2-ND ORDER SEXTUPOLE EFFECTS ON THE DYNAMIC APERTURE IN HERA–e

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Abstract

During the first year after the luminosity upgrade HERA–e was operated in a mode for which the accessible area in transverse tune space was determined by resonances driven by sextupoles in 2-nd order. It turned out that with typical total incoherent beam–beam tune shifts $(0.05, 0.08)$ for 2 IPs this space was too small for stable operation. We have used 2-nd order Canonical Perturbation Theory (CPT) to analyze the impact of the increased sextupole strengths in the upgraded lattice on the relevant resonance strengths and the detuning. Moreover, we have studied whether it is possible to compensate the resonances with localized octupole schemes (6 independent magnets in one straight section) to 1-st and 2-nd order, computed the resulting detuning and compared the result with 6D tracking.

CHROMATIC SEXTUPOLES IN HERA–E AFTER THE LUMINOSITY UPGRADE

To achieve the horizontal equilibrium emittance of 22nm which is necessary to meet the specifications of the luminosity upgrade [1], the phase advance in the arcs had to be increased from 60° to 72° per FODO cell. Thus stronger excitation of the chromatic sextupoles was needed to compensate the increased natural chromaticities. For the 72° optics a sextupole scheme [2] with two independent families per plane was chosen to minimize the impact of the direct sextupole resonances ($3Q_x$ and $Q_x + 2Q_y$) and the off–momentum dynamic aperture in the absence of collisions. However, under collisions with 100mA protons of $\epsilon_{p,x}^{\mathrm{norm},1\sigma} = \epsilon_{p,y}^{\mathrm{norm},1\sigma} = 5\pi$ mm mrad and with positron beta–functions at both IPs of $\beta_{e,x} = 0.63$m and $\beta_{e,y} = 0.26$m the total incoherent beam–beam parameters are $\xi_{e,x} = .046$ and $\xi_{e,y} = .074$. Fig. 1 shows the positron tune necktie caused by the beam–beam tune spread for the nominal fractional luminosity tunes $[Q_x] = .236$ and $[Q_y] = .319$ (referring to non–colliding bunches) of the run year 2002. The following non–skew sum–resonances of 4-th order cross or bound the populated tune space : $4Q_x$ (limits the non–coll. tune to the right), $2Q_x + 2Q_y$ (crosses the necktie) and $4Q_y$ (limits the central tune of the colliding bunches from below). As a matter of fact even with moderate proton currents of 60–80mA operational conditions where quite uncomfortable. Tiny deviations of the positron tunes away from their optima immediately degraded the luminosity and the positron lifetime. Since the relevant 4-th order resonances are most strongly driven by the chromatic sextupoles in 2-nd order CPT, we used a code[3] based on

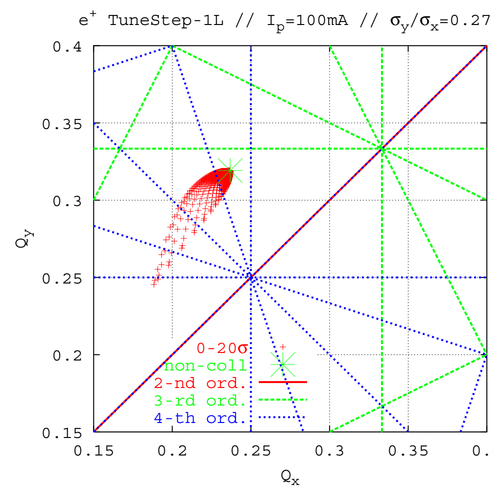

Figure 1: The tune necktie for the working point of 2002 with collisions with a 100mA proton beam

2-nd order CPT to compute their strengths in the old and new optics and analyzed potential cures.

