

# BEAM LOADING IN THE RF DEFLECTOR OF THE CTF3 DELAY LOOP

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## Abstract

In this paper we describe the impact of the beam loading in the RF deflectors on the transverse beam dynamics of the CTF3 Delay Loop. The general expression for the single passage wake field is obtained. A dedicated tracking code has been written to study the multi-bunch multi-turn effects. A complete analysis for different machine parameters and injection errors is presented and discussed. The numerical simulations show that the beam emittance growth due to the wake field in the RF deflectors is small.

## INTRODUCTION

The first stage of the bunch train compression scheme in the CLIC test Facility CTF3 [1] is realized in the 42 m circumference Delay Loop (DL) by using a transverse RF deflector (RFD) at 1.499 GHz. The process is illustrated in Fig. 1. The bunch timing of subsequent “even” and “odd” batches coming from the LINAC is adjusted in such a way that they have a phase difference of  $180^\circ$  with respect to the 1.499 GHz. The RFD deflects every “even” batch of 210 bunches into the DL, and, since the circumference of the DL corresponds to the length of one “even” batch of bunches, after one turn, it insert this batch between bunches of the following “odd” batch.

In this paper we investigate the beam loading effects in the RFD, which may result in transverse bunch emittance growth, particle loss, and degradation of the Drive Beam quality thus reducing the effectiveness of power conversion at 30 GHz. In particular, in par. 1 we will derive a general expression for the wake field generated by the single bunch passage in the RFD. In par. 2 and 3 we will illustrate the results obtained by a tracking code written to study the multi-passage effects in the case of a perfect injection of the bunches or considering different injection errors, respectively.

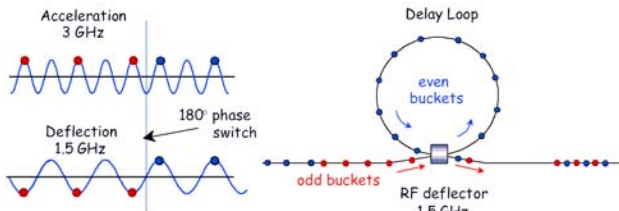


Fig. 1: Sketch of the bunch frequency multiplication in the DL.

## SINGLE PASSAGE WAKE FIELD

The design options for the DL RFD are widely illustrated in [2]. The final device will be a standing wave double cavity working on the  $TM_{110}$  mode at the frequency of 1.499 GHz. The main parameters of the RFD and of the DL are reported in Table 1.

Table 1: main parameters of the DL and RFD

Beam Energy [MeV]	150 (min)-300 (max)
Bunch length [mm]	1-3
DL length [m]	42
RFD frequency [GHz]	1.499 GHz
Quality factor Q	3200
R/Q per cell [ $\Omega/m$ ]	27

Referring to the case of a simple single cell deflecting cavity (Fig. 2a), the field component in the plane  $\vartheta = 0$  are given by [3]:

$$\begin{cases} E_{Dz} = E_0 J_1 e^{j\omega t} \\ E_{Dr} = 0 \\ E_{D\theta} = 0 \end{cases} \quad \begin{cases} B_{Dz} = 0 \\ B_{Dr} = 0 \\ B_{D\theta} = -j\omega a / p_{11} c^2 E_0 J_1' e^{j\omega t} \end{cases} \quad (1)$$

where  $J_1 = J_1(p_{11}r/a)$  and  $J_1' = J_1'(p_{11}r/a)$  with  $p_{11}$  first zero of the  $J_1$  Bessel function. In (1) we have considered the cavity like a pure pillbox cavity neglecting all field components given by the presence of the beam pipe tubes ( $E_{Dr}, B_{Dz}$ ). For the scope of this work and, in general, for small beam pipes, these components can be neglected.

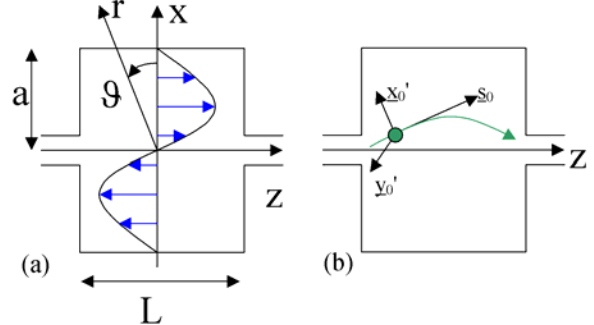


Fig. 2: (a) Sketch of a RFD cavity with the  $E_{Dz}$  field lines; (b) sketch of a particle passing through the RFD.

