MEASUREMENT OF MULTIPOLE STRENGTHS FROM RHIC BPM DATA*

R. Tomás, M. Bai, W. Fischer, *Brookhaven National Laboratory, Upton, NY 11973, USA*
A. Franchi, G. Rumolo, *GSI, Darmstadt, Germany*

Abstract

Recently resonance driving terms were successfully measured in the CERN SPS and the BNL RHIC from the Fourier spectrum of BPM data. Based on these measurements a new analysis has been derived to extract multipole strengths. In this paper we present experimental measurements of sextupolar and skew quadrupolar strengths carried out at RHIC. A non-destructive measurement using an AC dipole is also presented.

INTRODUCTION & THEORY

In [1] Normal Form and Lie algebra techniques were used to describe the motion of a particle confined in an accelerator in presence of non-linearities. The particle position x_1 as function of the turn number N at a certain location (indexed by 1) was given the following form,

$$\begin{aligned} x_1(N) &= \sqrt{\beta_{x1}}\ \Re\Big\{\sqrt{2I_x}e^{i(2\pi\nu_x N+\psi_{x_1})} - \\ & \quad 2i\sum_{jklm} jf^{(1)}_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}\times \\ & \quad e^{i[(1-j+k)(2\pi\nu_x N+\psi_{x_1})+(m-l)(2\pi\nu_y N+\psi_{y_1})]}\Big\} \end{aligned} \quad (1)$$

where $I_{x,y}$ are the horizontal and vertical actions, $\nu_{x,y}$ are the tunes, ψ_{x_1,y_1} are the initial phases and $f^{(1)}_{jklm}$ are the generating function terms. The generating function terms are directly related to the Hamiltonian terms $h^{(1)}_{jklm}$ as follows,

$$f^{(1)}_{jklm} = \frac{h^{(1)}_{jklm}}{1-e^{-i2\pi[(j-k)Q_x+(l-m)Q_y]}}\ . \quad (2)$$

In [2] it was found that these terms experience a characteristic variation around the accelerator lattice: their amplitude remains constant in sections free of multipoles and shows abrupt jumps at the locations of these sources. The analytical expression describing these abrupt changes is given by

$$\begin{aligned} f^{(2)}_{jklm} &= e^{-i[(k-j)\Delta\phi_x+(m-l)\Delta\phi_y]}\Big[f^{(1)}_{jklm} \\ & \quad -\sum_{q=1}^{n} e^{i(k-j)\phi_{xq}+i(m-l)\phi_{yq}}h_{q_{jklm}}\Big]\ , \end{aligned} \quad (3)$$

where $f^{(2)}_{jklm}$ is the generating term at a second location, $\Delta\phi_{x,y}$ are the horizontal and vertical phase advances between the two locations, the summation extends only over the multipoles placed between the two locations, $\phi_{xq,yq}$ are the phase advances between the q^{th} multipole and the first location and $h_{q_{jklm}}$ are real quantities proportional to the strength of the q^{th} multipole and to the product $\beta_{xq}^{\frac{j+k}{2}}\beta_{yq}^{\frac{l+m}{2}}$, see [3] for a more detailed expression.

In [2] the measurement of amplitudes and phases of generating function terms was successfully achieved at two accelerators: the CERN SPS and the BNL RHIC. This measurement together with eq. (3) opens the possibility of measuring magnet strengths. Indeed, if there is only one multipole between the two locations 1 and 2, its strength can be directly inferred knowing the betatron functions. Nevertheless there are two limitations to this approach:

1. The existence of several multipoles between the two locations avoids the measurement of particular strengths. When this is the case an integrated strength is obtained, namely the summation in eq. (3).

2. The measurement of f_{jklm} at one location needs of two BPMs separated by about 90° for the momentum reconstruction. If non-linearities exist between these two BPMs, f_{jklm} can only be measured up to an error in the order of the strength of the non-linearities. This error is usually smaller than f_{jklm} but would not be negligible when measuring magnet strengths.

The first limitation is unavoidable given the BPM configuration. The second one is overcome by adopting another approach using three BPMs. This new method follows.

