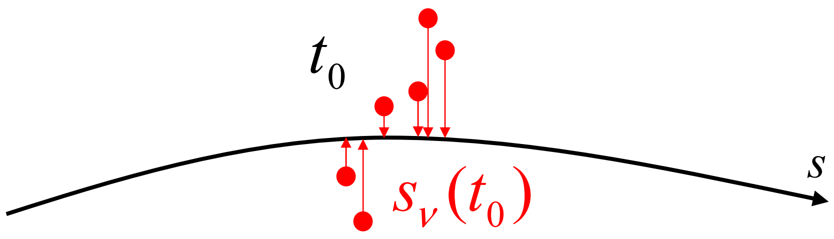
no SC effect,
1d E-field without γ^{-2} singularity:

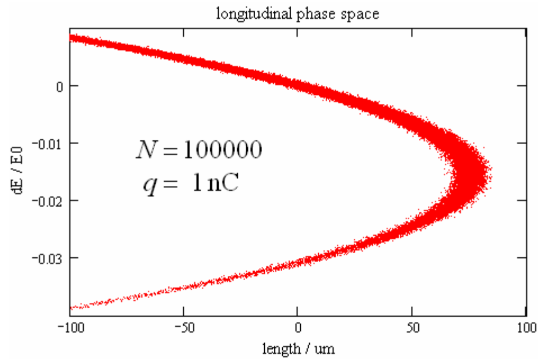
$$E^{(\lambda)}(s, t_0) = \int \lambda'(u + s - ct_0) K(s, u)\, du$$

no transverse dependency
of longitudinal forces

very low numerical effort

… “CSR” codes: 1d



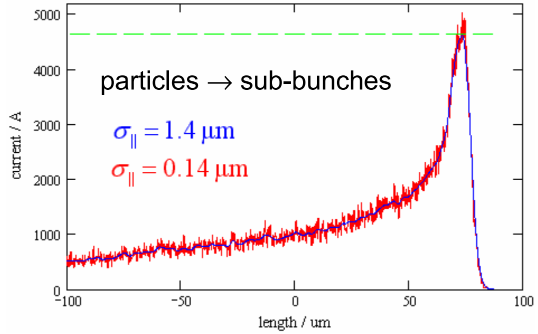


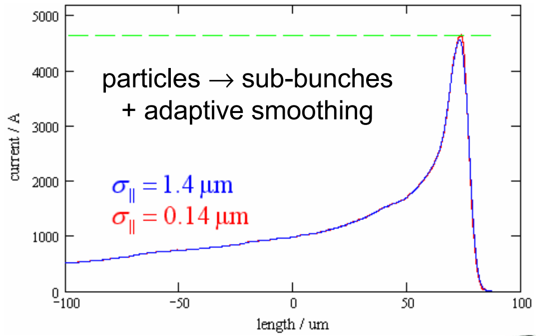
e.g. CSRtrack-1d

a) trajectory: general sequences of arcs and lines

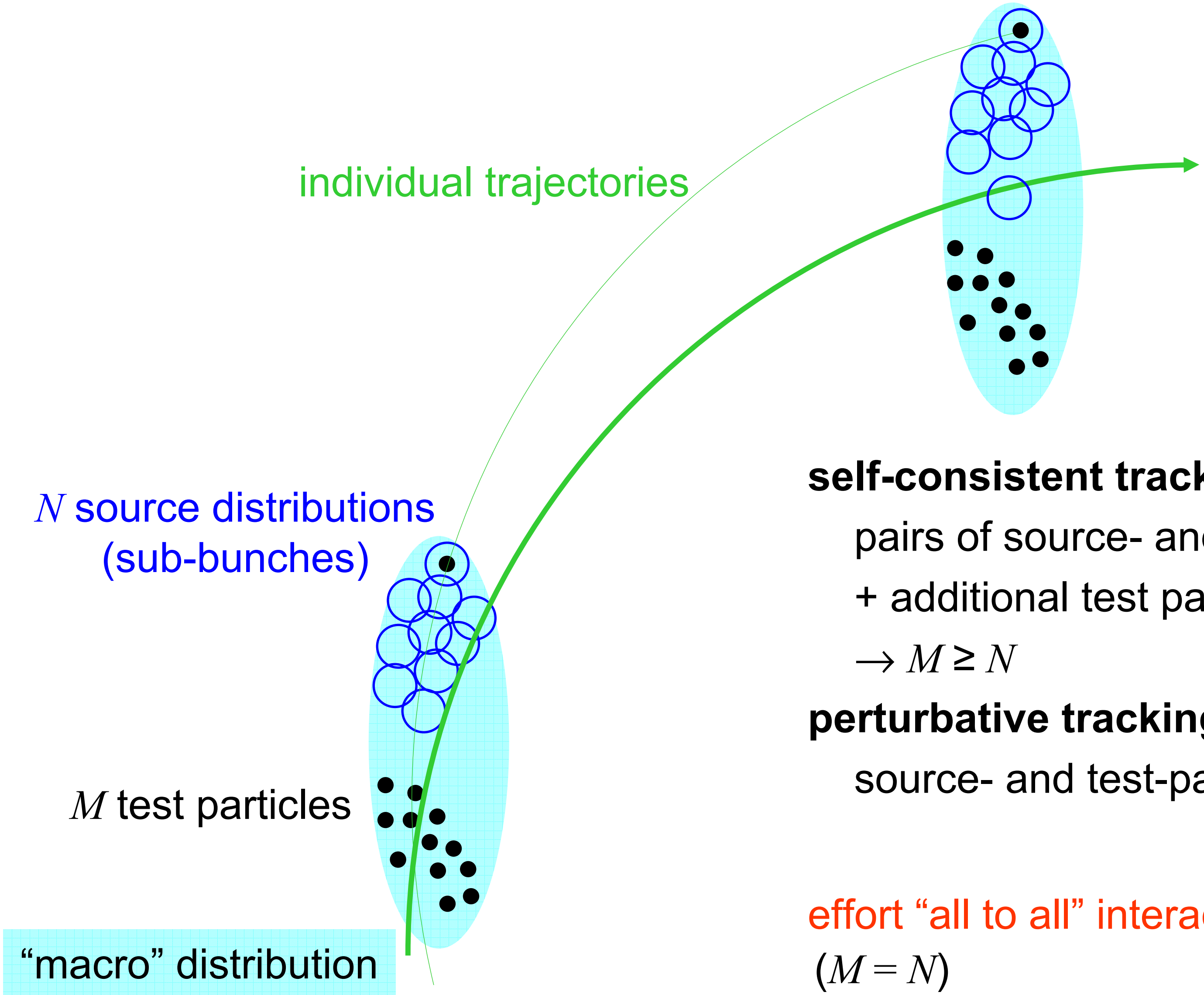
b) shielding: PEC planes

c) smoothing:
crucial for suppression of artificial μ-bunch effects
sub-bunches & density dependent adaptive filters





“CSR” codes: sub-bunch approach



self-consistent tracking:

pairs of source- and test-particles

\+ additional test particles

$\rightarrow M \geq N$

perturbative tracking:

source- and test-particles independent

effort “all to all” interaction = $N^2 \; E_{\mathrm{s\_to\_s}}$

$(M = N)$

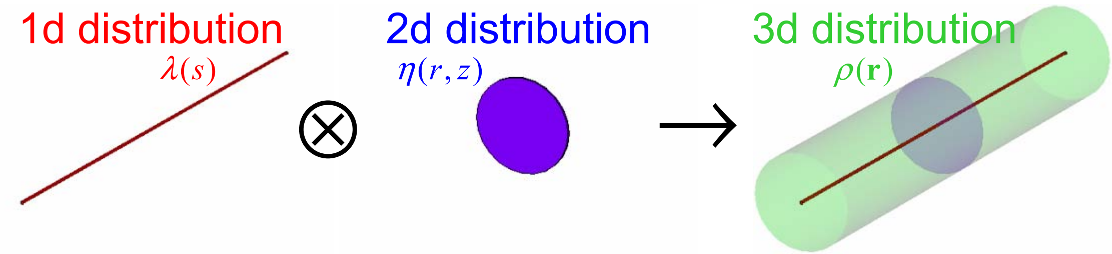
reduction of effort:
calculation of sub-bunches

in general:
3d sub-bunch needs 3d integration $\mathbf{E}(\mathbf{r},t) = \int \frac{\mathbf{Q}(\mathbf{r}',t')}{\|\mathbf{r}-\mathbf{r}'\|} \boxed{dV'}$

in particular:
reduction to 1d integrals for some types of sub-bunches

1d-bunch ⊗ 2-transverse distribution (approximation)	spherical, Gaussian (exact theory)

