# SPECIFIC BEAM DYNAMICS IN SUPER-BUNCH ACCELERATION

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#### Abstract

Proof-of-principle experiments of the induction synchrotron make progress at the KEK 12-GeV PS, in which a super-bunch will be accelerated by a long step-voltage generated in the induction accelerating gaps. Issues in beam dynamics of super-bunch acceleration, such as a stochastic motion induced by the barrier voltage and longitudinal emittance growth due to a droop and a ripple in the accelerating voltage, have been theoretically examined.

#### **INTRODUCTION**

Super-bunch acceleration is a key feature in an induction synchrotron [1]. In the induction synchrotron, superbunches confined in the longitudinal direction by a pair of barrier voltages are accelerated with long induction stepvoltage pulses (see Fig. 1). Experiments for proof-of-



Figure 1: Schematic view of the induction synchrotron.  $V_c$  is the barrier voltage for the longitudinal confinement and  $V_a$  is the accelerating voltage.

principle (POP) of the induction synchrotron are going to be performed step by step using the KEK 12-GeV PS [2]. In the first step, a single bunch that is captured in the RF bucket will be accelerated with the induced step-voltage alone. 3 newly developed induction accelerating cavities with an output voltage of 2kV/each were installed [3]. As the second step, an induction barrier experiment is planned, where 1-9 booster RF bunches are injected into the main ring, immediately captured in the induction barrier bucket, and then merged into a single super-bunch. In the last step, a super-bunch will be accelerated up to the flat-top energy with the induction voltage.

A super-bunch confined in the barrier bucket has a notable feature: extremely slow synchrotron oscillation with drifting between the barriers and the quick reflection in the barrier. This is distinguished from that of the conventional RF bunch. Following beam dynamics issues originated by this property have been addressed; 1) emittance blow-up due to stochastic motion caused by the barrier, 2) asymmetric acceleration by a droop voltage, which is indispensable in the induction accelerator system, 3) delicate reaction of super-bunch against various perturbations in the accelerating voltage, and 4) effects of slow mixing on various coherent instabilities. In this paper, the results of study on issue 1) - 3) are presented. Due to the page limitation, the results of issue 4) will be given elsewhere [4].

## **BARRIER BUCKET ACCELERATION**

A barrier voltage for the super-bunch beam is defined by using the step-function,  $u_x(t) = 0$  (t < x) and 1  $(t \ge x)$ , as  $V_{c}(\phi) = V_{step} f(\phi) [u_{-\phi_{0}}(\phi) + u_{\phi_{0}}(\phi) - 1]$ , where  $\phi = \{h \int \omega_s dt\} \mod (2\pi), 2\phi_0$  is the core-length of the super-bunch,  $T_s = 2\pi/\omega_s = C_0/(\beta c)$  is the revolution period of the synchronous particle,  $C_0$  is the circumference of a ring,  $\beta$  is the relativistic beta of the design particle, c is the velocity of light, h is the harmonic number and  $V_{step}$  is the peak voltage.  $f(\phi)(|f(\phi)| \le 1)$  represents a rising or falling profile of the barrier voltage. The sign of the voltage must be changed beyond the transition to maintain the phase stability, just as in a conventional RF synchrotron. The accelerating voltage with the perturbation  $\Delta V(\phi)$  is defined as  $V_a(\phi) = V_0 + \Delta V(\phi)$ , where  $V_0$  is the designed step-voltage for the acceleration. The machine parameters of the KEK 12GeV-PS are listed in Table 1. The temporal

Table 1: Machine parameters

h	9
Transition $\gamma$	6.63
Circumference	339 m
Injection energy	500 MeV
Extraction energy	12 GeV
$V_0$	10 kV

evolution of the energy difference from the synchronous energy,  $E_s$ , and the fractional phase for the particle of interest are given by the following difference equations:

$$\Delta E_{n+1} = \Delta E_n + e \left[ V_c \left( \phi_n \right) + \Delta V \left( \phi_n \right) \right], \quad (1)$$

$$\phi_{n+1} = \phi_n + 2\pi\eta_{n+1}h \frac{\Delta E_{n+1}}{\beta_{n+1}^2 (E_s)_{n+1}},\tag{2}$$

where  $\eta$  is the slippage factor. In order to manifest the pure effects of the perturbations, nonlinear kinematic terms are not taken account of in this paper. A set of difference equations, Eq. (1) and (2), gives a theoretical basis of the following discussion.

#### **DIFFUSION DUE TO BARRIER VOLTAGE**

In the case of the super-bunch confined by barriers, a stochastic motion can easily occur. A criterion for the stochastic motion is given in the context of Liapnov number;  $\tau < \tau_c = |\eta| V_{step} C_0 / (4\beta^3 c E_s)$ , where  $\tau$  is the rise and fall time of the barrier voltage [5]. It is clear that the small  $\tau$  or the large  $V_{step}C_0$  easily induce the stochastic motion. It finally results in the large emittance growth through the diffusion process as shown in Fig. 2. The sim-



Figure 2: (a) Diffusion width dependent on  $V_{step}C_0$  in the  $\Delta p/p$  direction and (b) the beam distribution after the diffusion ( $V_{step}C_0 = 29$  kV [km] and  $\tau = 6\tau_c$ ). Particles are initially set on -a < t < a and  $\Delta p/p = 0.4$  [%] only.  $f(t) = (|t| - a)/\tau (|t| < a + \tau)$  and  $1 (|t| \ge a + \tau)$ .  $a = \phi_0/(h\omega_s) = 50$  [nsec].

ulation suggests that the diffusion occurs below  $\tau \sim 100\tau_c$ , then reaches the maximum around  $\tau \sim 10\tau_c$ .

