WAKEFIELD CALCULATIONS FOR TTF-II

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Abstract

In this paper we estimate long- and short-range wake functions for new elements to be used in TESLA Test Facility (TTF) - II. The wake potentials of the LOLA-IV structure and the 3rd harmonic section are calculated numerically for very short bunches and analytical approximations for wake functions in short and long ranges are obtained by fitting procedures based on analytical estimations. The numerical results are obtained with code ECHO [1] for high relativistic Gaussian bunches with RMS deviation up to 0.015 mm. The calculations are carried out for the complete structures (including bellows, rounding of the irises and the different end cell geometries) supplied with ingoing and outgoing pipes. The low frequency spectra of the wake potentials is calculated using the Prony-Pisarenko method.

LONGITUDINAL WAKE FUNCTION OF LOLA-IV STRUCTURE

The LOLA-IV transverse deflecting cavity has to be used as a diagnostic for measuring the length of very short bunches in the TTF-II.

The LOLA structure consists of 104 cells. The gap g for the middle cells is equal to 29.1338 mm and for the end cells - to 29.0957 mm. The irises with radius $a = 22.4409\,mm$ are rounded and have the thickness 5.842 mm. The initial part of the geometry is shown in Fig 1. The total length of the structure is equal to $\sim 3.6m$.

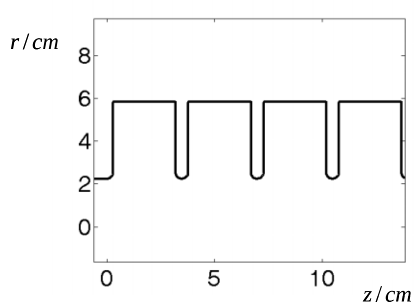

Fig 1. The geometry of the LOLA structure.

The calculated longitudinal wake potentials (solid lines) together with analytical approximations (2) (dashed lines) are shown in Fig. 2.

The LOLA cavity can be treated as a periodic structure of finite length. As shown in Ref. [2] the high frequency behavior of impedance is complicated. The high frequency dependence of impedance for an infinite periodic structure is $\omega^{-3/2}$ that differs essentially from finite structure $\omega^{-1/2}$ behavior. As the LOLA structure has a finite length there is a transition region where the frequency behavior of impedance changes from $\omega^{-3/2}$ to $\omega^{-1/2}$. The above argument explains the complicated oscillated behavior of the wake potentials for the shortest bunches (see Fig.2). To describe the oscillations, the one cell solution [3] is modified by a cosine factor preserving the original asymptotic behavior.

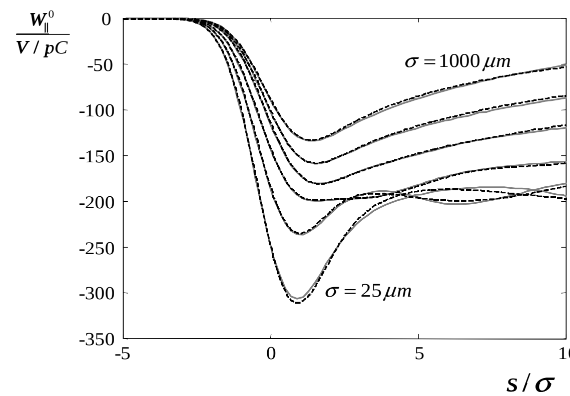

Fig 2. Comparison of numerical (solid lines) and "analytical" (dashed lines) longitudinal wake potentials in the LOLA structure for Gaussian bunches with $\sigma = 25, 50, 100, 250, 500, 1000\mu m$.

To find an analytical approximation of the wake function a fitting process was used. As an analytical model we used a combination of the modified one cell and periodic structure solutions:

$$w_\parallel^0(s) = -\theta(s)\left[Ae^{-\sqrt{s/s_0}} + B\frac{\cos(\omega s^\alpha)}{\sqrt{s} + Cs^\beta} \right]. \qquad (1)$$

The first term in equation (1) describes a periodic $O(1), s \to 0,$ behavior and as shown in Ref. [2,3] the coefficient can be estimated as

$$A = \frac{Z_0 c}{\pi a^2} L_{total} = 260 \cdot 10^{12},\ s_0 = 0.41\frac{a^{1.8} g^{1.6}}{L^{2.4}} = 4.8 \cdot 10^{-3}.$$

The second term in equation (1) should describe a finite structure $O(s^{-0.5}), s \to 0,$ behavior as well as oscillations seen in Fig. 2. As shown in Ref. [4] the coefficient B can be estimated as $B = Z_0 c\sqrt{L_{total}}\left(\sqrt{2}\pi^2 a\right)^{-1} = 0.7 \cdot 10^{12}$.

Fitting the wake function in form (1) to the numerical wake potentials shown in Fig.2 the following analytical expression is obtained (in [V/pC])

$$w_\parallel^0(s) = -\theta(s)\left[257.6e^{-\sqrt{\frac{s}{3.96 \cdot 10^{-3}}}} + 1.16\frac{\cos(1760 s^{0.72})}{\sqrt{s} + 1600 s^{1.23}} \right] \qquad (2)$$

The numerical coefficients in the equation are consistent with above analytical approximations.