The three BPM method

Assume that Fig. 1 represents the BPM and sextupole configuration of certain segment of an accelerator. In the

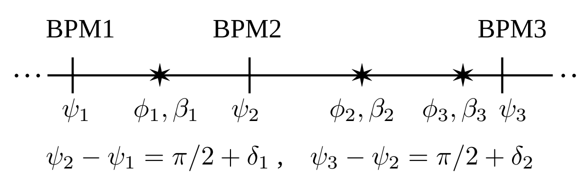

Figure 1: Segment of an accelerator lattice. BPMs and sextupoles are shown with their relevant twiss parameters.

figure δ_1 and δ_2 have been introduced for later convenience. A local observable is constructed from the BPMs turn-by-turn signals as follows,

$$\chi(N) = \frac{\hat{x}_1(N)}{\cos\delta_1}+\hat{x}_2(N)\big(\tan\delta_1+\tan\delta_2\big)+\frac{\hat{x}_3(N)}{\cos\delta_2}\ , \quad (4)$$

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where the hat means that the signal has been normalized to the amplitude of the fundamental betatron oscillation. These amplitudes and the phase advances δ_1 and δ_2 are obtained from the Fourier transform of the signals. For an ideal uncoupled linear machine $\chi(N) = 0$ for any N and for any set of three BPMs. Furthermore in presence of multipoles distributed around the ring, $\chi(N)$ only depends on those non-linearities placed between the three BPMs. The analytical proof of these statements and the following expressions is out of the scope of this paper. The equation that relates $\chi(N)$ and the local sources is given by,

$$\chi(N) = 2\sum_{j>k,l>m} j|\chi_{jklm}|(2I_x)^{\frac{j+k-2}{2}}(2I_y)^{\frac{l+m}{2}} \times \quad (5)$$
$$\cos\Big(((1-j+k)\nu_x + (m-l)\nu_y)2\pi N + \psi_{jklm}\Big) ,$$

where the local terms χ_{jklm} and the phases ψ_{jklm} are defined as

$$\chi_{jklm} = \sum_{q=1}^{n} e^{i[(1-j+k)\phi_{xq}+(m-l)\phi_{yq}]} SEN(\phi_{xq}) h_{q_{jklm}}$$
$$\psi_{jklm} = (1-j+k)\psi_{x1} + (m-l)\psi_{y1} + \arg(\chi_{jklm}) , \quad (6)$$

where the summation extends over the multipoles in between the 3 BPMs, $h_{q_{jklm}}$ are the quantities proportional to the strengths already introduced in eq. (3), and the function $SEN(\phi_{xq})$ is defined as

$$\begin{cases} \sin\phi_{xq}\sqrt{1+\tan^2\delta_1} & \text{if } \phi_{xq} < \psi_2-\psi_1 \\ \sin(\phi_{xq}-\delta_1-\delta_2)\sqrt{1+\tan^2\delta_2} & \text{if } \phi_{xq} > \psi_2-\psi_1 \end{cases} \quad (7)$$

Note that the above expressions largely simplify when $\delta_1 = \delta_2 = 0$, giving $\chi(N) = \hat{x}_1(N)+\hat{x}_3(N)$ and $SEN(\phi_{xq}) = \sin\phi_{xq}$.

We have constructed a local observable $\chi(N)$ that depends both on local magnet strengths and the distribution of the three BPMs. The Fourier coefficients of this observable provide the local terms χ_{jklm} which are similar to the Hamiltonian terms but strictly local. Therefore the measurement of these terms represents a means of finding lattice imperfections or unexpected multipoles in an accelerator.

RHIC MODEL

In order to compare results from the measurements of sextupolar components to predictions a MADX model of the RHIC yellow injection lattice has been constructed. The interaction regions (IRs) have been modeled as described in [6] using the corresponding magnet measurements. Some dipoles in the IRs do not have magnetic measurements. No sextupolar components have been assumed for them. The arcs contain the chromaticity sextupoles and the sextupolar components of the superconducting dipoles. The arc dipoles have been sliced into 8 slices and the corresponding sextupolar multipoles have been placed in between.