CANONICAL PERTURBATION THEORY

We start from the transverse Hamiltonian $H(x, x', y, y'; \theta) = H_0(x, x', y, y'; \theta) + H_1(x, y; \theta)$ where H_0 represents the linear, unperturbed and uncoupled motion and $H_1 = \sum_{n,m<1} a_{n,m}(\theta) x^n y^m$ represents the perturbation due to distributed multipoles. By applying the symplectic transformation generated by $x(\Psi_x, J_x, \theta) = \sqrt{2J_x\beta_x(\theta)} \cos(\Psi_x + \Phi_x(\theta))$ $y(\Psi_y, J_y, \theta) = \sqrt{2J_y\beta_y(\theta)} \cos(\Psi_y + \Phi_y(\theta))$ we transform the unperturbed Hamiltonian $H_0(\vec{\Psi}, \vec{J}; \theta) = 0$, i.e. the new variables $\vec{J}$, $\vec{\Psi}$ are *both* slow for small perturbation H_1. Here the $\Phi_i(\theta) = Q_i\theta + \psi_i(\theta)$ are the betatron phase advances with $\psi_i(\theta + 2\pi) = \psi_i(\theta)$. Introducing the abbreviations $\vec{\alpha} := (n, m; \nu, \mu)$ and $Q_{\vec{\alpha},k} := \nu\, Q_x + \mu\, Q_y + k$, and performing a Fourier transform w.r.t. θ we obtain

$$H_1(\vec{\Psi}, \vec{J}, \theta) = \sum_{\vec{\alpha},k} h_{\vec{\alpha},k}\, J_x^{\frac{n}{2}} J_y^{\frac{m}{2}}\, e^{i(\nu\Psi_x + \mu\Psi_y) + Q_{\vec{\alpha},k}\theta} \qquad (1)$$

where

$$h_{\vec{\alpha};k} := \frac{1}{2\pi} \int_{-\pi}^{+\pi} d\theta\, H_{\vec{\alpha}}(\theta)\, e^{-ik\theta} \qquad (2)$$

$$H_{\vec{\alpha}}(\theta) := \binom{n}{\frac{n-\nu}{2}} \binom{m}{\frac{m-\mu}{2}} a_{n,m}(\theta) \times \left(\frac{\beta_x(\theta)}{2}\right)^{\frac{n}{2}} \left(\frac{\beta_y(\theta)}{2}\right)^{\frac{m}{2}} e^{i(\nu\phi_x(\theta) + \mu\phi_y(\theta) - k\theta)} \qquad (3)$$

The key idea of CPT is to apply a symplectic near–identity transformation ($\vec{\Psi}, \vec{J} \to \vec{\Xi}, \vec{I}$) to eliminate as many as pos-

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sible $\vec{\Psi}$ and θ dependent terms, thereby approximating the orignal system by an integrable one. Such a transformation can be found (to all orders CPT) if a) the perturbation is sufficiently small and b) the tunes Q_x and Q_y fulfill a set of diophantine conditions, i.e. if $\min_{k\in\mathbb{Z}} Q_{\vec{\alpha},k}$ is non–zero and decays sufficiently *slow* with the resonance–order $|\nu|+|\mu|$. Here we are interested in the opposite case where there are some resonant $\vec{\alpha}_r$ so that $\exists k \in \mathbb{Z}$ with $Q_{\vec{\alpha}_r,k} \approx 0$. Note that a) if $\vec{\alpha}_r = (n, m, \nu_r, \mu_r)$ is resonant then so is $(n, m, j\nu_r, j\mu_r)$ with $j \in \mathbb{Z}$ and b) all the *detuning terms* (with $\nu = \mu = 0$) are "resonant" in the sense that they cannot be eliminated, however they do not destroy integrability. Some tedious algebra shows [4] that the *resonant normal form* to 2-nd order is given by the Hamiltonian