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Fig. 3 shows wake function (5) together with numerical wake potentials (solid gray lines) outlined earlier in Fig.2. We see that the wake function tends to be an envelope function to the wakes.

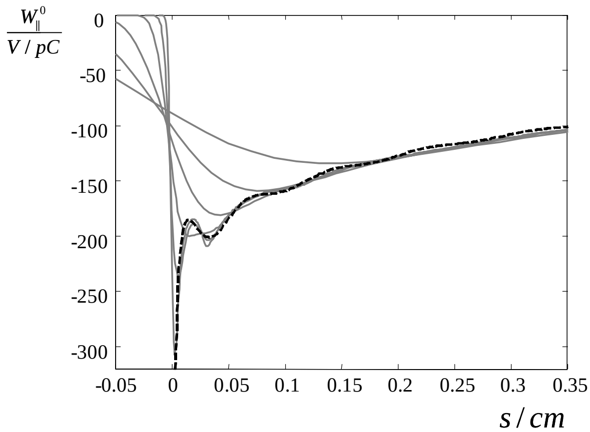

Fig 3. The "analytical" longitudinal wake function (dashed line) for the LOLA and numerical (solid lines) wake potentials for bunches with $\sigma = 25-1000\mu m$.

As the next check the above results can be compared to the analytical estimations given in Ref. [5], p.243. The impedance of structure consisting of N pillboxes at the high frequency limit can be estimated by an expression

$$Z_\parallel(\omega) = \sum_{k=1}^{N} (Y_1(\omega) + Y_2(\omega,k))^{-1} , \qquad (3)$$

$$Y_1(\omega) = a\pi \left(Z_0 c\right)^{-1} \left(1 + i\ sign(\omega)\right) \left(\pi |\omega| c g^{-1}\right)^{0.5} ,$$

$$Y_2(\omega,k) = Z_0^{-1} \alpha \sqrt{k-1} \tan^{-1} \left(\alpha (2\sqrt{k})^{-1}\right) ,$$

$$\alpha = (1 + i\ sign(\omega)) a \sqrt{\pi |\omega| (Lc)^{-1}} .$$

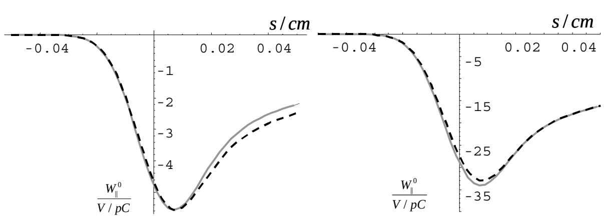

Fig 4. Comparison of numerical (solid line) and "analytical" (dashed line) longitudinal wake potentials for one cell (left) and ten cells (right) of the LOLA structure.

Fig. 4 shows comparison of the numerical and analytical (3) wake potentials in one and ten middle cells of the LOLA cavity for a bunch with $\sigma = 100\mu m$. Equation (3) is obtained in Ref. [5] for square irises. To take into account weakening of the wake fields due to rounding of irises an effective iris radius $\bar{a} = 1.13a$ is used in calculations and coincidence of the numerical and analytical curves can be seen in Fig. 4.

For the complete 104 cells LOLA structure the results are shown in Fig.5 for bunches with $\sigma = 100\mu m$ (left) and $\sigma = 25\mu m$ (right). We see that equation (3) is not able to describe transient behavior presented by cosine term in Eq.(1).

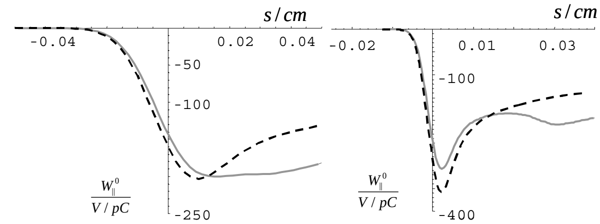

Fig 5. Comparison of numerical (solid line) and "analytical" (dashed line) longitudinal wake potentials for $\sigma = 100\mu m$ (left) and $\sigma = 25\mu m$ (right) in the LOLA.

To estimate long range wake fields the wake potential for Gaussian bunch with $\sigma = 1mm$ is calculated for distance up to 2 meter after the bunch.

The longitudinal wake function can be approximated by expression (in [V/pC])

$$w_\parallel(s) = -\theta(s) \left[2\sum_{i=1}^{\infty} K_i \cos(\frac{2\pi}{c} f_i s) + R(s) \right] . \qquad (4)$$

To obtain an approximation of the long-range wake function we keep in Eq. (4) only a finite number of addends corresponding to the lowest frequencies.

Table 1. The lowest frequencies and their amplitudes for long-range longitudinal wake function of the LOLA structure.

f_i , GHz	2.11	5.09	5.57	6.93	7.62	10.5	11	11.7
K_i , 10^{12}	34.6	3.26	5.76	4.7	6.4	1.3	2.6	2.8

The values are obtained using the Prony-Pisarenko algorithm [7] and are given in Table 1.

