MODIFIED POLARIZABILITIES AND WALL IMPEDANCE FOR SHIELDED PERFORATED BEAM PIPES WITH GENERAL SHAPE

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Abstract

We exstend Gluckstern results for the electric and magnetic polarizabilities of pumping holes in the wall of a circular liner surrounded by a co-axial circular shield to arbitrary non-circular geometries. The result is used for evaluating the pertinent beam coupling impedances in terms of a general perturbative formula using equivalent impedance boundary conditions at the liner's wall.

INTRODUCTION

The effect of pumping holes on beam dynamics and stability is a critical issue and has been carefully investigated, both theoretically [1] and experimentally [2]. We introduced accurate analytic approximants for beam coupling impedances in pipes with complex geometries and/or constitutive properties based on impedance (Leontóvich) boundary conditions [3], which cover the case of perforated walls [4]. The mentioned approach allows to account easily for the effect of a co-axial shielding tube, by suitable modification of the relevant electric and magnetic hole polarizabilities in the impedance boundary conditions. Such modified polarizabilities were first introduced and computed by Gluckstern [5] and re-derived in [4], for the special case of circular co-axial geometries. In this paper we extend the analysis to general co-axial geometries.

BEAM COUPLING IMPEDANCES IN PERFORATED LINERS FROM IMPEDANCE BOUNDARY CONDITIONS

The specific longitudinal and transverse beam coupling impedances $Z_{0,\|}(\omega)$ and $\overline{\overline{Z}}_{0,\perp}(\omega)$ of a *simple, unperturbed* pipe (e.g., circular, perfectly conducting) assumed known, can be related to those $Z_{\|}(\omega)$, $\overline{\overline{Z}}_{\perp}(\omega)$ of another pipe differing from the former by some *perturbation* in the boundary geometry and/or constitutive properties, as follows [3]:

$$Z_{\|}-Z_{0,\|}=\frac{\epsilon_0}{\beta_0 c Q^2}\left\{\oint_{\partial S}\left[\beta_0 E_n^{(irr.)}(\vec{r},0)+\frac{1}{\beta_0}E_n^{(sol.)}(\vec{r},0)\right]\cdot\right.$$

$$\left.\cdot\frac{Z_w}{Z_0}E_{0n}^{(irr.)*}(\vec{r},0)d\ell-\oint_{\partial S}E_{0z}^{*}(\vec{r},0)E_n^{(irr.)}(\vec{r},0)d\ell\right\}, \quad (1)$$

$$\overline{\overline{Z}}_{\perp}(\omega)-\overline{\overline{Z}}_{0,\perp}(\omega)=\frac{\epsilon_0}{\beta_0 c Q^2 k_0}\left\{Y_0\oint_{\partial S}Z_w\nabla_{\vec{r}_0}E_{0n}^{(irr.)*}(\vec{r},\vec{r}_0)\otimes\right.$$

$$\otimes\nabla_{\vec{r}_1}\left[\beta_0 E_n^{(irr.)}(\vec{r},\vec{r}_1)+\beta_0^{-1}E_n^{(sol.)}(\vec{r},\vec{r}_1)\right]d\ell+$$

$$\left.-\oint_{\partial S}\nabla_{\vec{r}_0}E_{0z}^{*}(\vec{r},\vec{r}_0)\otimes\nabla_{\vec{r}_1}E_n^{(irr.)}(\vec{r},1)d\ell\right\}_{\vec{r}_1=\vec{r}_0=0}, \quad (2)$$

where $c=(\epsilon_0\mu_0)^{-1/2}$ is the speed of light in vacuum, $Y_0=1/Z_0=(\epsilon_0/\mu_0)^{1/2}$ is the vacuum characteristic admittance, $k_0=\omega/c$ is the wavenumber, ϵ_0 and μ_0 being the vacuum permittivity and permeability, β_0 is the relativistic factor, Q is the total beam charge , $\vec{E}_0^{(sol.)}$, $\vec{E}_0^{(irr.)}$ are the solenoidal and irrotational parts of the electric field in the *unperturbed* pipe, Z_w is the (Leontóvich) pipe-wall complex characteristic impedance, and ∂S is the pipe cross-section boundary. The first integral term on the r.h.s of (1) and (2) accounts for the effect of the finite wall conductivity, and is nonzero if and only if Z_w is not identically zero on ∂S. The second integral on the r.h.s. of (1) and (2), on the other hand, accounts for the effect of the geometrical perturbation of the boundary, and is non-zero if and only if the *unperturbed* axial field component E_{0z} is not identically zero on ∂S[1].