$$K(\vec{\Xi}, \vec{I}, \theta) = \sum_{\vec{\alpha}_r, k_r} \left(h_{\vec{\alpha}_r,k_r} + \frac{g^{(x)}_{\vec{\alpha}_r,k_r}}{I_x} + \frac{g^{(y)}_{\vec{\alpha}_r,k_r}}{I_y} \right) \times I_x^{\frac{n_r}{2}} I_y^{\frac{m_r}{2}} e^{i(\nu_r \Xi_x + \mu_r \Xi_y + Q_{\vec{\alpha}_r,k_r}\theta)}, \quad (4)$$

where the *1-st order driving terms* $h_{\vec{\alpha}_r,k_r}$ are the resonant terms from (1) and the $g^{(x)}_{\vec{\alpha}_r,k_r}$ and $g^{(y)}_{\vec{\alpha}_r,k_r}$ are the *2-nd order driving terms* given by

$$g^{(x)}_{\vec{\alpha}_r,k_r} = -\sum_{\vec{\alpha}'+\vec{\alpha}''=\vec{\alpha}} \frac{(n''+2)\nu'}{8\pi^2} \int_{-\pi}^{+\pi} d\theta \int_{-\pi}^{+\pi} d\theta' \times H_{\vec{\alpha}''}(\theta) H_{\vec{\alpha}'}(\theta') f_{\vec{\alpha}'}(\theta), \quad (5)$$

where $f_{\vec{\alpha}'}(\theta)$ is a known function of θ which depends on whether $\vec{\alpha}'$ is resonant or not, and the $g^{(y)}_{\vec{\alpha}_r,k_r}$ are computed analogously except for $(n''+2)\nu' \to (m''+2)\mu'$. Note that (5) implies that sextupoles with 1-st order driving terms of resonance–order 3, generate 2-nd order driving terms up to resonance–order 6.

RESONANCE WIDTHS

We now assume that the system is close to a resonance $\nu Q_x + \mu Q_y + k \approx 0$ and that all but one driving terms for that resonance vanish. Then the system is well approximated by the *single resonance* Hamiltonian $K^{\vec{\alpha},k}(\vec{\Xi}, \vec{I}; \theta) = r_{\vec{\alpha},k} I_x^{\frac{n^*}{2}} I_y^{\frac{m^*}{2}} e^{i(\nu\Xi_x+\mu\Xi_y+Q_{\vec{\alpha},k}\theta)}$, where $r := h, g^{(x)}$ or $g^{(y)}$ and $n^* := n$, if $r = h, g^{(y)}$ or $n^* := n-2$ if $r = g^{(x)}$ plus the analogous for m^* if $r = h$ or $g^{(x)}$. Now we introduce $R := |r|$ and $\chi := \arg(r)$ and make the system autonomous by a canonical transformation that winds back the *slow* rotation through $Q_{\vec{\alpha},k}$. The new Hamiltonian (re-using H, $\vec{\Psi}$ and $\vec{J}$) is given by

$$H^{\vec{\alpha}k}(\vec{\Psi}, \vec{J}) = \vec{\Delta}^{\vec{\alpha},k} \cdot \vec{J} + R_{\vec{\alpha},k} J_x^{\frac{n^*}{2}} J_y^{\frac{m^*}{2}} \cos(\nu\Psi_x + \mu\Psi_y + \chi_{\vec{\alpha},k}), (6)$$

