# SIMULATION OF MICE USING G4MICE

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# Abstract

In the Muon Ionisation Cooling Experiment (MICE) muons will be fired through a cell of a muon ionisation cooling lattice. The results of the experiment will be used to optimise the cooling channel of a future Neutrino Factory. Upstream and downstream of the MICE cooling channel are two spectromenters is which will be used to measure the position of each muon in six-dimensional phase space. The behaviour of a beam of muons will be studied by collecting particles into bunches offline. The experiment will be run with a number of different input beams, magnet configurations, RF configurations and absorber types. We present the simulated detector and cooling performance of the MICE cooling channel using the G4MICE simulation code for a range of configurations. We detail the simulation of different absorber and field models and examine the cooling efficacy of the channel in the baseline case.

# SIMULATION REQUIREMENT FOR MICE

The MICE collaboration aims to observe ionisation cooling of muons for the first time. In the experiment, pions are generated at a target in the ISIS proton synchrotron and collected in the MICE beam line. Pions decay to muons in a 4T solenoid before propagating through a series of quadrupoles. Residual pion contamination of the muon beam downstream of the quadrupole channel are rejected by means of a Cerenkov detector and two time of flight counters (TOFs). A scintillating fibre tracking detector is used to measure the muon momentum and position, while the TOF hodoscope immiediately upstream of the upstream tracker is used to measure the muon time-of-arrival. Transport of the muon beam through the MICE cooling channel is achieved using a series of solenoids. Liquid hydrogen absorbers are used to reduce the muon energy, cooling the beam; RF cavities restore the energy lost in the absorbers. Downstream of the MICE cooling channel, a second spectrometer, including a scintillating fibre tracker, a third TOF hodoscope and a calorimeter is used to measure the muon phase space.

The experiment is simulated in the G4MICE software package based on GEANT4 [1]. G4MICE is designed to perform a full simulation of both accelerator and detector elements of MICE. The software package performs two functions: firstly, detector simulations will be used to determine the detector performance; and secondly, the Monte Carlo distributions will be compared to the results of the MICE experiment. These comparisons will allow an accurate simulation to be developed that can be used to optimise the design of the cooling channel for the Neutrino Factory.

## **COOLING CHANNEL**

The scintillating-fibre tracking detectors sit in regions of uniform magnetic field. The beam in the tracker solenoids is matched to the MICE cooling channel that sits between them. The cooling channel is made up of an SFoFo field generated by eight superconducting coils. Absorbers are positioned in the middle and at each end of the channel; the absorbers are 350 mm long liquid-hydrogen vessels. Two linacs, each consisting of four RF cavities and a superconducting 'coupling' coil sit between the absorber vessels. The MICE experiment will be built up in stages, enabling the detectors to be calibrated and each stage to be studied in turn. The final stage of MICE is visualised in Figure 1. MICE is described in detail in the MICE Technical Reference Document [2].



Figure 1: G4MICE visualisation of the central MICE detectors and the cooling apparatus. Time of flight counters are shown in yellow, tracking detector components in dark red, magnet bodies in blue, absorber windows in turquoise and the electron muon calorimeter in green. The distance between the time of flight counters is 13.2 metres.

## Solenoid Model

Solenoids are modelled using a number of current carrying concentric cylindrical sheets. A field map is generated on a rectangular grid and subsequently an interpolation is made between points on the grid to generate the field at a particular point. Alternately a field map can be read directly from a file. The analytic solution for the field from a single current sheet is given by [3]. By default, G4MICE uses 10 sheets for each solenoid, which gives errors in the field of less than 0.1 % relative to a field generated with a very large number of sheets [4]. The dominant source of error is the interpolation algorithm.

G4MICE performs a linear interpolation in r and a cubic interpolation in z to generate the field at an arbritrary space point. The difference between the field evaluated using this algorithm and the analytic calculation is shown for the two components of the field in Figure 2. The largest difference in  $B_r$  is 1 % and in  $B_z$  is 0.4 %. It should be noted that this peak error occurs well away from the beam centre; and a smaller grid spacing can be used if a higher precision is required.



Figure 2: The percentage errors on the field components due to interpolation as functions of position in a typical coil for radial and axial components, relative to the coil centre.

