DYNAMIC APERTURE STUDIES FOR PETRA III

Yongjun Li*, Klaus Balewski, Winfried Decking
Deutsches Elektronen Synchrotron (DESY), Notkestrasse 85, 22607 - Hamburg, Germany

Abstract
PETRA III is a low-emittance storage ring dedicated to synchrotron radiation. For efficient injection in the top-up mode, the dynamic aperture has to be larger than 30 mm-mrad in the horizontal plane. This paper presents the choice of tunes and the optimization of the sextupole configuration. Tracking simulations have been performed, including the non-linear effects of 20 four-meters-long damping wigglers and a representative set of undulators. Misalignment and multipole errors are considered as well, leading to specifications for the magnet design and alignment procedure.

INTRODUCTION
The 2.3 km long PETRA storage ring at DESY will be converted into a dedicated light source (PETRA III) with 1 mm-rad beam emittance [1]. One octant will be reconstructed into a DBA structure to accommodate 13-14 insertion devices. The main parameters of PETRA III are listed in Table 1.

For efficient injection in the top-up mode, the required dynamic aperture has to be larger than 30 nm-mrad in the horizontal plane to accommodate the injected beam with 350 nm-rad horizontal emittance. The vertical aperture is limited by the small physical gap (7mm) of the insertion device vacuum chambers inside the DBA octant.

Table 1: Main Parameters of PETRA III.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy/GeV</td>
<td>6</td>
</tr>
<tr>
<td>Circumference/m</td>
<td>2304</td>
</tr>
<tr>
<td>Current/mA</td>
<td>100</td>
</tr>
<tr>
<td>Emittance (H./V.)/nm-rad</td>
<td>1.0/0.01</td>
</tr>
<tr>
<td>Number of insertion devices</td>
<td>14</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>960</td>
</tr>
</tbody>
</table>

LATTICE DESCRIPTION
The whole ring is composed of one DBA octant and seven FODO achromat octants. The emittance of the bare machine (without damping wigglers) is 4.5 nm-rad, which can be further reduced down to 1 nm-rad with the help of totally 80 meters of damping wigglers, which are located inside two long straight sections. 14 insertion devices, including one 20 meters long undulator, will be installed in the DBA octant. The chromaticity correction is implemented with sextupoles which are located only inside the seven FODO octants because of the small dispersion values inside the DBA octant.

The linear and nonlinear effects of the damping wigglers have been studied carefully [2]. For the linear lattice design damping wigglers and undulators are modelled as linear transfer matrices.

OPTIMIZATION OF SEXTUPOLE CONFIGURATION
Simulation studies show the dynamic aperture to be mainly limited by chromaticity sextupoles located inside the FODO octants. Thus an optimization of the sextupole configuration based on normal form technique [3] has been performed. The basic steps are:
1. Constructing nonlinear one-turn-map in Lie algebra language;
2. Concatenating one-turn-map using BCH theorem and similarity transformation;
3. Using normal form technique to obtain nonlinear coefficients to desired order;
4. Constructing a merit function of sextupoles strength with different weight coefficients to be optimized;
5. Minimizing the merit function of sextupoles strength with nonlinear least squares.

The merit function to be optimized is defined as

\[
F(\lambda) = w_x (K_x(\lambda) - \xi_x)^2 + w_y (K_y(\lambda) - \xi_y)^2 + \sum_n w_n (K_n(\lambda) - \xi_n)^2 + w_{xx} (Q_{xx}(\lambda))^2 + w_{xy} (Q_{xy}(\lambda))^2 + w_{yy} (Q_{yy}(\lambda))^2.
\]

Here \(K_x, K_y\) are the chromaticities created by sextupoles, \(\xi_x, \xi_y\) are the natural chromaticities, \(K_n\) are the nonlinear coefficients, \(Q_{xx}, Q_{xy}, Q_{yy}\) are the coefficients of tune dependence on amplitude, which are second order in sextupoles strength, and \(w_\cdot\)s are the weight factors.

The sequence of sextupoles inside each FODO octant is \(5 (S1 - S2 - S3 - S4)\).

The optimization result is listed in Table 2. The strength of S1 and S3 are nearly equal, and so are S2 and S4, due to the particular 72 deg phase advance of the FODO cell.

Table 2: Sextupole Strength of Optimization.

<table>
<thead>
<tr>
<th>Sextupole</th>
<th>K2L (m⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-0.61940484</td>
</tr>
<tr>
<td>S2</td>
<td>0.75713146</td>
</tr>
<tr>
<td>S3</td>
<td>-0.65309156</td>
</tr>
<tr>
<td>S4</td>
<td>0.78349853</td>
</tr>
</tbody>
</table>

Principally this optimization procedure can be used in an iterative way to re-optimize beta-functions at the sextupoles’ location, phase advance between them and machine tunes [4]. In the case of the PETRA III design, the phase advance is chosen as 72 deg per FODO cell, so
the first order coefficients are cancelled very well. Tune
dependence on amplitudes is a second order effect, which
usually can be compensated by so-called harmonic
sextupoles located at dispersion-free sections. Because
each FODO octant in PETRA III contains 20 chromaticity
sextupoles, it is very difficult to compensate their
nonlinear geometrical effects by few sextupoles located in
dispersion-free sections. The attempt of using harmonic
sextupoles in PETRA III fails to give a good solution.
Table 3 gives the optimization result of using three
families of harmonic sextupoles S5, S6 and S7 in the
sequence:

\[
S5 - S6 - 5 \times (S1 - S2 - S3 - S4) - S7.
\]

The optimized values for harmonic sextupoles are close
to zero, which means they are not helpful to enlarge the
dynamic aperture. Further investigations show that
increasing the number of harmonic sextupoles does not
yield better results.