RHIC EXPERIMENTS

During 2004 RHIC gold operation experiments to measure magnet strengths from BPM data were carried out in a similar way as in [4]. Transverse betatron oscillations were excited either by injecting off orbit or by driving forced oscillations with the aid of an AC dipole. 1024 turn-by-turn BPM data were recorded after every transverse excitation. All experiments were performed at injection energy. The tunes were moved closer to the third order resonance to enhance the sextupolar resonances, Qx=0.31 and Qy=0.22. Chromaticities were $Q'_{x,y} \approx -2$ units, where the prime denotes the derivative with respect to the relative momentum deviation. For the presented measurements no IR correction circuits were used.

Prior to the data analysis the malfunctioning BPMs were removed as reported in [5]. Yet a new failure mode of the BPM system had to be pursued: few BPMs report on a different turn number than the rest. To find these faulty BPMs the phase advance between consecutive BPMs as measured from the Fourier transform is compared to that predicted by the model. Those few BPMs having a larger deviation from the model than the rest were rejected.

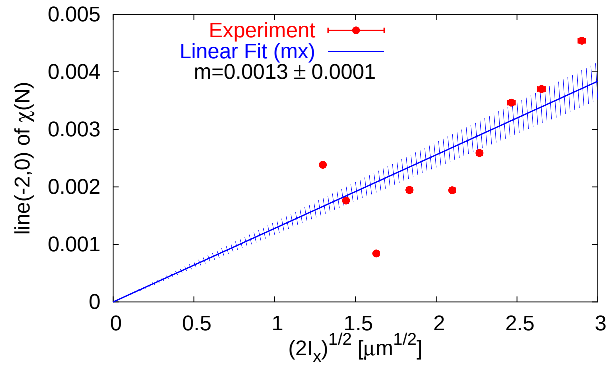

Figure 2: Amplitude of the spectral line with frequency $-2\nu_x$ (line (-2,0)) from the Fourier spectrum of $\chi(N)$ versus $\sqrt{2I_x}$.

Measurement of χ_{3000} from kick data

The measurement of χ_{3000} is performed in a similar way to that of f_{3000} as described in [2]. A line constrained to go trough the origin is fitted to the amplitude of the spectral line with frequency $-2\nu_x$ (line (-2,0)) from the Fourier spectrum of $\chi(N)$ versus $\sqrt{2I_x}$. An example of this fit is shown in Fig. 2 for a particular set of three BPMS. The effect of beam decoherence has to be taken into account as described in [2]. If the centroid oscillations are damped due to amplitude detuning the line(n,0) is reduced by a factor of |n|. Therefore $|\chi_{3000}|$ is given by one sixth (from eq. (5)) of the slope of the previous fit times two (only in presence of decoherence). The measurement of $|\chi_{3000}|$ around RHIC yellow ring is shown in Fig. 3 with a comparison to the model. The horizontal error bars of the plot are used to delimit the segment of the lattice occupied by the 3 BPMs (vertical lines are also used at the edges of the segment). The center dot corresponds to the location of the middle BPM. Model and experiment show good agreement in the arcs. Discrepancies arise in the IRs partly due to the fact that the model is not complete in these regions.

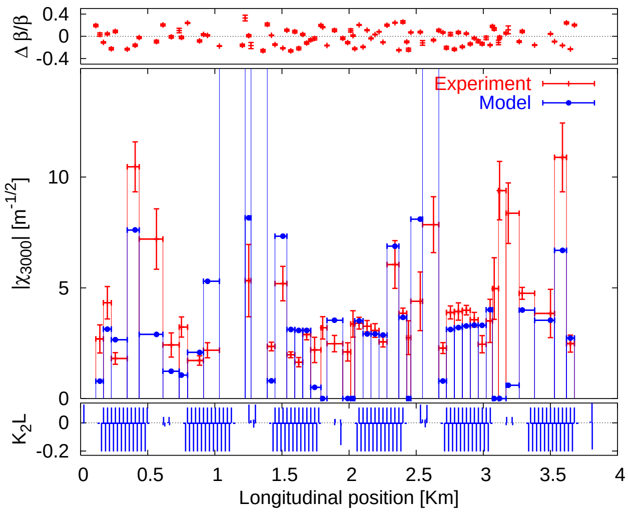

Figure 3: Measurement of $|\chi_{3000}|$ from kick data. The top plot shows the beta beating. The middle plot shows $|\chi_{3000}|$ around the ring with a comparison to the model. The bottom plot shows the sextupolar components of the ring.