Equations (1), (2) can be applied to the case of perforated walls. In [4] we shew, following both an heuristic argument and a rigorous boundary value problem approach, that for near-to-grazing incidence, a plane perforated conducting surface can be described using a Leontóvich b.c., with wall-impedance

$$Z_w=-jk_0Z_0n_\sigma(\alpha_m+\alpha_e), \quad (3)$$

where α_e, α_m are the (internal) electric and magnetic hole polarizabilities [6] and n_σ the hole surface-density. Equation (3) can be used for a *non-planar* perfectly conducting perforated surface provided the further condition: $|(Z_0/Z_w)\,k_0\,R_S|\gg 1$ is satisfied, where R_s is the (local) smallest radius of curvature of the perforated surface S, according to [7].

Let ℓ the arc-lenght along the pipe cross-section contour, assume N_λ (uniformly spaced) holes per unit pipe lenght[2], and let $n_\sigma=N_\lambda\delta(\ell-\ell_h)$, Then, from (1) and (3),

$$Z_{\|} \;=\; -\,jZ_0k_0N_\lambda(\alpha_e+\alpha_m)\,e_n(\ell_h)e_n^*(\ell_h), \quad (4)$$

where $e_n(\ell_h)=(Q/\epsilon_0)^{-1}E_n(\ell_h)$, $E_n(\ell_h)$ being the (normal) electric field produced by an axial beam with total charge Q at the hole's position. Equation (4) reproduces Kurennoy's result valid for general transverse geometries, obtained using a different rigorous (modal) approach [8].

[1]The beam impedances are obviously independent of the total beam charge, as the field in (1) is proportional to Q.

[2]For the very definition of beam impedance to apply, the holes should be (at least piecewise) uniformly-distributed in the longitudinal direction.

MODIFIED POLARIZABILITIES

For an infinite[3] perforated beam pipe surrounded by a co-axial shield (e.g., the LHC cold-bore) eq. (3) still holds, provided we use *modified* polarizabilities:

$$\alpha_{e,m} = \alpha^{(i)}_{e,m} + F\alpha^{(e)}_{e,m}, \tag{5}$$

where the suffixes e, m refer to the electric and magnetic polarizabilities, while the superfixes $(e), (i)$ denote the external and internal ones. This result was first obtained by Gluckstern [5] for the special case of circular co-axial geometry, and rederived in [4], following a different route. Here we extend this result to pipes with general transverse geometry, by providing an appropriate explicit expression for F, viz.:

$$F = \frac{-(\alpha^{(e)}_e + \alpha^{(e)}_m)}{(\alpha^{(i)}_e + \alpha^{(i)}_m) + jN_\lambda^{-1}\delta^*_S \dfrac{\oint_{\partial S_c} |e_{cb}|^2 d\ell}{|e_{cb}(\vec{l}_h)|^2}}, \tag{6}$$

where $\delta_S = (|k_0|Z_0)^{-1} Z^*_{wall}$ is the complex EM penetration depth into (both, assumed equal) coaxial region walls, ∂S_c is the (complete) contour of the co-axial region cross-section, e_{cb} is the transverse electromagnetic (henceforth TEM) eigenfunction in the co-axial region, ℓ_h identifies the hole position on the liner cross section boundary, and N_λ the longitudinal hole density.

The effect, e.g., of a non-zero wall thickness can be readily included [9], [10], [11].