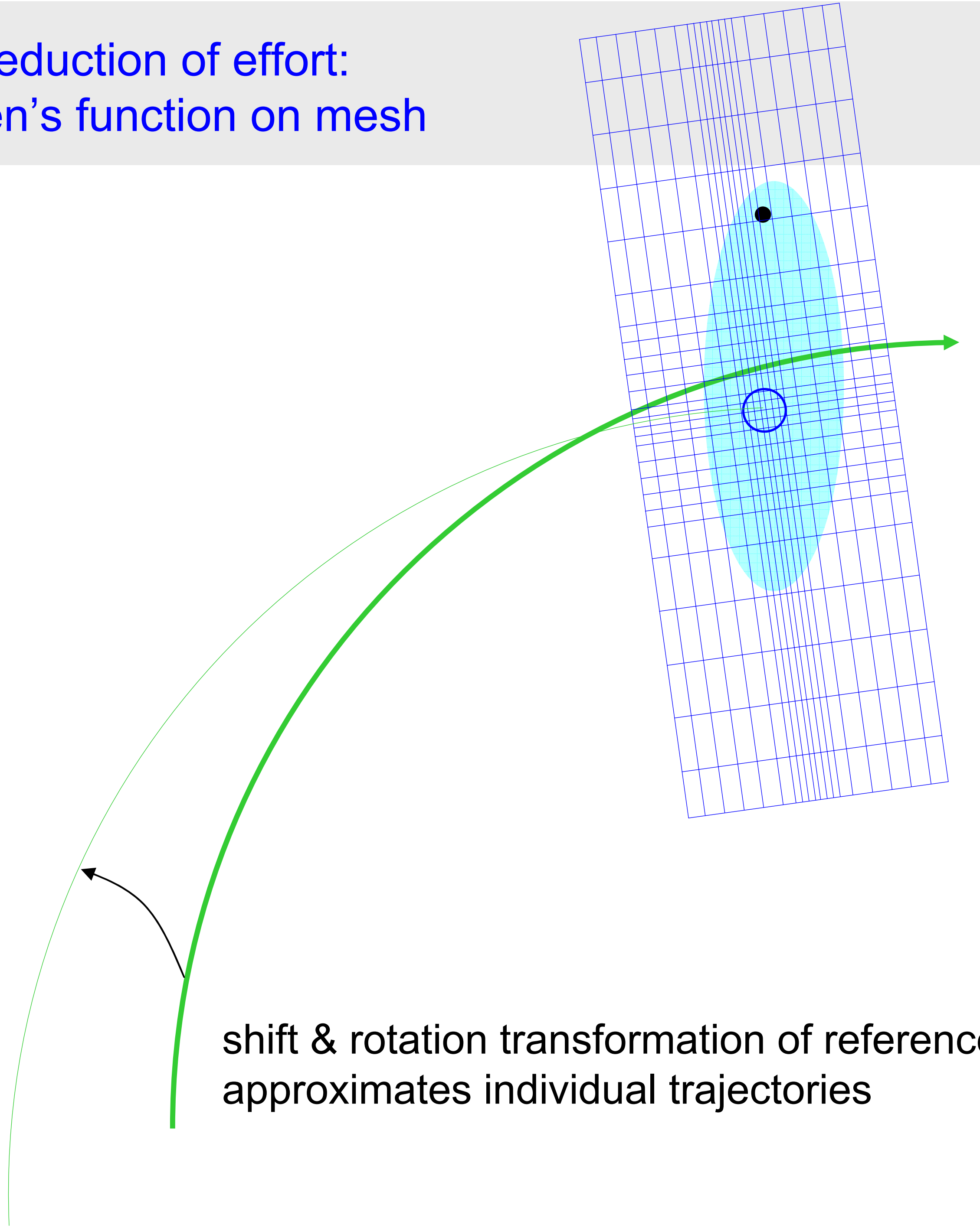
1d-bunch ⊗ 2-transverse distribution
(approximation)



used in TRAFIC[4] and CSRtrack

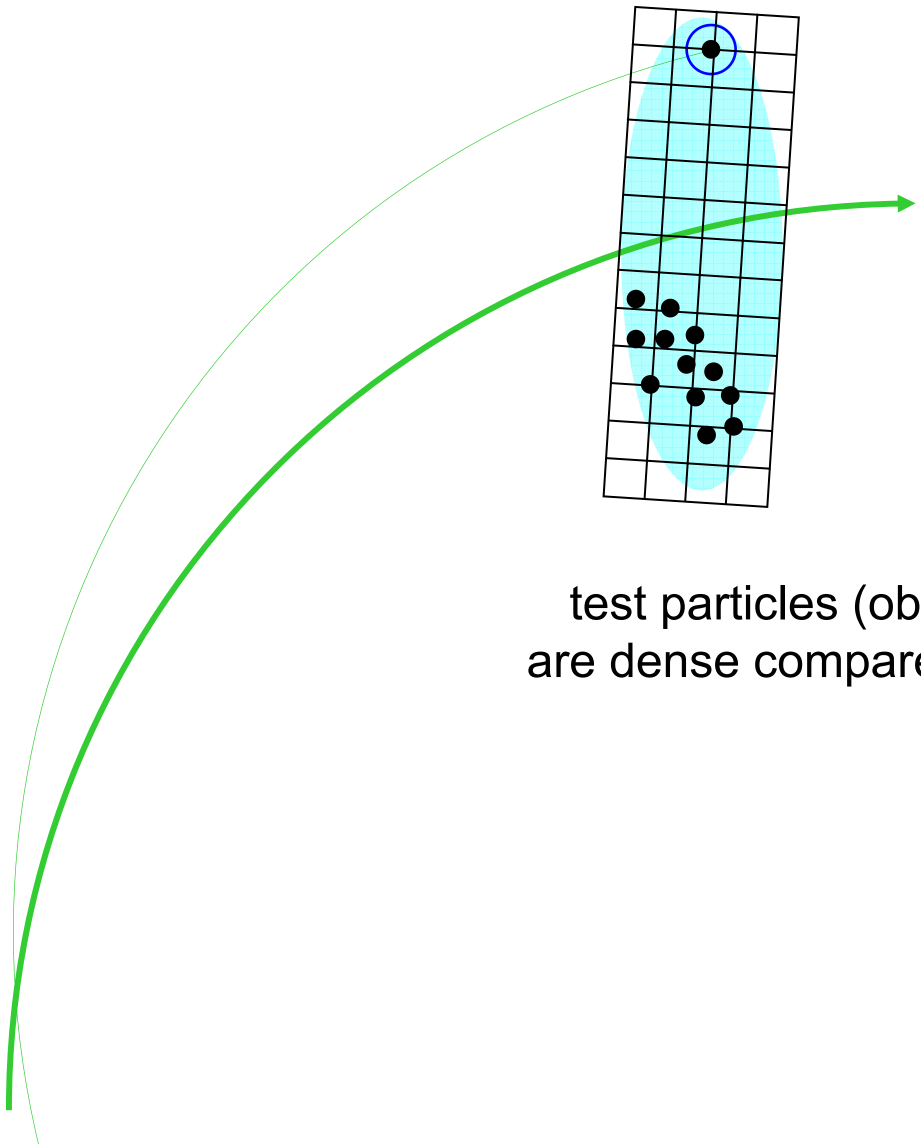
spherical, Gaussian
(exact theory)

reduction of effort:
Green's function on mesh



shift & rotation transformation of reference trajectory
approximates individual trajectories

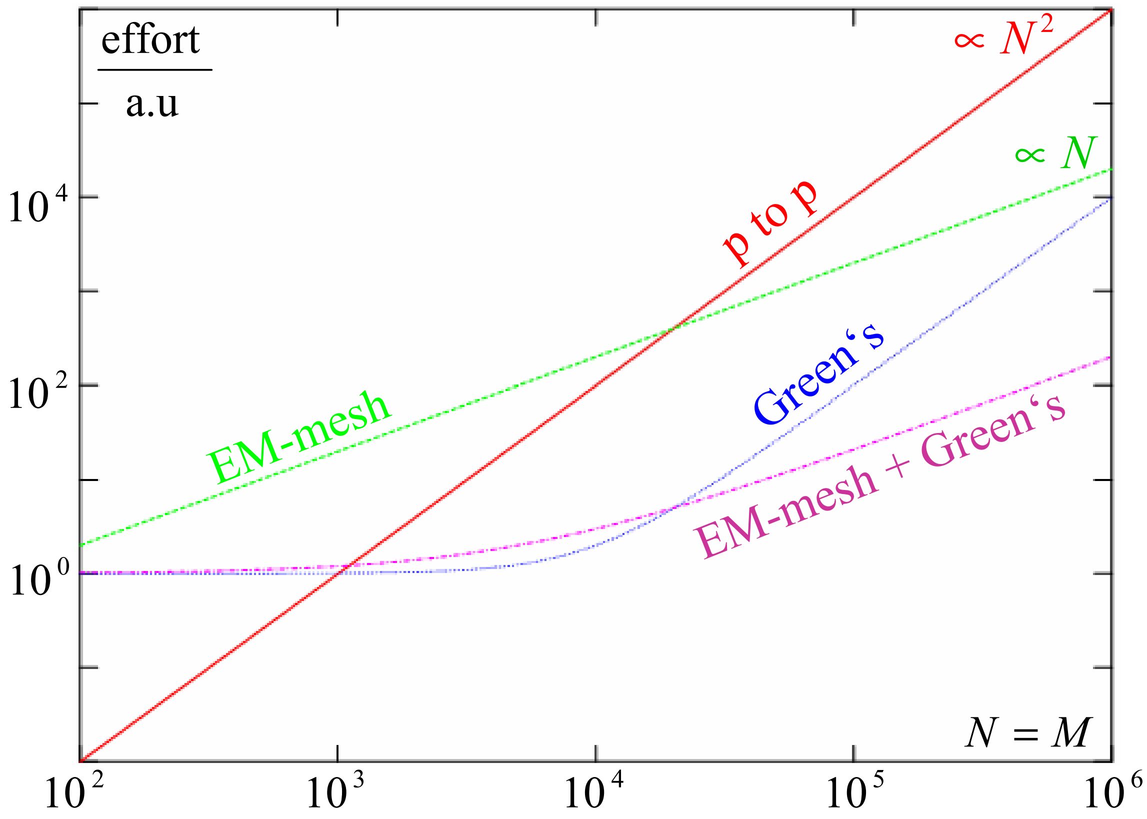
reduction of effort:
EM field on mesh



test particles (observer positions)
are dense compared to field variation

… "CSR" codes: sub-bunch approach

scaling of effort
(simplified)



Zeuthen benchmark chicane

ICFA Beam Dynamics Mini-Workshop, Berlin-Zeuthen 2002, `http://www.desy.de/csr`

CSR workshop 2002

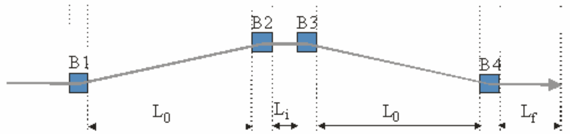
Workshop | Benchmark | Elegant | P. Emma Prog. | R. Li Prog. | TraFiC4 | Tredi | Info / Contact | Links

← computed by many CSR codes

still a reference for new developments

e.g. Maxwell-Vlasov solver

The example consists of a simple four-bend chicane with parameters similar to the one required for the compression stages of the LCLS (at 5 GeV) or TESLA XFEL (at 500 MeV). It is meant to be a compromise between academic benchmarking and more practical issues. The compressor is depicted in the figure below and its parameters are gathered in the first table. Click on the graphics to download a MAD input deck.