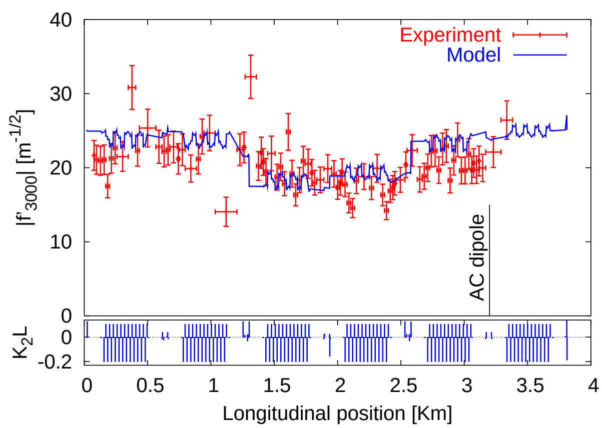

Figure 4: Measurement of $|f'_{3000}|$ with an AC dipole. The bottom plot shows the sextupolar components of the ring.

Measurements using an AC dipole

An AC dipole drives transverse beam oscillations at a frequency close to the betatron tune. The non-destructive measurement of resonance driving terms using an AC dipole was proposed in [7]. The main finding of this paper was that the resonance driving terms in presence of an AC dipole, f'_{jklm}, differ from the natural resonance driving terms f_{jklm} in a quantity that increases with the separation of the driving and the betatron tunes. We have measured f'_{jklm} for the first time around the RHIC yellow lattice. The measurement is shown in Fig. 4 together with a prediction from the model. The horizontal error bars delimit the locations of the two BPMs. The few points that show a discrepancy have a large horizontal error bar. This large separation of the BPMs compromises the reconstruction of the momentum, introducing an error of the order of the non-linearities within this region.

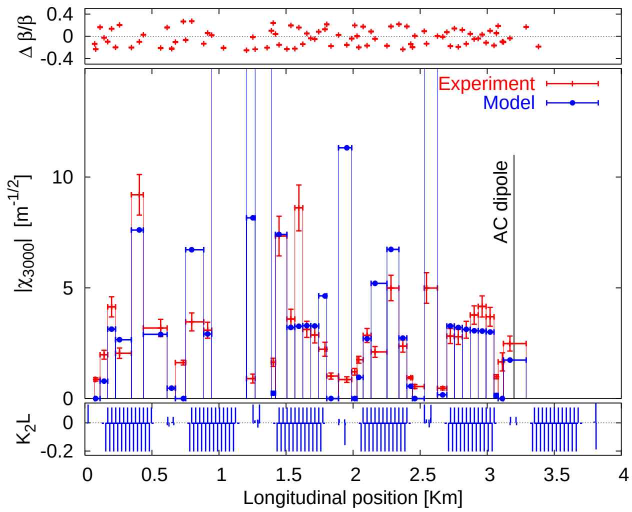

Figure 5: Measurement of $|\chi_{3000}|$ with an AC dipole. The bottom plot shows the sextupolar components of the ring.

The resonance terms f'_{jklm} tend to f_{jklm} when the driving tune approaches the betatron tune. Therefore for certain tune separations the measurement of magnet strengths gives similar results with or without an AC dipole. We have measured $|\chi_{3000}|$ proceeding in the same way as in the previous section but using AC dipole data. No decoherence factor has to be taken into account. The result is shown in Fig. 5. The agreement is similar or better than for the kick case. This demonstrates the feasibility of this kind of measurement.

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