TRANSVERSE WAKE FUNCTION OF LOLA-IV STRUCTURE

In this section we repeat the exercise for transverse case. The calculated transverse wake potentials (solid lines) together with analytical approximations (dashed lines) are shown in Fig. 6.

To find an analytical approximation of the wake function a fitting process was used. As an analytical model we used a combination of the one cell and periodic structure solutions:

$$w_\perp^1(s) = \theta(s) \left[A s_1 \left(1 - (1 + \sqrt{s/s_1}) e^{-\sqrt{s/s_1}}\right) + B\sqrt{s} \right] . \qquad (5)$$

The first term in equation (5) describes a periodic $O(s), s \to 0$, behavior. The expression for estimation of the coefficients, suggested in Ref. [6], results in

$$s_1 = 0.169\frac{a^{1.79}g^{0.38}}{L^{1.17}} = 2.4\cdot 10^{-3},\, As_1 = \frac{4Z_0 c}{\pi a^4} s_1 L_{total} = 4951\cdot 10^{12}.$$

The second term in equation (5) should describe a finite structure $O(s^{0.5}), s \to 0,$ behavior. As shown in Ref. [4] the coefficient B can be estimated as

$$B = 2a^{-2} Z_0 c\sqrt{2L_{total}}\left(\pi^2 a\right)^{-1} = 5467\cdot 10^{12}.$$

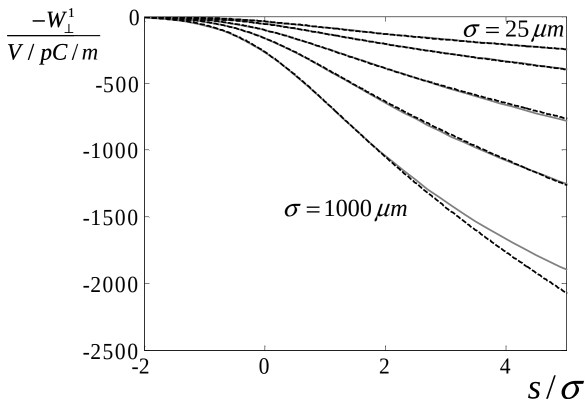

Fig 6. Comparison of numerical (solid lines) and “analytical” (dashed lines) transverse wake potentials for the LOLA structure.

Fitting the wake function in form (5) to the numerical wake potentials shown in Fig.6 the following analytical expression is obtained (in [V/pC/m])

$$w_\perp^1(s) = \theta(s)\left[10200\left(1-\left(1+\sqrt{\frac{s}{11.7\cdot 10^{-3}}}\right)e^{-\sqrt{\frac{s}{11.7\cdot 10^{-3}}}}\right)+9200\sqrt{s}\right]$$

The numerical coefficients in the equation are consistent at least in order with analytical approximations.

The long range transverse wake function is given in Ref.[8].

WAKE FUNCTIONS OF THE 3RD HARMONIC SECTION

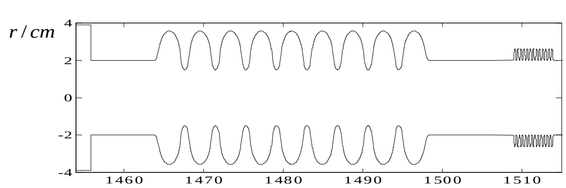

Fig 7. The part of the geometry of the 3rd harmonic section. Only one cavity is shown. The whole structure includes 4 cavities and 3 bellows.

For the phase 2 of the TESLA test facility (TTF 2) it has been planed to use a cavity section operated at three times the 1.3 GHz frequency of the existing TTF1 cavities to compensate nonlinear distortions of the longitudinal phase space. The 3rd harmonic section consists of 4 TESLA-like (but reduced by factor 3) cavities with 13-fold bellows in between. The iris radius is $a = 15\,mm$. The cavities are connected with pipes of the radius $a_p = 20\,mm$. On both ends of the section there are step transitions from a_p to $b = 39mm$ as shown in Fig 7.

Fitting analytical wake function models [10] to the numerical wake potentials the analytical expression are obtained (in [V/pC] and [V/pC/m], correspondingly)

$$w_\parallel^0(s) = -\theta(s)\left[318e^{-\sqrt{\frac{s}{8.4\cdot 10^{-4}}}} + 0.9\frac{\cos(5830 s^{0.83})}{\sqrt{s}+195s} + 0.036\delta(s)\right],$$

$$w_\perp^1(s) = \theta(s)\left[\begin{array}{l} 2232\left(1-\left(1+\sqrt{\frac{s}{0.56\cdot 10^{-3}}}\right)e^{-\sqrt{\frac{s}{0.56\cdot 10^{-3}}}}\right)+ \\ +5441\sqrt{s}+88.5 \end{array}\right].$$

The analytical models, estimation of the coefficients and long-range wake functions of the 3rd harmonic section are described in details in Ref. [8].

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