Parameters	*Symbol*	*Value*	*Unit*
Bend magnet length (projected)	L_b	0.5	m
Drift length B1->B2 and B3->B4 (projected)	L_0	5.0	m
Drift length B2->B3	L_i	1.0	m
Post chicane drift	L_f	2.0	m
Bend radius of each dipole magnet	R	10.35	m
Bending Angle	ϕ	2.77	deg
Momentum compaction	R_{56}	-25	mm
2nd order momentum compaction	T_{566}	+37.5	mm
Total projected length of chicane	L_{tot}	13.0	m
Vertical half gap of bends	g	2.5,5	mm

The electron beam description:

The input electron beam will test two different examples: (1) a uniform, and (2) a Gaussian distribution for the temporal profile, where the initial rms length is the same in both case ($FWHM_{uniform}$=2Sqrt[3]*rms). The transverse phasespace is assumed to be gaussian in either case. The beam should have a perfectly linear time-energy "chirp" (the bunch head has lower energy than the tail). Therefore the time and energy distribution will be identical. In addition a very small uncorrelated ("slice") energy spread should be added with, for example, a Gaussian distribution.

Parameter	*Symbol*	*Value*	*Unit*
Nominal energy	E_0	0.5/5.0	GeV
bunch charge	Q	0.5, 1.0	nC
incoherent rms energy spread	$(\Delta E)_{\dots}$	10	keV

energy = 500 MeV / 5GeV

charge = 0.5 nC or 1nC

compression factor = 10

(600 A → 6 kA)

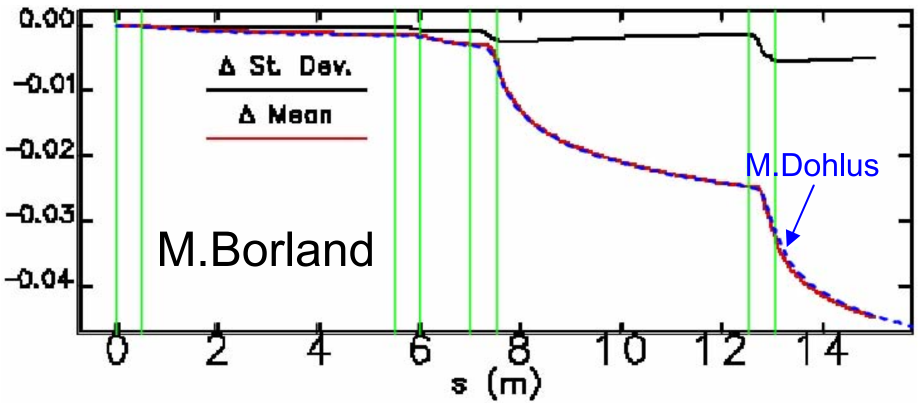
shape = Gaussian / rectangular

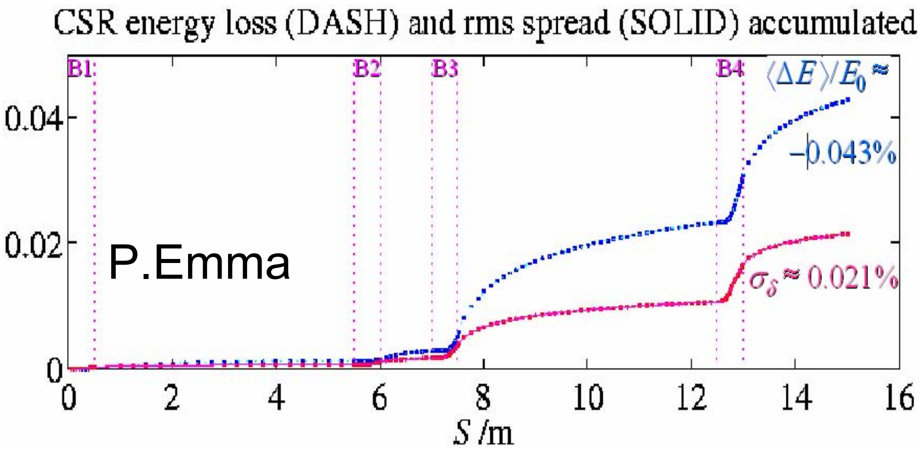
… Zeuthen benchmark chicane

longitudinal phase space

5GeV, 1nC, Gaussian

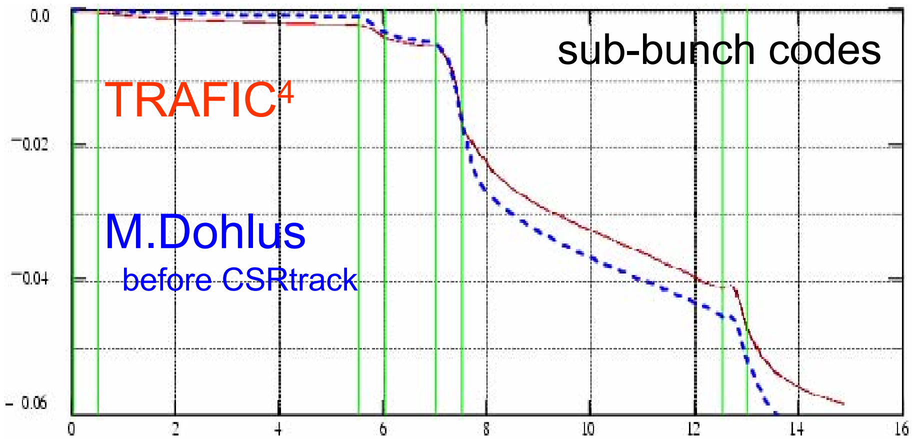
1d codes

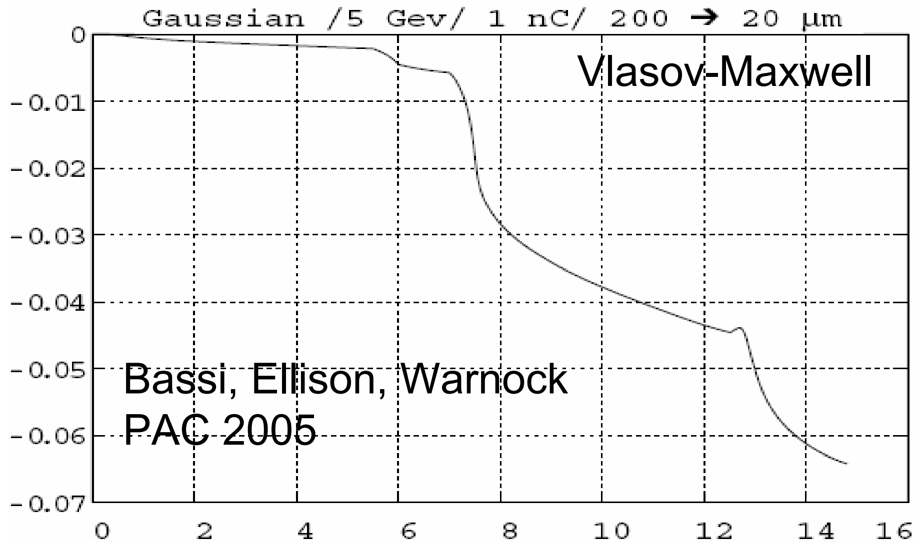




agreement between 1d codes

e.g. relative loss @ 14m ≈ 0.04%





relative loss @ 14m ≈ 0.06%

(differences due to transverse beam dim.?!)