The general expression, in frequency domain, of a field excited by a charge passing through a cavity [4] is:

$$\underline{E}_{exc} = E_{exc} \underline{z}_0 = -\frac{1}{1+jQ\delta} \frac{E_{Dz} \int_{cavity} J_z \cdot E_{Dz}^* dV}{2P_T} \underline{z}_0 \quad (2)$$

where  $\delta = (\omega_r / \omega - \omega / \omega_r)$ ,  $P_T$  is the total dissipated power in the cavity plus external load,  $Q$  is the quality factor and  $J_z$  is the longitudinal component of the electric density current corresponding to the charge. Assuming a charge passing in  $z=0$  at the time  $t=0$  (see Fig. 2b) whose density current is  $\underline{J} = q\delta(x')\delta(y')\exp(-j\omega z/c)\underline{s}_0(z)$  we have that:

$$E_{exc} = -\frac{1}{1+jQ\delta} \frac{E_{Dz} \int_{cavity} q e^{-j\frac{\omega}{c} z_1} E_{Dz}^* [r_q(z_1)] dz_1}{2P_T} \quad (3)$$

where,  $r_q(z_1)$  is the particle trajectory. In (3) we have neglected the transverse component of the particle

velocity ( $\beta_0 \equiv z_0$ ). The longitudinal voltage induced in the cavity is given by:

$$V_{exc z} = \int_{cavity} E_{exc z} e^{j\frac{\omega}{c}z} dz \quad (4)$$

By means of the Panofsky-Wenzel theorem it is possible to calculate the transverse deflecting voltage induced in the cavity:

$$V_{exc r} = \frac{qc}{j\omega} \cdot \frac{1}{1+jQ\delta} \int_{cavity} \frac{dE_{Dz}}{dr} e^{j\frac{\omega}{c}z} dz \frac{\int_{cavity} E_{Dz}^*(r_q) e^{-j\frac{\omega}{c}z_1} dz_1}{2P_T} \quad (5)$$

Considering bunches passing near the cavity axis, it is possible to develop to the first order in  $r$  the  $E_{Dz}$  field in the form  $E_{Dz} \equiv E'_{0Dz} r$ . Moreover, considering a generic trajectory in the deflector of the type  $r_q = a_1 z^2 + a_2 z + a_3$  we obtain:

$$V_{exc r} \equiv K(R_1 a_1 + R_2 a_2 + R_3 a_3) = V_1 a_1 + V_2 a_2 + V_3 a_3 \quad (6)$$

where:

$$K = qc / (2 j\omega P_T) / (1 + jQ\delta) \int_{cavity} E'_{0Dz} e^{j\frac{\omega}{c}z} dz$$

$$R_1 = \int_{cavity} E'_{0Dz} z_1^2 e^{-j\frac{\omega}{c}z_1} dz_1, R_2 = \int_{cavity} E'_{0Dz} z_1 e^{-j\frac{\omega}{c}z_1} dz_1$$

$$R_3 = \int_{cavity} E'_{0Dz} e^{-j\frac{\omega}{c}z_1} dz_1$$

The corresponding induced wake fields in case of a cavity symmetric with respect to the plane  $z=0$  are:

$$V_{1,3}(t) \equiv q_L c R_{1,3} / Q e^{\frac{\omega_r}{2Q} t} \sin(\omega_r t) \quad (8)$$

$$V_2(t) = q_L c R_2 / Q e^{\frac{\omega_r}{2Q} t} \cos(\omega_r t)$$

Bunches whose trajectories have zero angle at the center of the deflector ( $a_2=0$ ) generates a pure wake field  $90^\circ$  out-of-phase, while particles whose trajectories have  $a_{1,3}=0$  generates a wake field in phase. This situation is illustrated in Fig. 3. In the figure we have considered the case of two recombined trains and we have plotted the wake field produced by the first bunch considered with a generic trajectory. The numerical values of the coefficient  $R_{1,2,3}$  can be easily calculated by e.m. codes.

## TRACKING CODE RESULTS

A dedicated tracking code has been written to study the multi-bunch multi-passage effects. The tracking code scheme is sketched in Fig. 4. Each gaussian bunch of  $\sigma_z = 3mm$  has been discretized in 9 equally spaced slices. In the code even the case of perfectly injected bunches ( $x_{in}=0$  and  $x'_{in}=7.5$  mrad with respect to the deflector axis) than the case of bunches injected with errors can be considered.