where $\vec{\Delta}^{\vec{\alpha},k}$ is constrained by $Q_{\vec{\alpha},k} - \nu\Delta_x^{\vec{\alpha},k} - \mu\Delta_y^{\vec{\alpha},k} = 0$. In passing we note that $H^{\vec{\alpha}k}$ and $\mu J_x - \nu J_y$ and are integrals of motion. The (sum-) *resonance width* on a torus described by $\vec{J}$ is defined as the $\|\vec{\Delta}^{\vec{\alpha},k}\|$ for which $H^{\vec{\alpha},k}$ has fixed points $\partial_{\vec{\Psi}} H = \partial_{\vec{J}} H = 0$ at $\vec{J}$. The reason is simply that these fixed points determine the approximate distance from $\vec{J} = 0$ outside which the motion becomes potentially unstable for a sum resonance. In the case of several non vanishing $R_{\vec{\alpha},k}$ with different n, m for the same ν, μ, k we approximate the width by the absolute value of the vector sum of the $\vec{\Delta}^{\vec{\alpha},k}$'s. Tab. 1 summarizes the results of the calculation for the luminosity optics of 2002 (`helumgj`), 2003 (`helumsm`) compared to the last (pre–upgrade) optics from 2000 (`helumiv7`). The resonances $4Q_x$ and $4Q_x$ are strongly enhanced and thus limit the available tune space, while $2Q_x + 2Q_y$, which crosses the beam–beam footprint in fig. 1, is only marginally affected. Note that before the upgrade (and in fact after mid 2003) collisions in HERA were established at positron tunes far away from the 4-th order resonances.

Table 1: Ratio of resonance widths of e^+-optics `helumgj` (2002) and `helumsm` (2003) w.r.t. `helumiv7` (pre–upgrade) computed with `CANO`

$\nu Q_x + \mu Q_y$	(yr.02)/(pre)	(yr.03)/(pre)
$4Q_x + 0Q_y$	3.07	2.24
$2Q_x + 2Q_y$	1.20	0.72
$0Q_x + 4Q_y$	2.85	18.35

OCTUPOLE CORRECTION

Since in 2002 it was not completely clear whether operating HERA–e at tunes in a save distance from the 4-th order resonances could be achieved [5], it was decided to look into possible schemes for weakening the strengths of the resonances $4Q_x$, $2Q_x + 2Q_y$ and $4Q_y$. The only technically feasible scheme seemed to place a small number of octupoles, which drive the 4-th order resonances already at 1-st order CPT, in some suitable places in the West straight section, i.e. to compensate accumulated effects of the distributed chromatic sextupoles globally with localized octupoles.

Sextupoles contribute the independent 1-st order driving terms $h_{3,0;3,0}$, $h_{3,0;1,0}$, $h_{1,2;1,2}$, $h_{1,2;1,0}$ and $h_{1,2;1,-2}$. Eq. (5) tells us which 2-nd order driving terms for 4-th order resonances they generate. However, (4) shows that some of them come with powers of $\vec{J}$ that are not the same as the corresponding 1-st order octupole terms. Therefore not all driving terms can be canceled globally throughout phase space. Tab. 2 lists these terms. Our strategy was

Table 2: 2-nd order driving terms due to sextupoles and 1-st order term due to octupoles. The sextupole terms are divided in terms with powers of the J_i being *incompatible* and *compatible* for global cancellation by octupoles. The index k is suppressed.

ν, μ	2-nd order sxt		1-st ord. oct
	incompat.	compat.	
4,0	$g^{(x)}_{4,2;4,0}$	$g^{(x)}_{6,0;4,0}$	$h_{4,0;4,0}$
2,2	$g^{(x)}_{2,4;2,2}$	$g^{(y)}_{2,4;0,4}$, $g^{(x)}_{4,2;2,2}$	$h_{2,2;2,2}$
0,4	$g^{(y)}_{2,4;0,4}$	$g^{(x)}_{2,4;0,4}$	$h_{0,4;0,4}$

to completely cancel the sextupole terms that have a $\vec{J}$–dependence that is compatible with octupoles and leave the other terms unchanged. To minimize the real and imaginary parts of 3 resonances at least 6 octupoles, located at positions with suitable beta–functions and betatron phase advances, are needed. Once such locations are found, solving a linear system of minimization constraints leads to the required integrated octupole strengths. Note that octupole strengths for our example turned out to be rather large. The result of comparing the resonance widths with and without octupoles in 2-nd order CPT is summarized in tab. 3. Instead of reducing the resonance widths, the octupoles increased them up to a factor of 2. The reason is fairly simple: Already the *number* of 2-nd order driving terms introduced through octupoles and cross terms of sextupoles with octupoles is typically a factor of two bigger than with the chromatic sextupoles only. Tab. 4 compares these numbers explicitly.

