DYNAMICAL EFFECTS IN CROSSING OF THE MONTAGUE RESONANCE

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Abstract

In high-intensity accelerators space-charge-induced emittance coupling, known as Montague resonance, is known to occur for small tune split, where it can lead to emittance equilibration. We show here by simulation that new phenomena arise for slow crossing of this resonance. In 2D coasting beams the crossing leads to practically pure exchange of emittances, in spite of the underlying nonlinearity, while the beam remains intrinsically self-matched. In 3D bunched beams an additional mixing effect by synchrotron motion is found, which suppresses complete exchange, depending on the speed of crossing.

INTRODUCTION

In a detailed analytical single-particle analysis Montague[1] pointed out that the space charge driven fourth order difference resonance may lead to emittance coupling. It can be avoided only by a sufficient splitting of tunes. In recent years interest in this resonance has returned both in high-intensity applications of linear accelerators as well as circular accelerators with detailed self-consistent simulation studies in Refs. [2, 3]. In linacs it was identified as "equipartitioning", a source of coupling, which can have the effect of transferring emittance from longitudinal to transverse (or vice versa) during a relatively small number of betatron periods only [5]. In circular accelerators as in the CERN Proton Synchrotron this resonance may influence the high-intensity performance. In the KEK synchrotron it was also observed during multi-turn injection and studied by simulation [6]. The scope of the present study is to show that for crossing through the stop-band new phenomena arise with significant differences between coasting and bunched beams.

For the simulations presented here we are largely relating to an experiment carried out during a high-intensity machine development time at the CERN Proton Synchrotron (PS) in September 2003 [7]. The number of protons in the single-bunch (200 ns long at 4σ) was 1×10^{12} at 1.4 GeV, where a flat-bottom was provided for carrying out the measurements. The vertical tune was kept fixed at $Q_y = 6.21$. Due to injection flexibility from the PS booster, the vertical emittance could be chosen about 1/3 of the horizontal emittance, which enabled observation of a pronounced exchange effect and led to a maximum space charge tune shift (in the bunch center) of $\Delta Q_y \approx 0.1$ and $\Delta Q_x \approx 0.06$. In the subsequent simulations we have, however, increased the intensity by a factor of 5, hence also the tune shifts, which enhances the space charge nonlinearity by the same factor and allows a general reduction of the required simulation times by a factor 5.

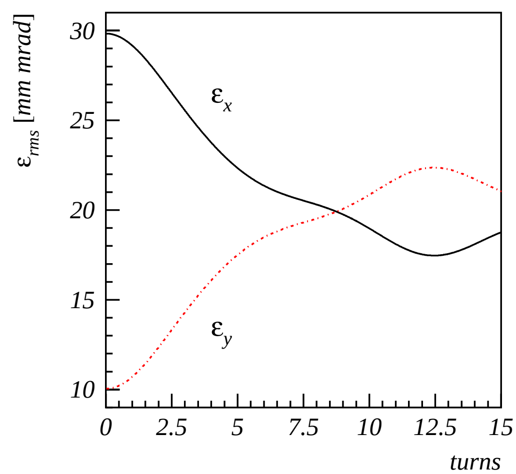

Figure 1: Rms emittances in 2D for a typical static working point ($Q_x = 6.19, Q_y = 6.21$).

2D SIMULATION

We first present results for coasting beam 2D simulations using the particle-in-cell code MICROMAP [9]. The lattice is approximated by constant focusing, and a Gaussian initial distribution is used. For static tunes the subject was extensively studied in Ref. [10]. A typical result is the finding of partial or full emittance equalization ("equipartition") within the stop-band of this resonance. This is shown in Fig. 1 for $Q_x = 6.19$. Results for slightly smaller tunes show even less emittance exchange, and none at all outside the stop-band, which has a width of about $\Delta Q_x \approx 0.2$ in our simulations. Note the short time scale of about 10 turns during which the coupling occurs, which is proportional to the space charge tune shift.

Dynamical crossing of the stop-band by a slow tune variation starting sufficiently distanced from the stop-band shows, however, a different behavior. In Fig. 2 we plot the result for a tune ramp over the interval $Q_x = 5.85 \rightarrow 6.45$ over 30 as well as over 100 turns. For the slow crossing over 100 turns emittances evolve smoothly with a cross-

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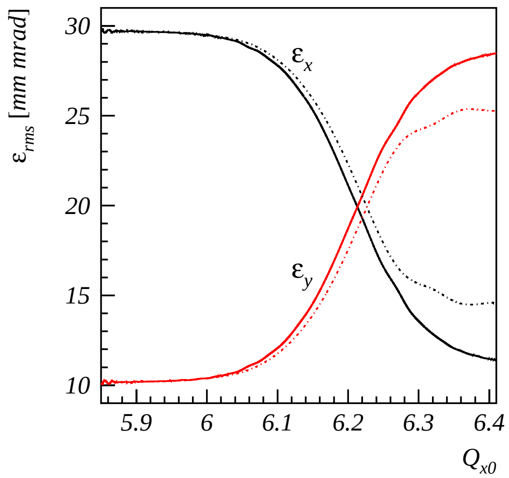

Figure 2: Rms emittance evolution in 2D for a tune ramp $Q_x = 5.85 \rightarrow 6.45$ over 30 (fast, dotted line) and 100 (slow, continuous line) turns.