The output positions and angles of the central slice of bunches with and without considering the effect of the beam loading are reported in Fig. 5. In the case of no beam loading the differences with respect to the “nominal values”  $x_{out}=0$  and  $x'_{out}=-7.5$  mrad are due to the finite filling time of cavities. In Fig. 6 it is reported the rms emittance referred to the central slice of each bunch in the same cases. It is clear that the effect of the beam loading is comparable with the effect of the finite filling time of

cavities while the effect in the beam emittance growth is a small fraction of the design emittance.

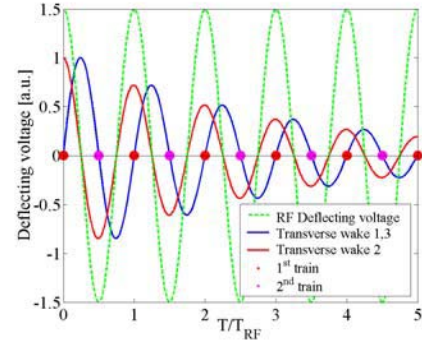


Fig. 3: Transverse wake fields induced by bunches with different trajectories.

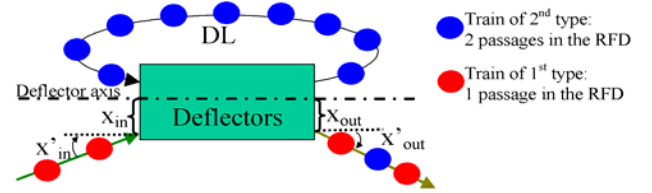


Fig. 4: Tracking code scheme.

All these results have been obtained considering a DL phase advance of  $\approx 260^\circ$ . Different phases advances gives different results as shown in Figs. 7 and 8 where the average positions, angles and rms emittances (with error bars) are plotted as a function of the phase advance. In all cases for the average positions and angles, the effects of the beam loading are comparable with the effect of the finite filling time of cavities. In particular, even in the case in the case of no beam loading, there is a compensation effect between the first and second passage through the deflector if the DL phase advance is  $\approx 180^\circ$ . Considering the rms emittances there is, also, a minimum of the beam loading effect if the DL phase advance is  $\approx 180^\circ$ .

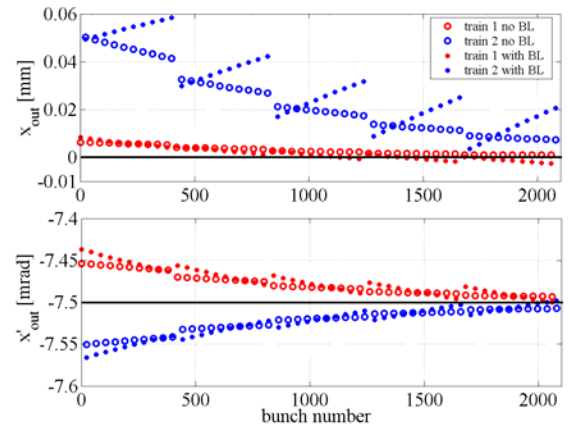


Fig. 5: Output positions and angles of bunches in case of perfect injection.

The effect of an injection error in position is shown in Figs. 9 and 10. Concerning the output positions and

angles the initial error is “transferred”, for the trains 1, at the exit of the deflectors, after the recombination, practically unchanged and slightly reduced for the train of type 2. Moreover the effect of the beam loading is that to increase of about 20% the spread around the average positions with respect to the case without beam loading effects. Concerning the r.m.s emittance the effect is practically the same for all injection errors. This is due to the fact that the effect of the wake generated in the first deflector is partially compensated by the second one.

