NON-LINEAR MODELING OF THE RHIC INTERACTION REGIONS*

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Abstract

For RHIC's collision lattices the dominant sources of transverse non-linearities are located in the interaction regions. The field quality is available for most of the magnets in the interaction regions from the magnetic measurements, or from extrapolations of these measurements. We discuss the implementation of these measurements in the MADX models of the Blue and the Yellow rings and their impact on beam stability.

IMPLEMENTATION OF MAGNET MEASUREMENTS IN THE BLUE AND YELLOW LATTICES

The Relativistic Heavy Ion Collider (RHIC) consists of two rings, Blue and Yellow. A positively charged particle travels clockwise in the Blue ring and counterclockwise in the Yellow ring. The particles from both rings perform head-on collisions at up to four interaction points. The magnet imperfections in the interaction regions represent the most relevant contribution to the deterioration of the single particle stability at top energy. These magnet imperfections were measured prior to the installation of the magnets in the RHIC tunnel at warm and cold temperatures [1]. The coefficients of the multipolar expansion of the magnetic fields, b_n and a_n, are available in RHIC's databases in units of 10^{-4} [2]. In this paper the convention is followed that b_0 and a_0 correspond to regular and skew dipole fields respectively. Utmost care has to be taken when implementing these measurements in a tracking model. The orientation of the magnet in the ring with respect to its orientation during the measurement determines sign changes for particular multipoles. Before giving detailed expressions on these sign changes it is worth understanding the effect of particular coordinate and polarity transformations:

- Horizontal reflection: $x \rightarrow -x$. A detailed derivation of the effect of this transformation on the multipolar coefficients is given in [2]. Being a'_n and b'_n the multipolar components in the transformed frame they are related to the original components by the following expressions,

$$b'_n = (-1)^n b_n \tag{1}$$
$$a'_n = (-1)^{n+1} a_n \tag{2}$$

- Vertical reflection: $y \rightarrow -y$. Following a similar derivation to that of [2] for the previous case the following relations can be established for this transformation,

$$b'_n = -b_n \tag{3}$$
$$a'_n = a_n \tag{4}$$

- Polarity change: Under a change of the polarity of the current fed to the magnet all the multipole components change sign,

$$b'_n = -b_n \tag{5}$$
$$a'_n = -a_n \tag{6}$$

The orientation of the magnets in the tunnel is available in RHIC's databases and has two possible values: 1 and -1. If the orientation is 1 the accelerator coordinate system of a particle traveling clockwise is the same one as that used in the magnet measurements. If the orientation is -1 the horizontal axis of the accelerator coordinate system of a particle traveling clockwise is opposite to that used in the magnet measurements while the vertical axis remains unchanged.

The information concerning the polarity of the magnet is extracted from the tracking model. During the measurements dipoles are powered for a positive charged particle to bend to the positive side of the horizontal axis of the coordinate system of the measurement. In the same way quadrupoles are powered to focus a positive particle. The orientation and the polarity information are combined to get the transformations that connect the coordinate system of the measurement to the coordinate system of the accelerator. Let b_n and a_n be the multipole components from the measurements and b'_n and a'_n the multipole components in the accelerator coordinate system of a clockwise particle. Then, for a dipole magnet,

$$b'_n = (ori)^n \ (pol) \ b_n \ ,$$
$$a'_n = (ori)^{n+1} \ (pol) \ a_n \ , \tag{7}$$

where ori is the orientation variable from the database and pol is 1 if the magnet polarity is positive and -1 if the polarity is negative. Similarly for a quadrupole,

$$b'_n = (ori)^{n+1} \ (pol) \ b_n \ ,$$
$$a'_n = (ori)^n \ (pol) \ a_n \ . \tag{8}$$

The former relations hold for particles traveling clockwise, which is the case of the Blue ring. A new set of expressions have to be derived for the Yellow ring since it

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contains particles traveling counterclockwise. The accelerator coordinate system for a counterclockwise ring has the same horizontal axis as in the clockwise case (it points outwards) but opposite vertical and longitudinal axes. Going from the clockwise system to the counterclockwise system implies a change of sign in the orientation and the polarity variables. The transformation due to the opposite vertical axes has also to be taken into account as described above. Therefore, the expressions relating multipole components for the Yellow ring are as follows,

$$\begin{aligned} b'_n &= (-ori)^n\ (pol)\ b_n \\ a'_n &= -(-ori)^{n+1}\ (pol)\ a_n \end{aligned} \tag{9}$$

for dipoles and

$$\begin{aligned} b'_n &= (-ori)^{n+1}\ (pol)\ b_n \\ a'_n &= -(-ori)^n\ (pol)\ a_n \end{aligned} \tag{10}$$