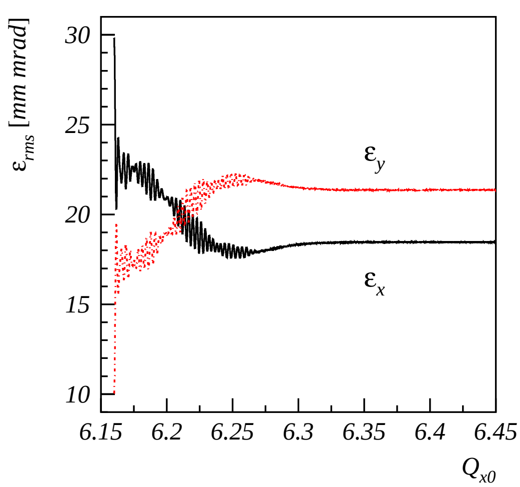

Figure 3: Rms emittance evolution in 2D for tune ramp $Q_x = 6.15 \rightarrow 6.45$ over 1000 turns.

ing at $Q_x = 6.21$ ($= Q_y$) and nearly full reversal beyond (the reversal is even more complete for crossing over 1000 turns). For the faster crossing over 30 turns the reversal is, however, less complete and less smooth. On the other hand, the emittance reversal is largely suppressed if the simulation is started well inside the stop-band, where rapid emittance equilibration occurs as is shown in Fig. 3. Note that the tune ramp in this case (taken over 1000 turns) is about a factor 20 slower than for the slower case in Fig. 2. The significant difference in behavior between Fig. 2 and Fig. 3 can be explained in the following way:

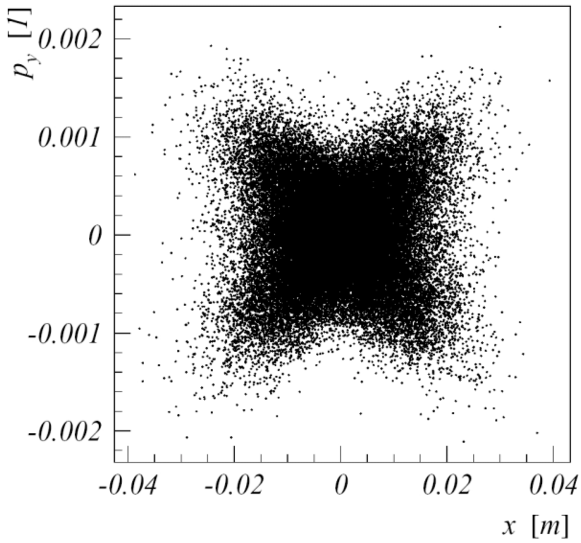

Figure 4: Correlated phase space plot inside the stop-band ($Q_x = 6.19$) showing a self-consistent imprint of the nonlinear space charge potential.

The slow – adiabatic – approach of the space charge resonance in Fig. 2 makes the distribution closely follow an intrinsically matched one, which is at all points fully self-consistent with respect to the Hamiltonian including the nonlinear space charge potential. In the case of Fig. 3 we are starting the simulation with the a distribution that is not intrinsically matched – only rms matched. Rapid action of the Montague resonance results, which brings the emittances closer together as shown in Fig. 1. This process is largely irreversible, hence leaving the stop-band adiabatically does not allow the emittances exchange to the extent found in Fig. 2.

In order to illustrate the nature of a fully nonlinearly self-matched solution, it is useful to take a closer look at the phase space distribution. Obviously, away from the resonance the nonlinear space charge potential is negligible, whereas it becomes significant at the resonance. There it causes a pronounced distortion of the phase space distribution reflecting the nonlinearity. This is shown in Fig. 4 for the correlated phase space $x - p_y$ related to the slow crossing in Fig. 2, where the four-fold symmetry is a direct imprint of the predominant fourth order space charge potential term. An important feature of Fig. 2 to underline here is that the rms emittances are equal at the point, where $Q_x = Q_y = 6.21$. This does not, however, imply "equipartition", since the emittance equilibration is fully reversible: for $Q_x > Q_y$ the emittance ratio is reversed, and the initial emittances are basically switched between the two planes since the correlated phase space keeps full memory of the initial emittance imbalance. After crossing – sufficiently distanced from the stop-band – the four-fold symmetry vanishes again gradually, and the phase space plots reflect the typical Gaussian behavior with elliptical contours.