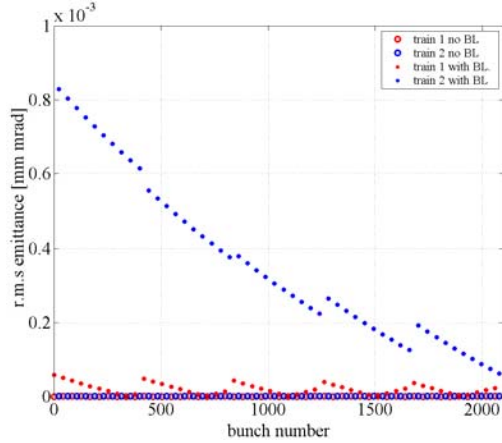


Fig. 6: rms emittance in case of perfect injection

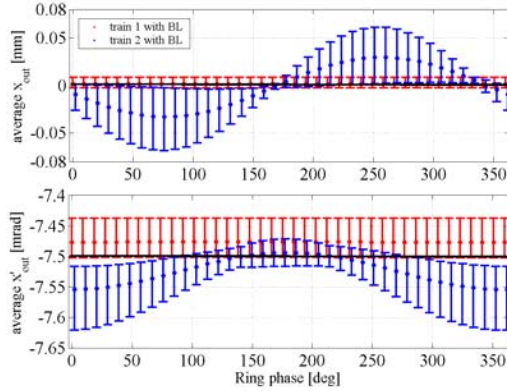


Fig. 7: Output positions and angles of bunches without injection errors as a function of the DL phase advance.

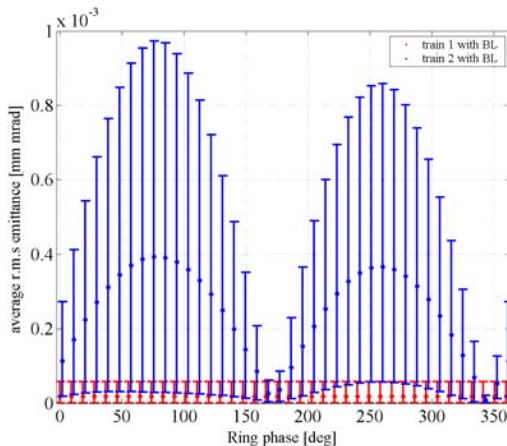


Fig. 8: r.m.s emittances without injection errors as a function of the DL phase advance.

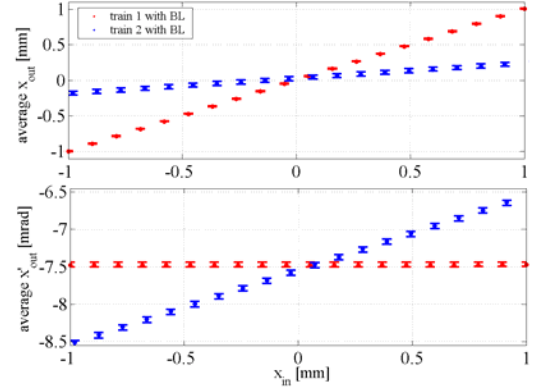


Fig. 9: Average output positions and angles (with error bars) of bunches in case of an injection error in position ( $x_{in}$ ) between  $\pm 1mm$ .

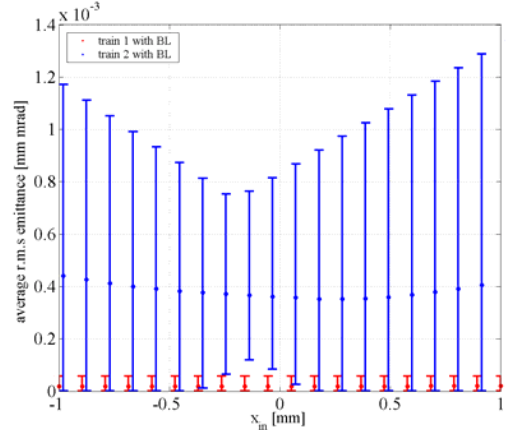


Fig. 10: rms emittances of bunches in case of an injection error in position.

## CONCLUSIONS

The effects on transverse beam dynamics of the beam loading in the CTF3 Delay Loop RF deflector have been studied. For this purpose:

- a) we have calculated the single passage transverse wake induced by a charge considering different trajectories in the deflector;
- b) we have written a tracking code to study the multi-bunch multi-passage effects.

The main results are:

- 1) in case of perfect injection the beam loading effects are comparable with the effect due to the finite filling time of deflector; 2) in case of injection errors the beam loading does not amplify significantly the initial error; 3) different phase advances of the DL give different results but, in any case, the beam loading effects are controllable.

## REFERENCES

- [1] “CTF3 Design Report”, CERN PS 2002-008, 2002.
- [2] F. Marcellini and D. Alesini, this conference.
- [3] see for example R.E. Collin, “Foundation for Microwave Engineering”, McGraw Hill, 1992.
- [4] G. Dôme, “RF Theory”, CERN 92-03, Vol. I, 1992.