… Zeuthen benchmark chicane

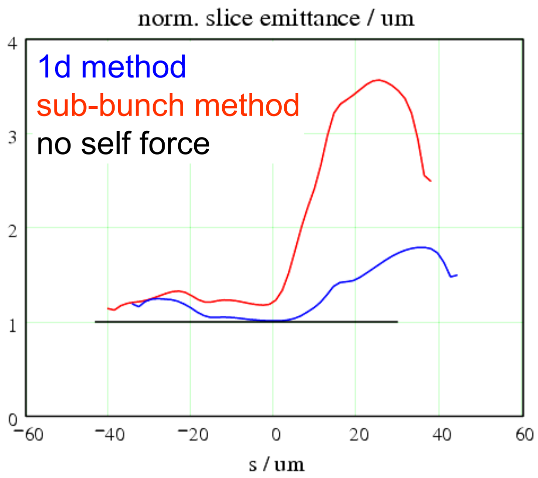
horizontal phase space

5GeV, 1nC, Gaussian

agreement between 1d and sub-bunch methods:
projected emittance
slice emittance

but: 500 MeV, 1nC, trapezoid
(stronger SC effects)

significant differences between 1d and sub-bunch methods:
projected emittance: 5 compared to 3
slice emittance:



part 1: CSR codes

part 2: simulation of BC systems

physical & numerical problems

example: SC effects after last BC

piecewise tracking, codes & tools

μ bunching

particle distributions

non linear effects in longitudinal phase space

examples: rollover compression

linearized compression

compensation in 2BC systems

conclusions

more physical & numerical problems

physical
numerical

- tracking with different methods (different particle descriptions)

- particle description (macro particles,
  ensembles, sub-bunch distributions
  phase space density)

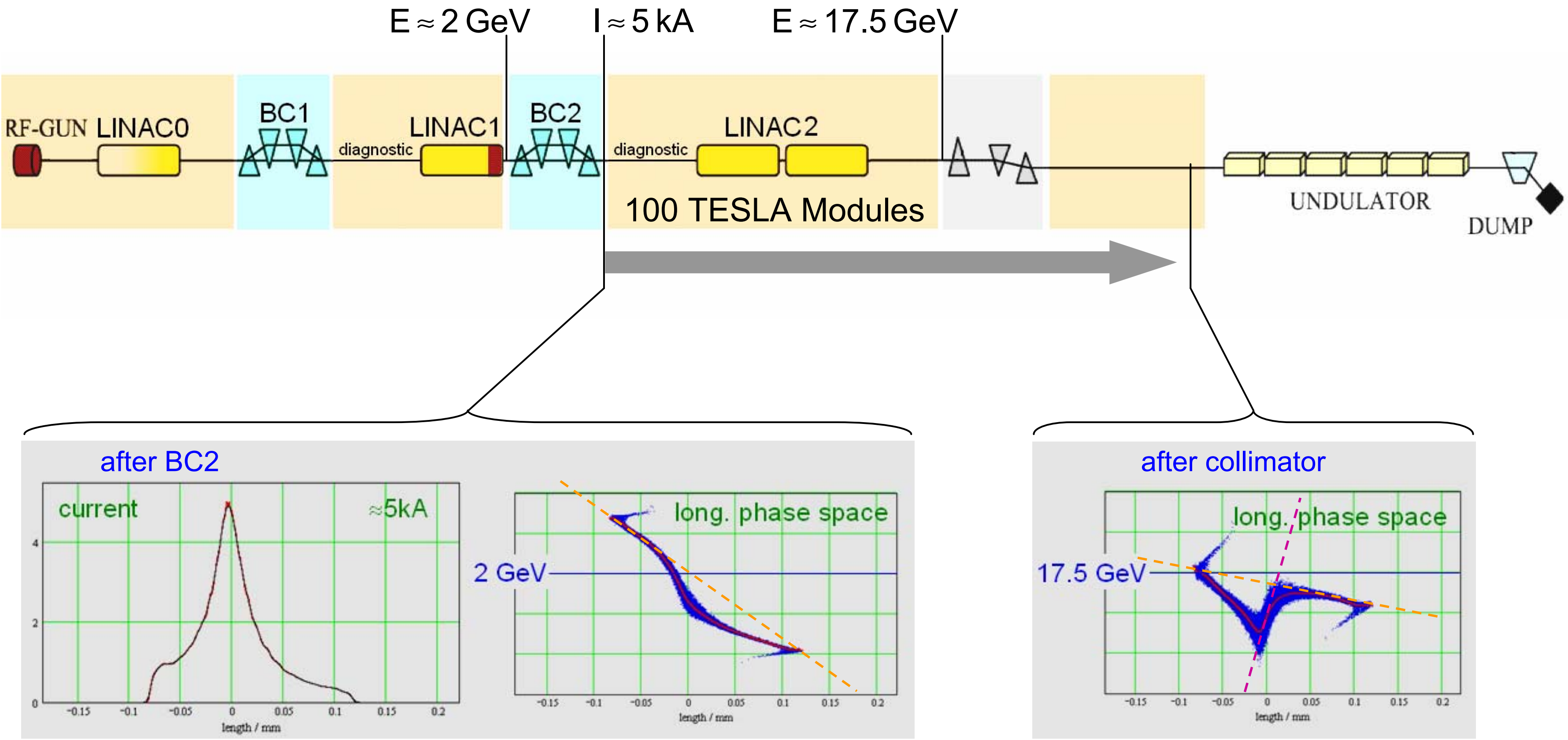
- μ-bunching $\rightarrow$ laser heater
  $\rightarrow$ decoupled investigation $\rightarrow$ amplification
  $\rightarrow$ noise suppression

- longitudinal sensitivity $\rightarrow$ a) controlled compression
  $\rightarrow$ b) “over” compression

- transverse: space charge Q shift

example: SC effects after last BC

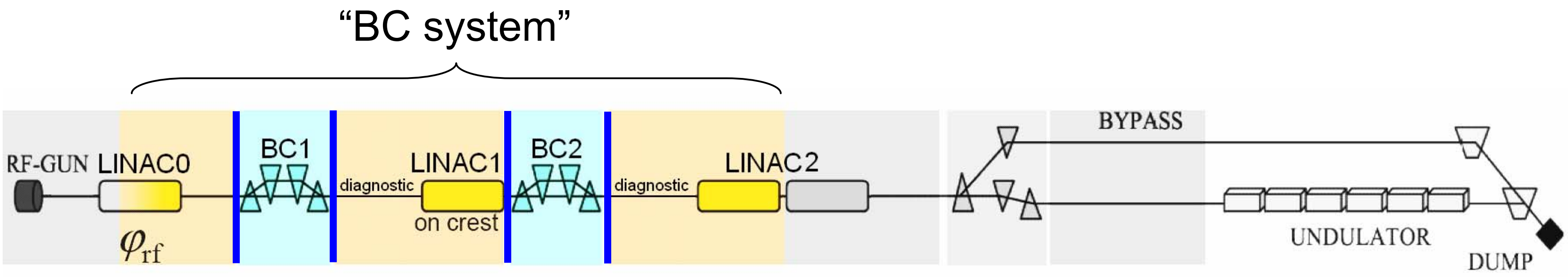
European XFEL:



negative chirp compensated by LINAC wakes

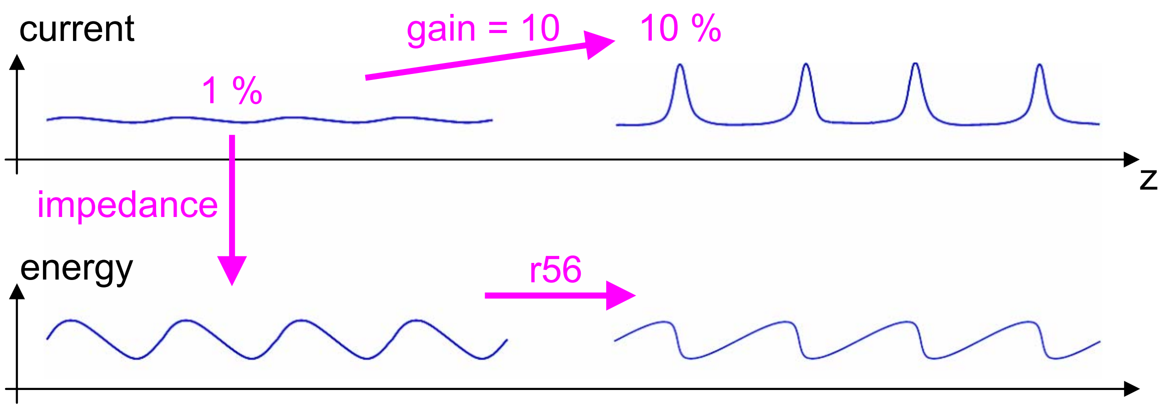
positive chirp induced by space charge !

piecewise tracking, codes & tools



linear trajectory codes (=LT codes)	Runge-Kutta tracker + Poisson solver PARMELA, ASTRA, GPT, … or ELEGANT + external SC calculation
CSR codes	Trafic4, TREDI, CSRtrack, Elegant, …
utility programs	conversions simple manipulations …

μ-bunching - amplification



picture based on: Z. Huang FLS2006

impedances (steady state):

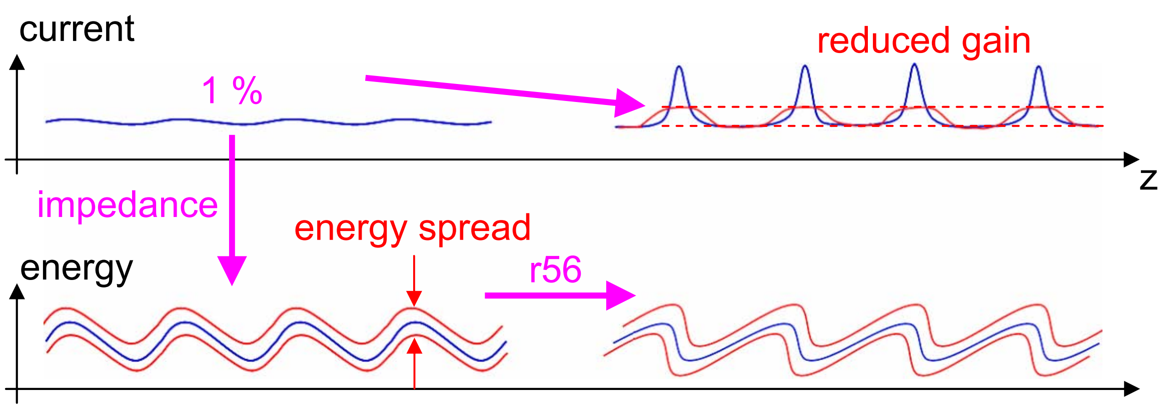
$$Z'_{sc}(k,\gamma,\ldots) \propto \frac{ik}{\gamma^2}\ln\left(\frac{\gamma}{k\sigma_r}\right)$$ "SC-instability"

$$Z'_{CSR}(k,R_{curv}) \propto \sqrt[3]{\frac{k}{iR_{curv}^2}}$$ "CSR-instability"