for quadrupoles. One can summarize all the previous expressions by introducing the variable $ring$ that takes the value of 1 for the Blue ring and -1 for the Yellow,

$$\text{for dipoles} \quad \begin{cases} b'_n = (ring)^n (ori)^n (pol)\ b_n \\ a'_n = (ring)^n (ori)^{n+1} (pol)\ a_n \end{cases}$$

$$\text{for quadrupoles} \quad \begin{cases} b'_n = (ring)^{n+1} (ori)^{n+1} (pol)\ b_n \\ a'_n = (ring)^{n+1} (ori)^n (pol)\ a_n \end{cases} \tag{11}$$

Tracking codes, like Teapot [3] or SixTrack [4], generally treat non-linear multipoles as thin lenses to ensure simplecticity. Due to the large variations of the beta functions within the interaction regions the magnets have been cut into a number of slices and the multipoles have been placed in between these slices. Two multipoles are placed at the ends of each magnet to account for the end fields. The number of slices has been left as a variable to be able to study the effect of slicing. The end fields were only measured for a few magnets at high intensity. End fields can be regarded as mainly systematic and for this reason generic end fields were used for all the quadrupoles. Almost all quadrupoles and dipoles had the integrated components measured and therefore the strength of the multipoles within the body were chosen to reproduce the integrated strength after having introduced the end components. For those magnets that did not have their multipole components measured at high intensity an extrapolation from the measurements at low intensity (warm measurements) to the top intensity was performed [5]. Seven out of twelve DX magnets, the nearest dipoles to the interaction points, still miss estimates of their multipole components.

IMPACT ON BEAM STABILITY

In this article the tune footprint and the DA (dynamic aperture) are used to probe the non-linear content of the lattices. All the studies were done assuming a normalized 95% emittance of 20 μm and a relative momentum deviation of 0.00027. The tune footprint shows how the tunes change with the transverse oscillations amplitudes. The tune footprints are plotted in Fig. 1 for the two lattices and two different slicing patterns, eight and two slices. If the magnet is split into two slices there are three multipoles introduced, two at the ends and one in the center. As seen on the plots, there is little difference between the two and the eight slices cases.

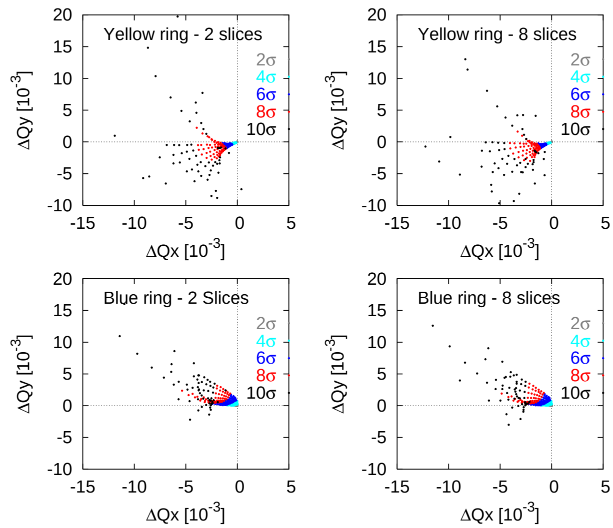

Figure 1: Tune footprints up to 10σ for the Yellow and Blue lattices for 2 different slice patterns.

The minimum DA is defined here as the lowest transverse amplitude that originates unstable motion for five transverse phase-space angles within a certain number of turns. In this paper we will choose either 10^5 or 10^6 turns to compute the DA. The DA for a set of working points around RHIC's past operation working point is shown in Fig. 2. These simulations contain synchrotron motion and have been performed for the eight slices lattice with the tracking code SixTrack. The maxima of the dynamic aperture curves observed around Qx=0.23 has been indeed the RHIC operation working point. The dominant resonances can be inferred from this plot as the fourth and the fifth order resonances at the edges of the plot and the ninth order resonance at about Qx=0.22.