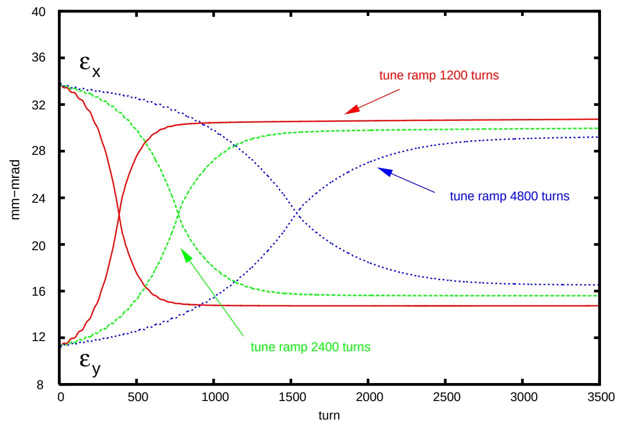

Figure 5: Rms emittances in 3D bunched beam for different tune ramps and fixed synchrotron period (100 turns).

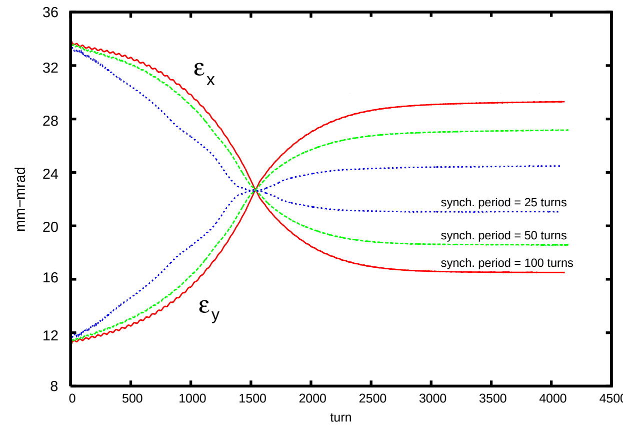

Figure 6: Rms emittances in 3D bunched beam for given tune ramp, but doubled and quadrupled synchrotron frequency.

3D SIMULATION

For the bunched beam simulation we use the fully 3D particle-in-cell code IMPACT [8] and employ 5 million simulation particles on a grid of 65x65x257 points assuming, for simplicity, a constant focusing lattice. Tests with different numbers of particles have shown a slow, non-negligible growth of about 5% per 1000 turns for all emittances, if only 1 million particles were used, which disappeared for more than 4 million particles. The bunch length was chosen to be 200 ns, but the synchrotron period was reduced to a value as low as 100 turns (6 times faster than in the experiment) by increasing the momentum spread of the simulation bunch by a factor 6 compared with the experiment. At the same time the space charge tune shift was also enhanced by the same factor (as in the 2D simulations) to equally speed up the effect of the Montague resonance. This re-scaling helped to cut down the simulation time (several days per job) by the same factor. In Fig. 5 we show results for the rms emittances, where a linear tune ramp $Q_x = 6.05 \rightarrow 6.55$ was realized in 1200, 2400 and 4800 turns. Surprisingly, the trend for slower tune ramps is opposite to the coasting beam case of Fig. 2: For slower ramps the emittance exchange is less effective. We explain this new feature as a result of mixing caused by the additional synchrotron motion, provided that the number of synchrotron periods over which the stop-band crossing occurs is large. The picture employed here is that synchrotron motion gradually erodes the coherence and phase space memory effect, which was made responsible for the emittance exchange in the coasting beam case. Note that in case of the continuous (red) line we have slightly over 3 synchrotron periods to reach the point of exact resonance ($Q_x = 6.21$), whereas this is doubled for the dashed (green) line and doubled again for the dotted (blue) line.

The effect is demonstrated further in the simulation of Fig. 6, where the tune ramp is held fixed, whereas the number of synchrotron periods per crossing is increased by shortening the bunch length and thus doubling the synchrotron frequency in two steps. Note that in Fig. 6 the continuous (red) line equals the slowest ramp case (dotted blue) in Fig. 5 ; the green dashed line to a case with half the synchrotron period (50 turns), and the blue dotted again halved (25 turns). Hence the latter case, with about 50 synchrotron periods up to the exact resonance point, shows that emittance reversal is nearly fully suppressed.

In summary, we have found that slow crossing through a nonlinear space-charge-induced resonance follows a self-matched solution in the absence of synchrotron motion. For bunched beams – with 3D PIC-simulations pushed to the extreme in terms of number of turns and particles– this appears to be suppressed. Experiments are needed to verify these predictions.

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