… μ-bunching - amplification

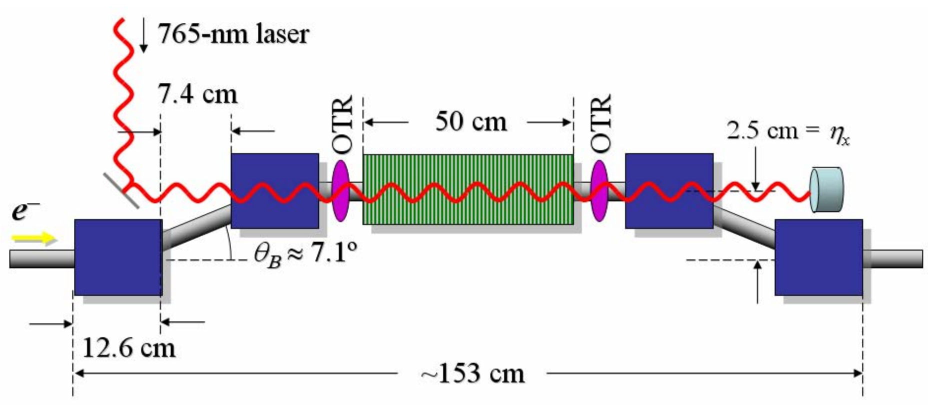
laser heater

proposed by E. Schneidmiller 2002



picture based on: Z. Huang FLS2006

‘laser heater’ System (LCLS layout)



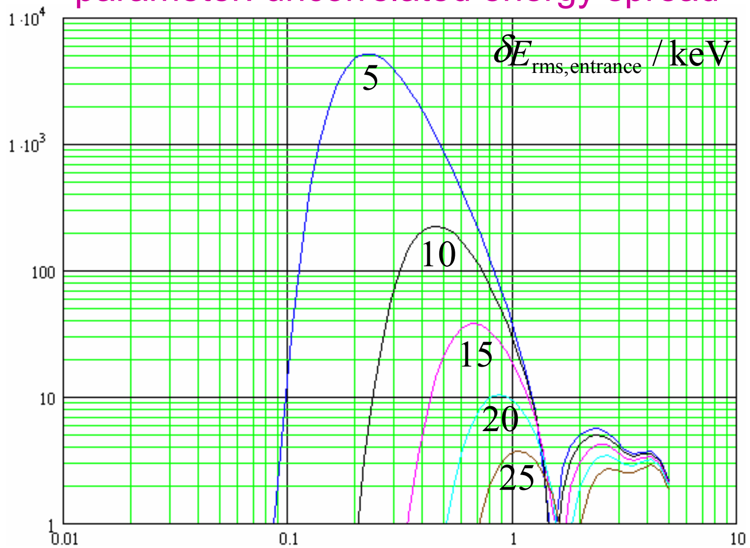
… μ-bunching - amplification
numerical aspects

1) it is difficult to simulate macroscopic & microscopic effects together
(very high resolution, very many particles required)

2) → separate investigation of μ-bunching
CSR: integral equation method (limited applicability)
projected method: modulated beam, 1- and 2-stage compression
SC: impedance + r56

example: European XFEL

gain vs. modulation wavelength
parameter: uncorrelated energy spread



wavelength/ mm (before bc1) →

3) s2e simulations without μ structure:
avoid artificial instability
e.g. due to shot noise of few macro-particles → noise reduction

particle distribution

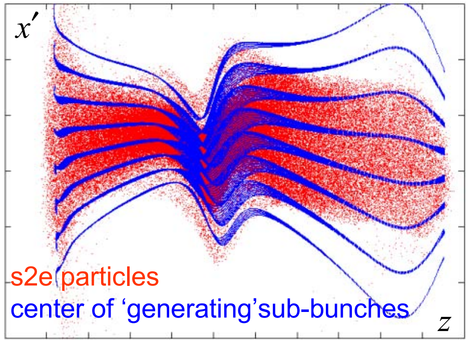
- use one particle set for complete simulation
  if possible!

- from injector simulation: N ~ 10^5 … 10^6 equal charged particles
  (semi)random in 6d phase space

- N is no problem for LT-codes and 1d-CSR codes;
  N is possible with sub-bunch CSR codes with mesh + parallel processing
  no hope for CSR codes with particle to particle interaction

- noise is a problem in general
  1d: binning, filtering, density adaptive methods
  mesh: use enough particles per cell

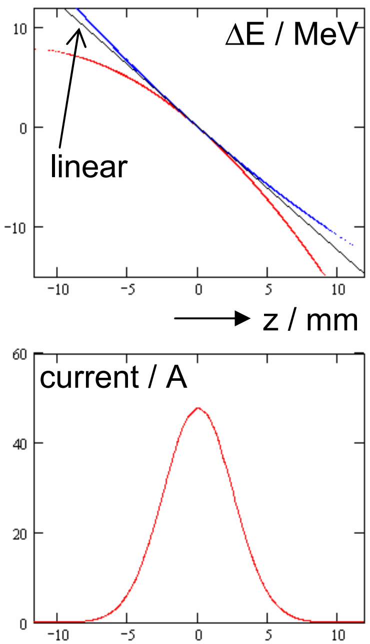
- ‘image’ technique
  create a ‘image’ distribution that has all essential
  properties of the original particle set but is smooth;
  track image distribution with self interactions;
  track original particles in the field of the ‘image’;