PERFORATED BEAM PIPE IN A CO-AXIAL SHIELD

We shall work in the spectral domain ($z - \beta ct \longrightarrow k$), and assume that the spectral content of the primary field is such that *only* the fundamental (TEM) mode may propagate in the coaxial region. The electric (magnetic) field component normal (tangential) to the wall will be denoted by E_0 (resp. H_0) and E_{cb} (resp. H_{cb}) in the liner and coaxial region (cold-bore), respectively. We shall assume the holes to be located at $\ell = \ell_h$ and $z = L, 2L, ...$

In the presence of an external co-axial tube, the elementary sources radiating *into* the liner will be Bethe dipoles at $\ell = \ell_h,\ z = lL$ with moments

$$\left\{ \begin{array}{l} \vec{P}_l = \hat{u}_n\epsilon_0\left[\alpha^{(i)}_e E_0(\ell_h, lL) + \alpha^{(e)}_e E_{cb}(\ell_h, lL)\right] \\ \vec{M}_l = \hat{u}_c\left[\alpha^{(i)}_m H_0(\ell_h, lL) + \alpha^{(e)}_m H_{cb}(\ell_h, lL)\right] \end{array} \right. , \tag{7}$$

Equation (7) can be equally written:

$$\left\{ \begin{array}{l} \vec{P}_l = \hat{u}_n\ \epsilon_0 \left[\alpha^{(i)}_e + \alpha^{(e)}_e\ F\right] E_0(\ell_h, lL) \\ \vec{M}_l = \hat{u}_c\ \left[\alpha^{(i)}_m + \alpha^{(e)}_m\ F\right]\ H_0(\ell_h, lL) \end{array} \right. , \tag{8}$$

which involves *only* the liner (unperturbed) fields, and corresponds to assuming that the liner stands in *vacuo*, and using the *modified* polarizabilities (5) for the liner's holes, to account for the effect of the coaxial region, in computing the perforated wall impedance

$$Z_w = -jZ_0k_0n_\sigma\left[\left(\alpha^{(i)}_e + \alpha^{(i)}_e\right) + F\left(\alpha^{(e)}_e + \alpha^{(e)}_m\right)\right]. \tag{9}$$

In order to evaluate F in eq. (8), we proceed as follows.

The TEM field set up in the coaxial region by a *single* hole at $\ell = \ell_h,\ z = lL$, i.e., by the Bethe dipoles with moments:

$$\left\{ \begin{array}{l} \vec{P}_l = \hat{u}_n\epsilon_0\left[\alpha^{(e)}_e E_0(\ell_h, lL) + \alpha^{(i)}_e E_{cb}(\ell_h, lL)\right], \\ \vec{M}_l = \hat{u}_c\left[\alpha^{(e)}_m H_0(\ell_h, lL) + \alpha^{(i)}_m H_{cb}(\ell_h, lL)\right] \end{array} \right. \tag{10}$$

can be written:

$$\left\{ \begin{array}{l} \vec{E}^{\ \pm}_{cb} = C^{\pm}_l\ \vec{e}_{cb}(\ell_h)\ e^{\mp jk_g(z-lL)} \\ \vec{H}^{\ \pm}_{cb} = \pm Y_0\ \hat{u}_z \times \vec{e}_{cb}(\ell_h)\ e^{\mp jk_g(z-lL)}, \end{array} \right. \tag{11}$$

where the superfix $\pm$ applies to $z \gtrless lL$ and k_g is the TEM propagation constant in the coaxial region. The coefficients $C^{\pm}_l$ are readily evaluated by resorting to Lorentz (reciprocity) theorem in the form [12]:

$$\iint_{\partial V} (\vec{E}_{cb} \times \vec{\mathcal{H}}^{\pm} - \vec{\mathcal{E}}^{\pm} \times \vec{H}_{cb}) \cdot \vec{n}\ d\Sigma =$$

$$-j\omega\mu_0\vec{\mathcal{H}}^{\pm}(lL) \cdot \vec{M}_l + j\omega\vec{\mathcal{E}}^{\pm}(lL) \cdot \vec{P}_l, \tag{12}$$

where V is a slice of the coaxial region limited by the planes $z = lL \pm \delta$ (δ as small as one wishes), ∂V is its boundary, and $(\vec{\mathcal{E}}^{\pm}, \vec{\mathcal{H}}^{\pm})$ is a source-free forward or backward TEM field

$$\vec{\mathcal{E}}^{\pm} = \vec{e}_{cb}(\vec{\rho})\ e^{\mp jk_gz},\quad \vec{\mathcal{H}}^{\pm} = \pm Y_0\hat{u}_z \times \vec{e}_{cb}(\vec{\rho})e^{\mp jk_gz}, \tag{13}$$

where $\vec{\rho}$ is the transverse position. Following [5] we shall make the assumption that the backward fields can be neglected. Indeed the TEM waves produced by the Bethe dipoles (holes) accumulate coherently in the forward direction, as they travel at the same speed as the primary field of an (assumed) ultrarelativistic particle in the liner. The hole spacing is assumed to be such that no backward phasing may occur within the spectrum of the primary field.