Table 3: Harmonic Sextupole Optimization.

<table>
<thead>
<tr>
<th>Sextupole</th>
<th>K2L (m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-0.62774759</td>
</tr>
<tr>
<td>S2</td>
<td>0.75538557</td>
</tr>
<tr>
<td>S3</td>
<td>-0.64480679</td>
</tr>
<tr>
<td>S4</td>
<td>0.78533184</td>
</tr>
<tr>
<td>S5</td>
<td>-0.01954510</td>
</tr>
<tr>
<td>S6</td>
<td>0.01404320</td>
</tr>
<tr>
<td>S7</td>
<td>0.01168470</td>
</tr>
</tbody>
</table>

**SIMULATION RESULTS**

The simulation studies were performed with the 6-D
tracking code SIXTRACK [5]. Several additional
subroutines were inserted into the original code to deal
with the tracking of wigglers and insertion devices [6].
The field map of damping wiggler is described by
Halbach formulæ including several longitudinal
harmonic modes. The nonlinear map of damping wiggler
is described as a forth order numerical generating
function (GF) [7]. The undulators have short period
length, so an analytical generating function only including
the fundamental mode is accurate enough for the
simulation study. It is worthwhile to notice that the tune
dependence on amplitude caused by undulators is [8]

\[
Q_y = \frac{dQ_y}{dJ_y} = \frac{\pi \beta_y^2 L}{4 \lambda^2 \rho^2},
\]

which requires small vertical beta-function in undulators,
specially for very long undulator. Here \( L \) is the undulator
length, \( \rho \) the bending radius, and \( \lambda \) the period length. A
low vertical beta-function section was designed to accommodate the 20 meters long undulator in the short
straight section North-East [2].

Multipole errors and misalignments of magnets will
cause dramatic reduction of dynamic aperture. The
influence of those errors on dynamic aperture gives
requirements for magnet field quality for the magnet
design, manufacture and installation. The existing
magnets of PETRA II will be re-used in the seven FODO
octants, their multipole errors have been analysed and measured [9]; while for the magnets of the new DBA
octant, the field imperfection is obtained by field
calculations and the measurement results of the prototypes. The field errors of dipoles and gradient errors
of quadrupoles are specified to create \( \sim 0.5 \) mm static orbit
distortion and \( \sim 10\% \) beta-beat at both planes. The
multipole coefficients are summarized in Table 4.

Table 4: Relative multipole coefficients of PETRA III
magnets at \( r = 50\)mm.

<table>
<thead>
<tr>
<th>Order</th>
<th>Dipole</th>
<th>Old Quadrupole</th>
<th>New Quadrupole</th>
<th>Sextupole</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1×10^-4</td>
<td>1×10^-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.947</td>
<td>7.45</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>1.379</td>
<td>1.75</td>
<td>1.0</td>
<td>1.03</td>
</tr>
<tr>
<td>5</td>
<td>3.663</td>
<td>1.60</td>
<td>0.1</td>
<td>2.51</td>
</tr>
<tr>
<td>6</td>
<td>2.386</td>
<td>1.51</td>
<td>1.5</td>
<td>0.49</td>
</tr>
<tr>
<td>7</td>
<td>1.445</td>
<td>0.43</td>
<td>0.1</td>
<td>0.34</td>
</tr>
<tr>
<td>8</td>
<td>6.834</td>
<td>0.34</td>
<td>0.3</td>
<td>0.27</td>
</tr>
<tr>
<td>9</td>
<td>3.151</td>
<td>0.43</td>
<td>0.0</td>
<td>3.07</td>
</tr>
<tr>
<td>10</td>
<td>3.000</td>
<td>3.23</td>
<td>0.5</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Another reason for dynamic aperture reduction comes
from the small physical aperture of undulators. Particles
with large horizontal amplitude are scraped by the small
vertical physical apertures due to linear and nonlinear
coupling effects. Tracking results show most of lost
particles are scraped at the location of high-beta
undulators, thus additional collimators are needed to
protect undulator poles.

Considering all the limitations mentioned above, tune
scanning was performed to choose suitable tunes in order
to avoid dangerous resonance lines, and at the same time,
to obtain large enough energy acceptance (or off-
momentum dynamic aperture) to maintain long beam life-
time. At present, the tune is chosen as 36.095/31.345. The
frequency map including all kinds of errors for one
random seed is given in Figure 1. The momentum
acceptance is larger than 1.5% which accounts for a
Touschek lifetime of 2 hrs at a single bunch intensity of
2.5 mA. Figure 2 shows the dynamic aperture for the
ideal machine, machine with errors and the required
aperture for high efficient injection. The ideal machine
(without multipole errors) has 50 mm-mrad dynamic
aperture in the horizontal plane. The reduction of aperture
caused by multipole errors, orbit distortion and beta-beat
is approximately 30%. Figure 3 displays another
frequency map with tune 36.145/31.345, in which case
some particles with large amplitude are located on the
resonance line \( Q_y = 2Q_y = n \).
Figure 1: Frequency map with tune (36.095/31.345) including magnet errors (one random seed).

Figure 2: Dynamic aperture of storage ring with and without errors.

Figure 3: Frequency map with tune (36.145/31.345) including magnet errors (one random seed). Some particles are located on the resonance line $Q_x - 2Q_y = n$.

**SUMMARY**

The dynamic aperture of PETRA III has been calculated and optimized. Tune scanning has been performed to choose suitable tunes to maintain large enough dynamic aperture and energy acceptance. Simulation results show that the lattice design, magnet misalignments and error specifications can satisfy top-up injection requirements.

**REFERENCES**