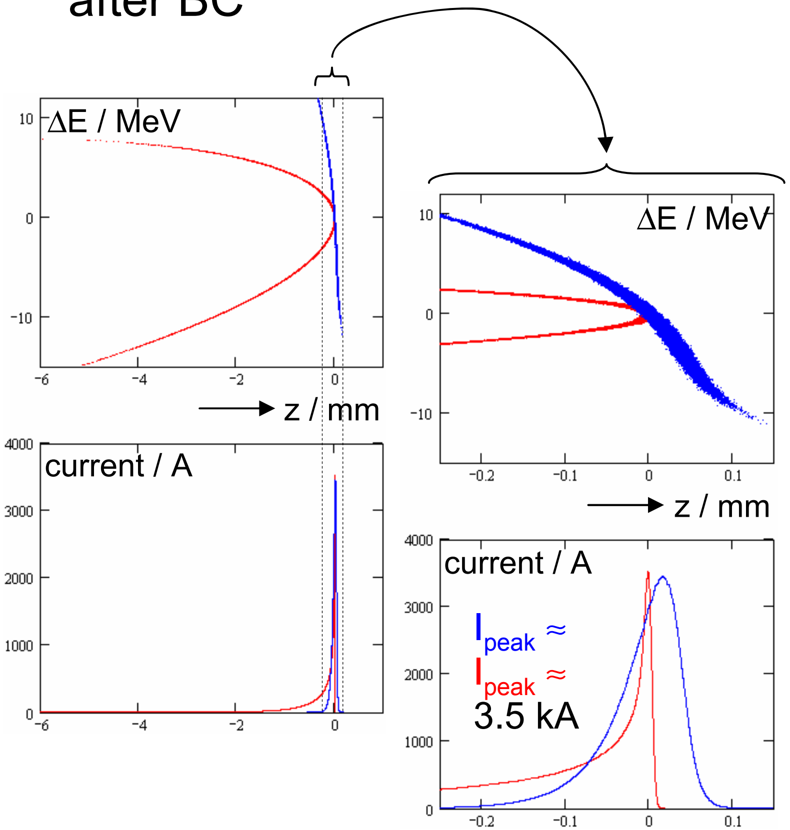
non linear effects in long. phase space

'controlled' or linearized compression
'rollover' compression

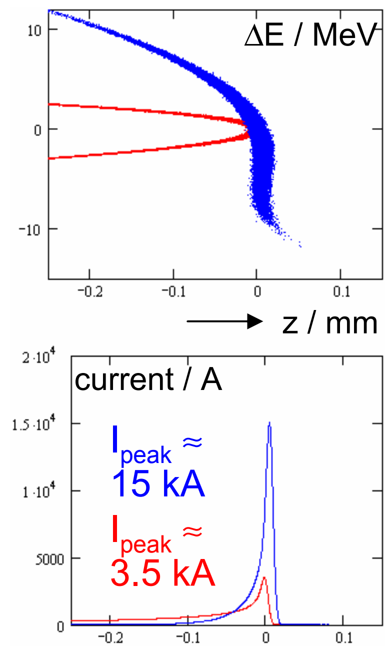
before BC



after BC

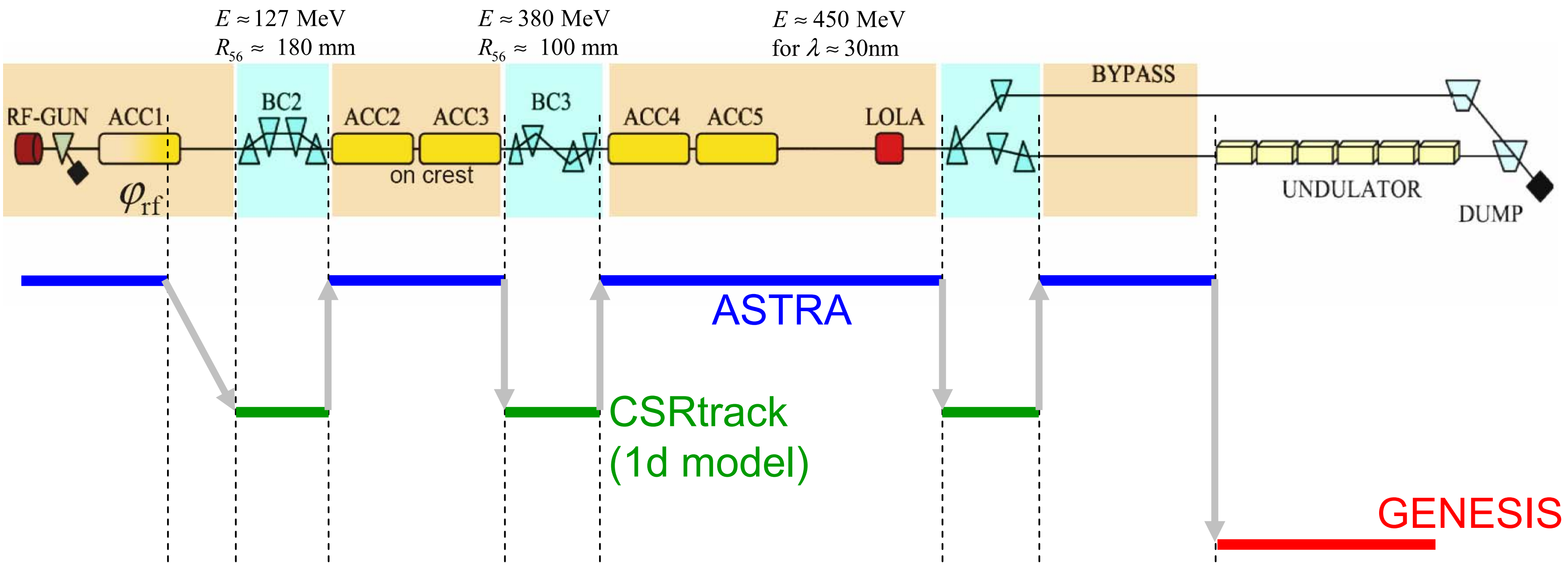


lost control:
magnet strength
changed by 0.5%

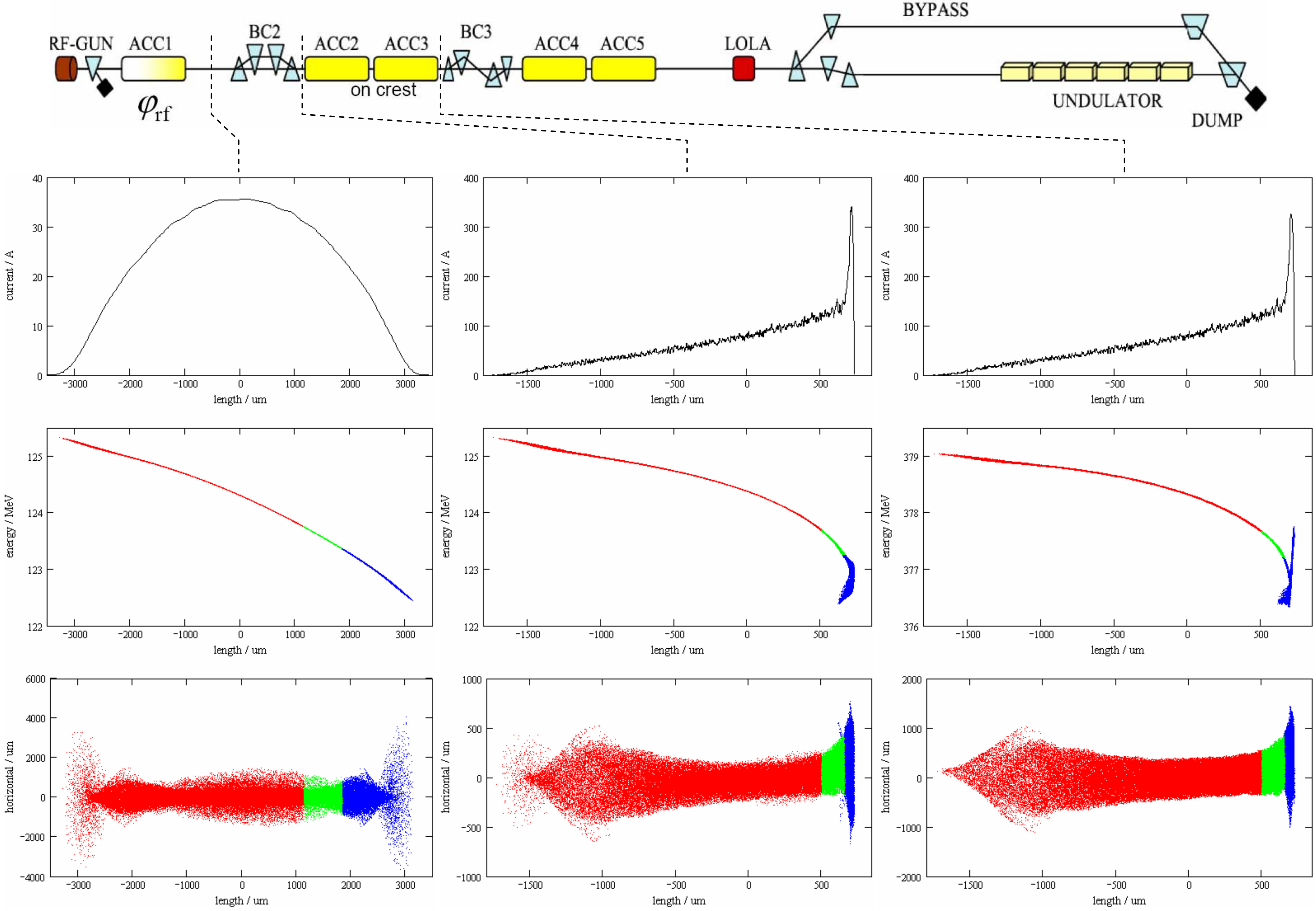


… rollover compression

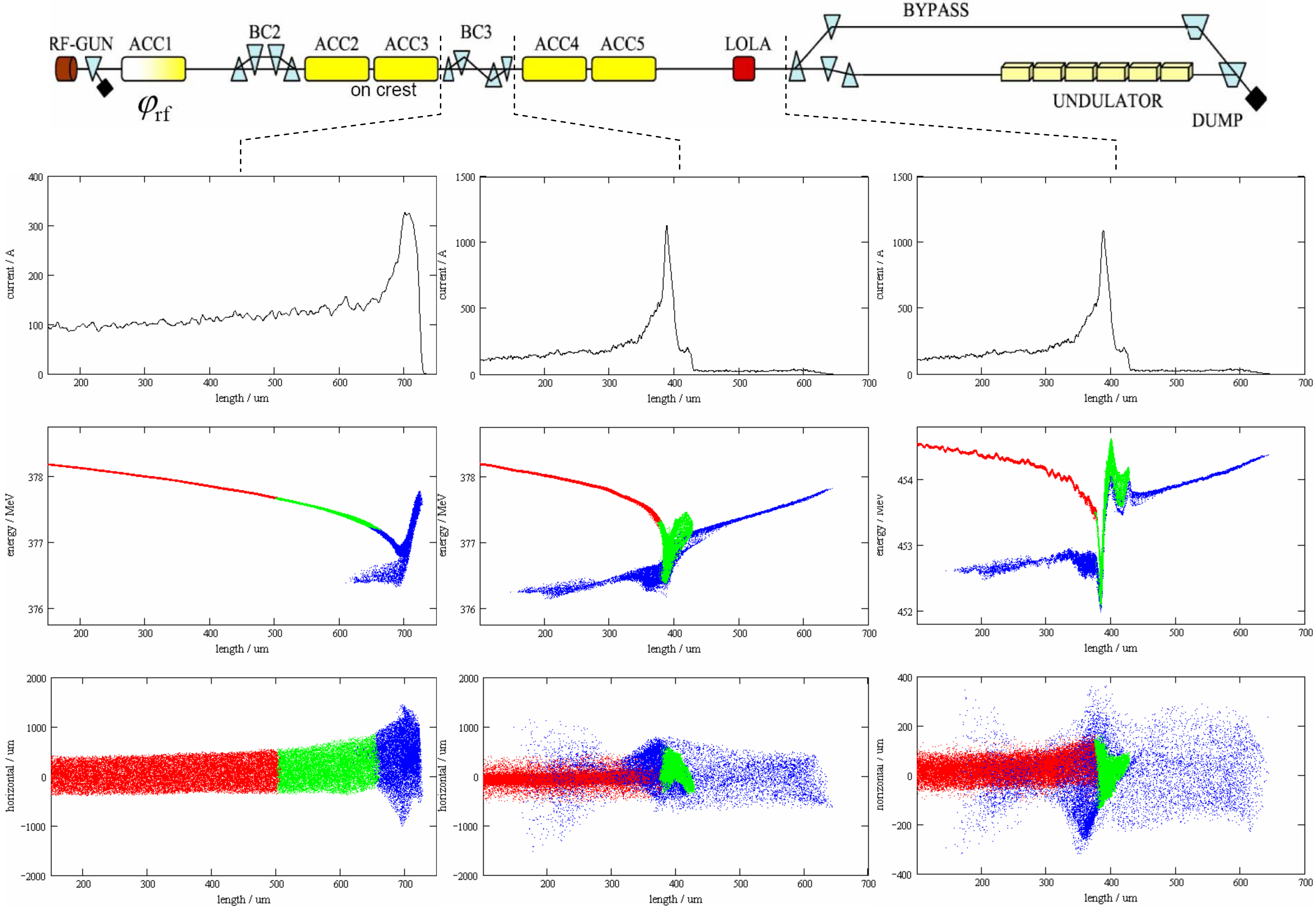
example: FLASH s2e simulation