Table 3: Ratio of resonance width of e^+-optics **helumsm** (2003) with chromatic sextupoles and octupole correction (SO) and octupole correction only (nO) w.r.t. chromatic sextupoles only (Sn) computed with CANO

$\nu Q_x + \mu Q_y$	nO/Sn	SO/Sn
$4Q_x + 0Q_y$	0.95	1.93
$2Q_x + 2Q_y$	0.14	1.14
$0Q_x + 4Q_y$	0.10	1.10

Table 4: Number of 2-nd order driving terms for sextupoles only (Sn), octupoles only (nO) and both (SO).

	$4Q_x$	$2Q_x + 2Q_y$	$4Q_y$
Sn	2	3	2
nO	4	4	3
SO	7	7	6

Another problem of the 6-octupole scheme is the strongly increased detuning. The detuning part of the 2-nd order normal form Hamiltonian is at most quadratic in the actions for sextupoles only while it contains cubic terms when octupoles are introduced. Fig. 2 shows the vertical detuning ΔQ_y vs. the orbital amplitudes. Obviously the main contribution comes from the octupoles (green). The sextupole contribution (red) is smaller and mostly opposite in sign, thus softening the total detuning (blue) slightly. We note that in principle it is possible to completely cancel all detuning due to octupoles once 3 more independent octupoles at suitable beta–functions and phases are introduced.

Finally the 6-octupole scheme was analyzed using a version of SIXTRACK. In a double loop particles with varying horizontal and vertical amplitudes and a fixed initial momentum deviation where tracked through the full 6D lattice with (black x-es) and without (red crosses) octupoles for 1024 turns. Fig. 3 shows the survival plot for both ensembles. The chosen 6–octupole scheme reduced the short term dynamic aperture by a factor 4–5.

The conclusion is that analytical methods (2-nd order CPT) and 6D symplectic tracking agree that compensating the effect of strong and distributed chromatic sextupoles by a localized group of the minimal number of octupoles is difficult if not impossible in HERA–e. However, the problem was solved [5] by moving the working point away from the 4-th order resonances.

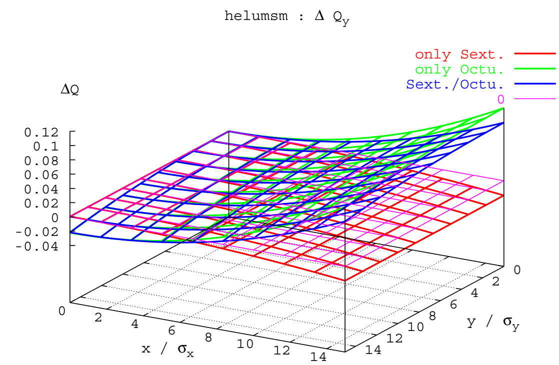

Figure 2: The vertical amplitude dependent tune shift in 2-nd order CPT due to chromatic sextupoles, the correcting octupoles and the combination of both.

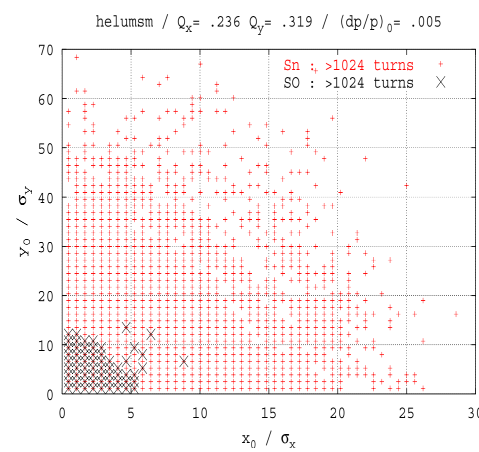

Figure 3: Survival plot for 6D tracking with SIXTRACK

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