In the following sections,  $V_{step}$  is set to 85 kV for capturing particles with  $\Delta p/p = 0.4$ % by the pulse of ~ 100 nsec, and  $\tau$  is set to  $\tau = 666\tau_c = 10$  nsec for avoiding this diffusion. In addition,  $\tau \sim 20$  nsec is realistic as the type of machine parameter.

#### EMITTANCE GROWTH DUE TO DROOP

The induction cavity is connected to a high-voltage pulse modulator through a matching cable, and driven at a repetition rate on the order of MHz [6]. When the step-voltage is inputted, some droop in the output voltage is unavoidable because of a finite inductance and resistance. This could give particles an extra defocusing effect in the longitudinal direction below the transition energy and a focusing effect above the transition energy.

If the droop voltage is assumed to be linear in time,  $\Delta V$  in Eq. (1) can be given by  $\Delta V = hk \int_0^{\phi} d\phi' / \omega_s$ . The potential energy of the synchrotron oscillation with the droop can be written in the following form [7]:

$$U(\phi) = -\frac{e\beta^2 E_s}{2\pi\eta\omega_s^2 h^3} \left[ \int V_c(\phi) \, d\phi + \frac{hk}{2\omega_s} \phi^2 \right]. \tag{3}$$

Eq. (3) implies that double wells are generated around  $\phi = \pm \phi_0$  before the transition energy and one shallow well is generated at its center after the transition energy (see Fig. 3). The mismatching induced by these wells through



Figure 3: Longitudinal potential energy.

the transition causes the emittance growth at the transition as shown in Fig. 4. Moreover, this emittance growth becomes larger as the droop size becomes larger, or the length becomes longer because the depth of the potential wells in Eq. (3) becomes deeper. This fact strongly suggests the importance of a droop correction for the acceleration of a super-bunch in an induction synchrotron with the transition energy. As a droop-compensating device, a technique to superimpose several pulse voltages of half-sine in time [8] is under consideration in a realistic manner.

### **EMITTANCE GROWTH DUE TO RIPPLE**

The long step-voltage for the super-bunch acceleration is subject to a ripple. The ripple frequency ranges from the accelerator operation frequency of 0.25 Hz to DC power supply inner frequency of kHz. These frequencies are far lower than the revolution frequency of the synchronous particle,  $0.7 \sim 1$  MHz. The potential energy with the ripple is written in the form of

$$U(\phi) = -\frac{e\beta^2 E_s}{2\pi\eta\omega_s^2 h^3} \left[ \int V_c(\phi) \, d\phi + v_p \phi \right], \quad (4)$$



Figure 4: Emittance growth dependent on k [kV/ $\mu$ sec] through the super-bunch acceleration, where  $\phi_0 = 7\pi$ [rad].

where the instantaneous  $\Delta V = v_p$  in Eq. (1) is nearly constant over many turns. The second term of Eq. (4) means that the slope in the bottom of the potential well slowly changes with  $v_p$ . It is predicted that the varying slope resonantly induces a large amplitude oscillation in the longitudinal direction when the ripple frequency is close to the synchrotron oscillation frequency. In a global scale, the resonant excitation appears as a dipole oscillation in the super-bunch. The large emittance growth can result in because of the filamentation mechanism in the barrier bucket.

Typical simulation result is shown in Fig. 5, where the frequency of 50 Hz and the amplitude of  $V_0/100$  are assumed in the ripple. As mentioned, the emittance growth is caused when the maximum frequency of the synchrotron oscillation crosses the line of 50 Hz (0.5 sec and 1.0 sec). Other simulation shows that the emittance growth is re-



Figure 5: Emittance growth and the maximum frequency of the synchrotron oscillation of particles in the super-bunch. The frequency becomes 0 at the transition.  $\phi_0 = \pi$ [rad].

markable at the low ripple-frequency below  $\sim 20$  Hz as shown in Fig. 6. This large emittance growth is strongly caused near the transition energy, where the frequency of the synchrotron oscillation is also small as shown in Fig. 4.

The induction synchrotron is realized with a digital feedback system [3, 9], where a digital signal is used as a trigger



Figure 6: Emittance growth dependent on the frequency of the ripple.

signal of the power modulator driving the induction cavity. At least, the simulation suggests that the idea is well to compensate effects of the ripple by monitoring the dipole oscillation.

#### CONCLUSION

Beam dynamic issues of the super-bunch, which are important to realize the induction synchrotron or super-bunch hadron collider [10], have been discussed. Some of them are intrinsic for induction acceleration and the others are characteristic for capturing in the barrier bucket. Their counter-measures have been studied too. They will be tested in the POP-experiments.

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