example: FLASH s2e simulation

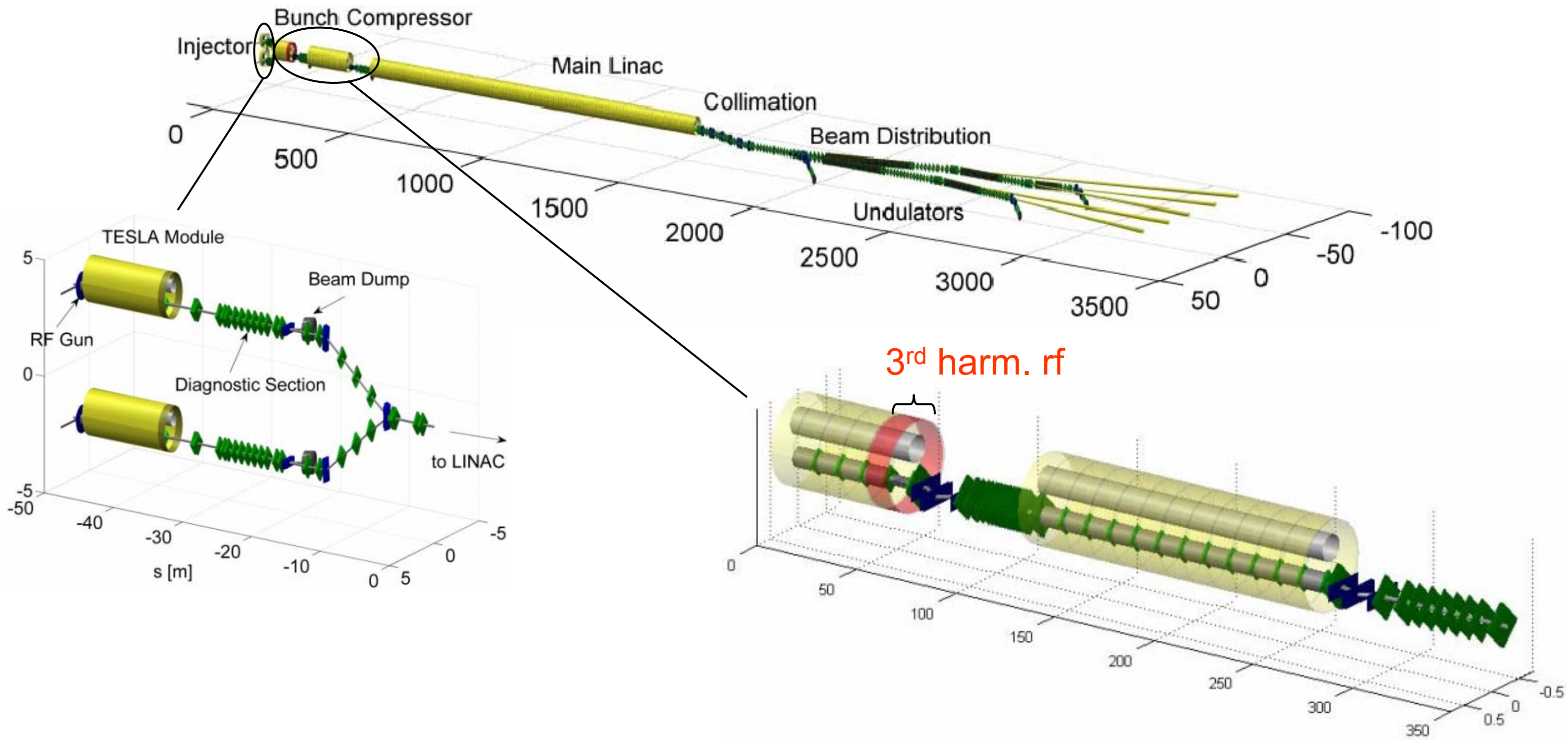


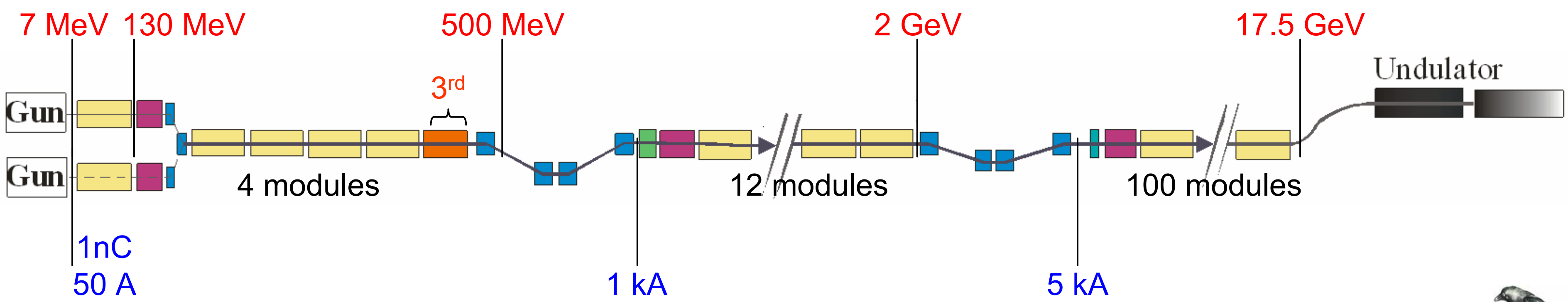
example: FLASH s2e simulation



controlled compression

example: European XFEL

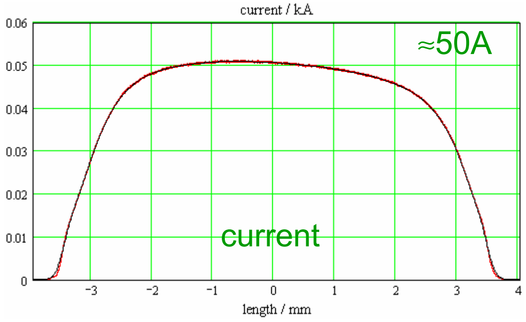


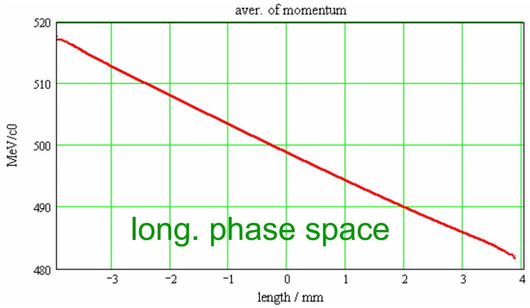


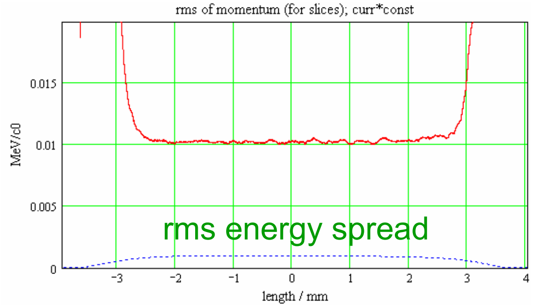
example: European XFEL

before BC1

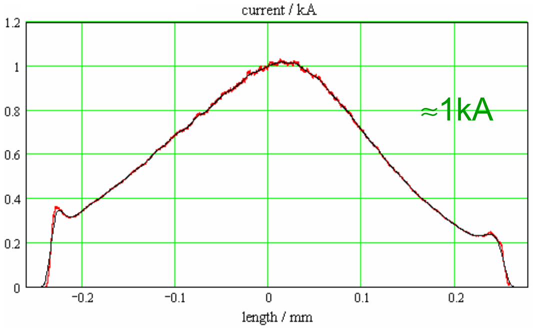
1.3GHz: 442.85 MV 1.42 deg
3.9GHz: 90.63 MV 143.35 deg