The total (forward propagating) field at $\ell = \ell_h$ and $z = pL$, is thus obtained by summing over all $l < p$:

$$E_{cb}(\ell_h, pL) = \sum_{l<p} \frac{-jk\ e_{cb}(\ell_h)}{2\displaystyle\int_{\partial V} e^2_{cb}(\vec{\rho})\ dS}\ e^{-jk_g(p-l)L} \cdot$$

$$\cdot\left[(\alpha^{(e)}_m + \alpha^{(e)}_e)E_0(\ell_h, lL) + (\alpha^{(i)}_m + \alpha^{(i)}_e)E_{cb}(\ell_h, lL)\right] \tag{14}$$

At this point we *postulate* $E_{cb}(\ell_h, pL) = FE_0(\ell_h, pL)$, and assume F as independent of p, because the (infinite) structure is invariant under the group of z-translations by multiples of L.

Then we use the obvious identity

$$E_0(\ell_h, lL) = E_0(\ell_h, pL)e^{-jk_g(p-l)L} \tag{15}$$

[3] More realistically, the argument applies to ring machines provided (bunch lenght) $\ll$ (ring circumference).

and formally sum the geometric series in (14),

$$\sum_{l<p} e^{j(k_g-k)(p-l)L} = \frac{1}{1-e^{j(k_g-k)L}} \approx \frac{-1}{j(k_g-k)L}, \quad (16)$$

where the last approximation is justified in view of the expected smallness of the exponent. To evaluate the difference $k_g - k$ we note that the free-space wavenumber k is equal to the loss-free TEM propagation constant in the coaxial region [12]. The first order formula [13] for the (longitudinal) wavenumber in a lossy-wall waveguide gives:

$$k_g - k = \ -jY_0 Z_w \frac{\displaystyle\oint_{\partial S_c} e_{cb}^2(\vec{\rho})d\rho}{2\ Re \displaystyle\int_{S_c} e_{cb}^2(\vec{\rho})dS}, \qquad (17)$$

where e_{cb} is the (unperturbed, perfect conductor) TEM eigenfunction in the co-axial region, ∂S_c is the (complete) contour of the co-axial region cross-section, and Z_{cb} is the (complex) characteristic impedance of the coaxial region walls.

From (14) to (17) after a little algebra we get

$$F = \frac{-(\alpha_e^{(e)}+\alpha_m^{(e)})}{(\alpha_e^{(i)}+\alpha_m^{(i)}) + jN_\lambda^{-1}\delta_S^* \dfrac{\oint_{\partial S_c} |e_{cb}|^2 d\ell}{|e_{cb}(\ell_h)|^2}}, \qquad (18)$$

where $\delta_S \ = \ (|k_0|Z_0)^{-1} \ Z_{wall}^*$. is the complex EM penetration depth into (both, assumed equal) coaxial region walls[4].

For a circular pipe with radius $r = b$ in a coaxial circular tube of radius $r = a$ equation (18) gives back [4]:

$$F = -\frac{\alpha_e^{(e)}+\alpha_m^{(e)}}{\alpha_e^{(i)}+\alpha_m^{(i)} + j\ sgn(k)\delta_s^* N_\lambda^{-1}(1+b/a)}, \quad (19)$$

which coincides with Gluckstern result [5].

COMPARISON WITH MEASUREMENTS

The parasitic losses in the co-axial region have been computed in [2] following a different approach, in terms of the equivalent co-axial transmission line current I_{cb}, and compared favorably with measurements. In view of the obvious relationship between this latter and the magnetic field in the co-axial region, one gets (for a pointlike bunch $I(z,t) = Q\beta c\delta(z-\beta ct)$): $|F| = |I_{cb}(z,\omega)/I(z,\omega)|$. The result obtained and validated in [2] is remarkably recovered, provided $|\alpha_e^{(i)} + \alpha_m^{(i)}| \ll |n_\sigma^{-1}\hat{\delta}(1+b/a)|$, which holds true under the assumptions made in [2].

[4] Note that in some previous papers [5] the *complex* character of Z_w is apparently overlooked.

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