In order to study the importance of a particular multipole the DA is computed for the lattice without that particular multipole. This has been done for all multipoles above the quadrupole at the same working point (0.22, 0.23) and results are shown in Fig. 3. The main conclusion after looking at this graph is that there is no single multipole that dominates the beam stability. This implies that to significantly increase the DA, and therefore the machine performance, various multipole corrector circuits have to be optimized. Identifying a minimum set of multipoles whose removal from the model significantly enlarges the DA has become a highly time consuming task. The following sets were already removed without observing a significant im-

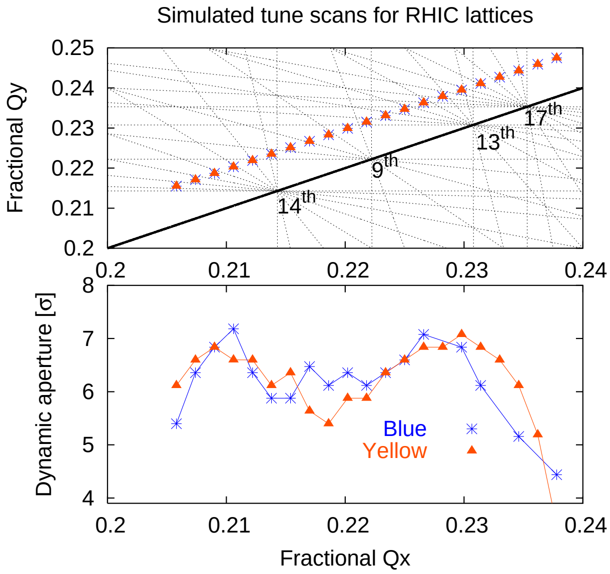

Figure 2: 10^6 Turn Dynamic aperture of the RHIC lattices for a set of working points. The top plot shows the location of the different working points on the tune diagram with the relevant resonance lines. The bottom plot shows the dynamic apertures for each of the working points.

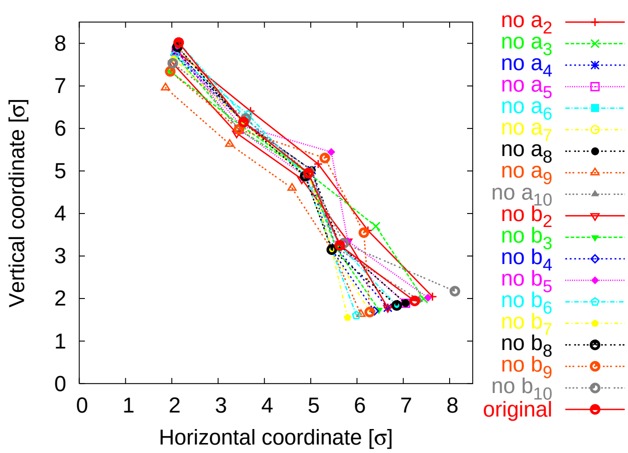

Figure 3: 10^5 Turn Dynamic aperture of the RHIC blue lattice after completely removing one multipole for each of the cases shown.

provement: $b_5 + a_2$, $b_5 + b_3 + a_2$, $b_5 + a_2 + b_2$, $b_5 + b_6 + b_7$, $b_8 + b_9 + b_{10}$ and $a_8 + a_9 + a_{10}$. Nevertheless the DA is above 10σ if all the multipole errors of order larger than 4 are removed. This could imply that all high orders are relevant and fruitful corrections might be unachievable due to the lack of the needed corrector circuits.

Another important question is the effect of the uncertainty of the magnetic field measurements and the possible change of the multipolar errors after quenches and thermal cycles [6]. The impossibility to strictly assess this issue led to assume reasonable random errors for all the multipoles. Multipoles with order below 5 have been assigned an uncertainty of 0.2 units and multipoles of order above

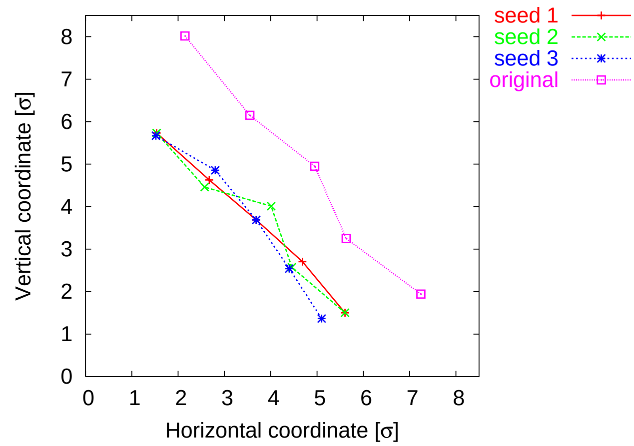

Figure 4: 10^5 Turn Dynamic aperture of the RHIC blue lattice after introducing random errors in all multipoles for three different seeds of the random distribution.

4 have been assigned an uncertainty of 0.1. Three lattices were randomly generated with three different seeds using the former rms values for the Gaussian distribution. The results from this study are shown in Fig. 4. All three seeds have a consistently lower DA than the original lattice's DA at about 6σ. This shows that the uncertainty of the multipole errors has a relevant impact in this kind of study and should be carefully studied.

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