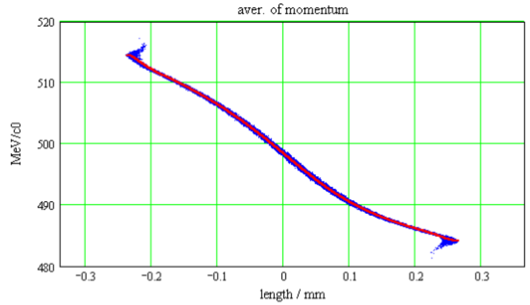


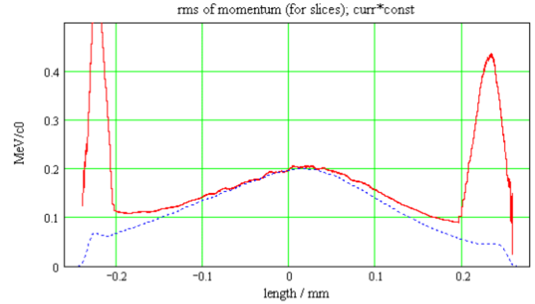




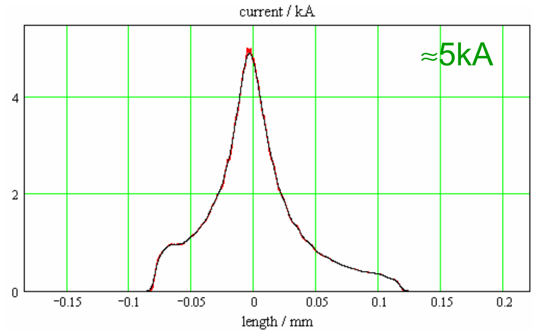
after BC1

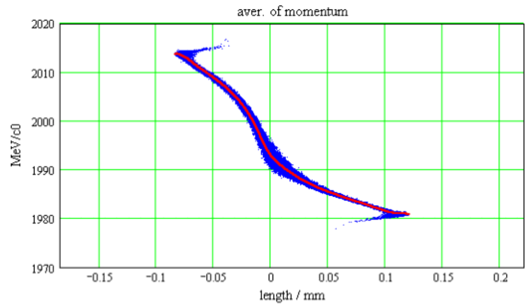


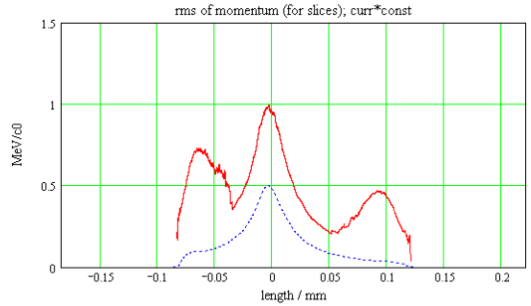




after BC2

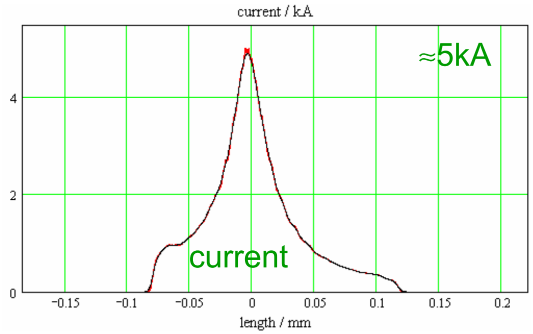


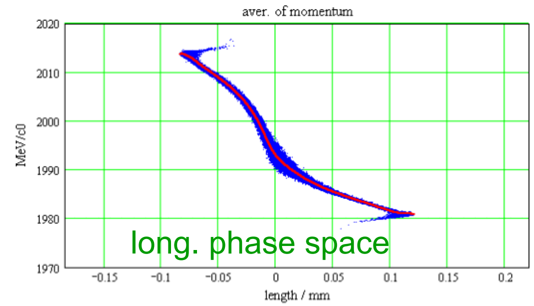


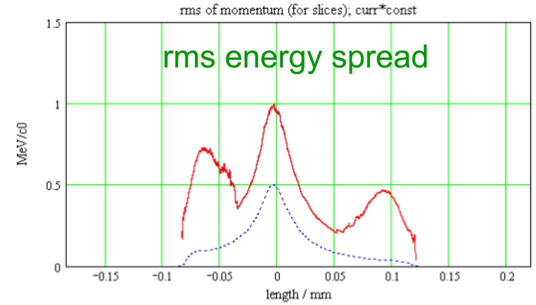


example: European XFEL

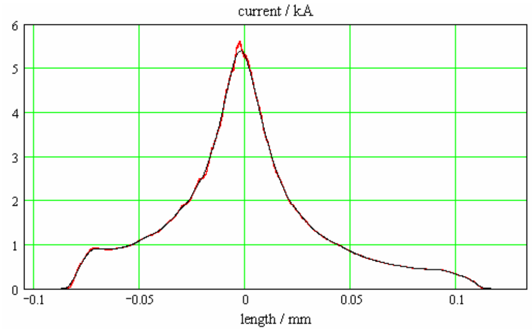
after BC2

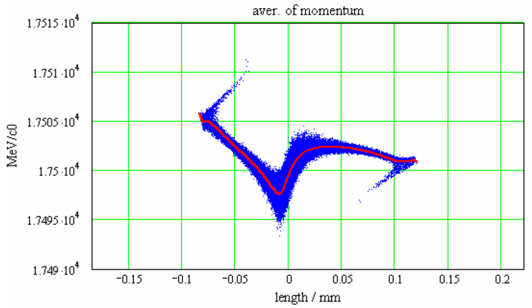


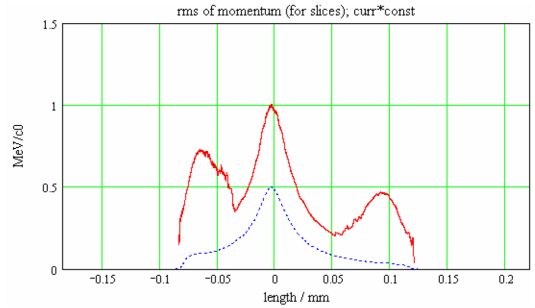




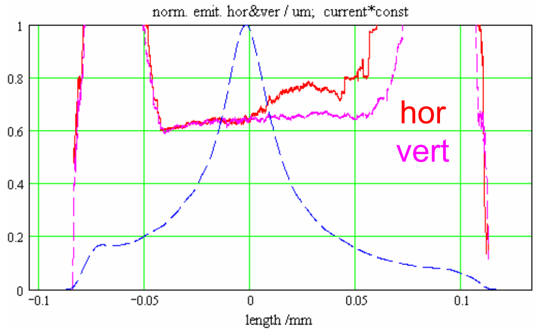
after collimator





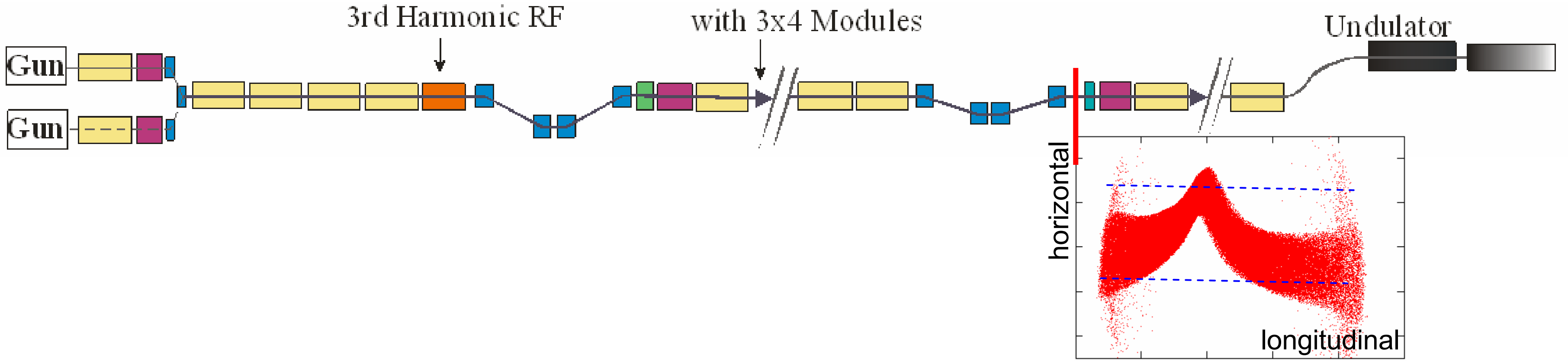


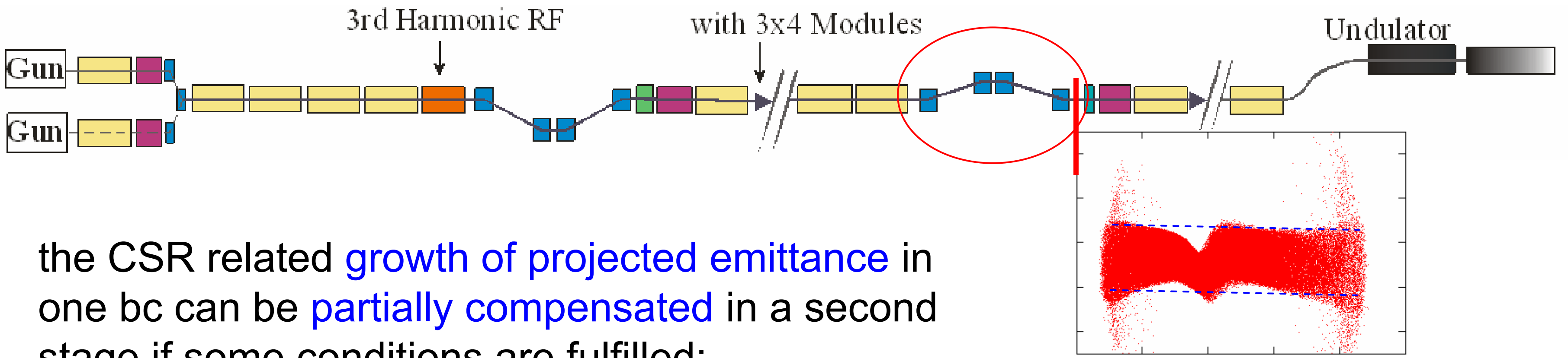
norm. emittance



compensation in 2-bc systems

shielding & resistive walls





the CSR related growth of projected emittance in one bc can be partially compensated in a second stage if some conditions are fulfilled:

right phase advance,

right compression ratio (chirp as well as r56),

no interference with other effects

to be checked: influence of shielding, resistive walls

Conclusion

part II

• effects in BC **systems** are challenging (many physical effects are involved)
• μ-bunching effects beyond the resolution of non-1d-codes
• several types of codes needed (LT- and CSR-codes)

part I

• 1d- and sub-bunch codes are available
  Vlasov-Maxwell approach and paraxial approximation under development
• resolution of sub-bunch method increased
• ‘CSR’ methods cover all important physical effects
  (SC, CSR, shape variation, shielding, resistive walls)

in reach: code that covers all effects