#### RF Model

The MICE RF cavities operate in the TM010 mode with a frequency of 201.25 MHz and an energy gain of 11 MeV in each string of four cavities. This is sufficient to restore the mean energy lost due to the liquid-hydrogen absorbers and absorber-vessel windows. The RF will be run either on-crest or at a phase of 40° from zero crossing, depending on the amount of RF power available. In G4MICE, the cavities are modelled by Bessel functions [5].

Phasing is achieved using an approximation to the ref-

erence trajectory generated by a reference particle in electrostatic fields. The field is approximated using a constant field in the z-direction given by the energy gain in the cavities. The peak field can then be calculated from the transit time factor and RF phase. It is found that this provides an accurate phasing of the RF cavities. It is possible to set the phases and peak voltage to arbritrary values if required.

### Absorber Models

The MICE liquid Hydrogen absorbers have a central width of 350 mm contained by aluminium windows. In order to minimise the window thickness and beam heating, a concave window is used. G4MICE offers hemispherical, torispherical or flat absorber windows as approximations to the window shape that will be used in MICE.

In ionisation cooling, the transverse emittance reduction through a material is given by [6]

$$\frac{d\epsilon_n}{dz} \approx -\frac{\epsilon_n}{\beta_{rel}^2 E} \frac{dE}{dz} + \frac{1}{2m} \frac{13.6^2}{L_R} \frac{\beta_\perp}{\beta_{rel}^3 E}.$$
 (1)

Here dE/dz is the energy lost per unit length due to ionisation,  $L_R$  is the material radiation length,  $\beta_{\perp}$  is the transverse beta-function and  $c\beta_{rel}$  is the particle speed. The first term describes cooling due to ionisation energy loss, while the second describes heating due to multiple scattering.

In Figure 3 the emittance reduction for a typical Neutrino Factory beam with transverse emittance of 6  $\pi$  mm rad and  $\beta_{\perp}$  of 333 mm in the absorber centre is shown and compared with a similar absorber simulated in the ICOOL particle tracking code [7].



Figure 3: The simulated emittance reduction due to a single liquid Hydrogen block for an absorber with torispherical ends (green), flat ends (blue) and flat ends simulated in ICOOL (red).

#### SIMULATION OF MICE

The cooling performance of MICE as simulated in G4MICE is demonstrated for the baseline case, a beam with initial transverse RMS emittance of 6  $\pi$  mm rad, RF operating at 90° and fields chosen such that  $\beta_{\perp}$  is 420 mm in the absorbers.

# Fields

The fields are controlled in MICE using 18 coils. In Figure 4 the on-axis magnetic field is shown for the baseline configuration, as calculated using the routine described above. Constant field regions are provided by three coils in each spectrometer, which contain the beam as it passes through scintillating-fibre spectrometers; matching coils match the beta function between the spectrometer and the cooling apparatus; focus coil pairs sit on either side of the absorber providing focussing on the absorbers; and coupling coils contain the beam as it passes through the RF cavities. In the baseline case, the field is antisymmetric about the centre of each absorber in order to conserve canonical angular momentum.



Figure 4: The on axis B-field in the z direction for the MICE baseline case.

## Beta function

The transverse beta function  $\beta_{\perp}$  is shown as calculated in G4MICE's linear beam optics package and particle tracking in Figure 5. In this configuration the beta function is focussed to be 420 mm in the absorbers. Effects due to the transverse and longitudinal emittance of the beam and the energy loss in liquid hydrogen and consequent gain in the RF cavities introduce a slight mismatch. The MICE collaboration plans to operate with several different settings to explore the cooling power, with  $\beta_{\perp}$  ranging from 70 mm to 420 mm in the absorbers.

#### Transverse Emittance

In Figure 6 the cooling power of MICE evaluated using G4MICE is shown for the configuration outlined above. A significant emittance reduction is observed in each of the absorbers. There is significant emittance growth due to non-linear effects due to the relatively high beam emittance.

# CONCLUSIONS

The simulation of different elements of the MICE beamline in G4MICE has been reviewed. The simulation of



Figure 5: The beta function through the MICE cooling channel from G4MICE's linear optics package (red) and from particle tracking (black).



Figure 6: The simulated emittance reduction through the full cooling channel.

coils, RF cavities and absorbers has been detailed. The full simulation of the MICE cooling channel has been described and shown to provide of order 10% cooling in the baseline case.

#### REFERENCES

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