# A METHOD TO MEASURE THE SKEW QUADRUPOLE STRENGTHS IN THE SIS-18 USING TWO BPMS 

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## Abstract

In the GSI synchrotron SIS-18 a new set of skew quadrupoles has been installed to improve the multi-turninjection. A new method based on the measurement of the resonance driving terms (RDT) has been proposed to crosscheck the nominal values and polarities of their gradients. Once a beam is transversely kicked, it experiences oscillations whose spectrum contains both the betatron tune line and secondary lines. The amplitude of each line is proportional to the strength of the multipoles, such as skew quadrupoles and sextupoles, present in the lattice. In this paper a recursive algorithm to derive the magnet strength from the spectral lines and the application of this method to the eight skew quadrupoles in the SIS-18 are presented.

## INTRODUCTION

The possibility of reconstructing the magnetic potential experienced by the beam in particle accelerators can be of great help during the machine commissioning and the routine maintenance. Once the corrector magnets such as skew quadrupoles and sextupoles are installed in the beam line, it is important to check that the power supplies generate the requested magnetic strengths and polarities.

So far beam-based nonlinear lattice modeling has been performed looking at the nonlinear optics (nonlinear chromaticity and amplitude depending detuning) [1], providing information about the global machine nonlinearities.

Once the beam has been transversally displaced, it experiences coherent oscillation which can be recorded turn-by-turn by beam position monitors (BPMs). If the lattice is purely linear the spectrum of these oscillations contains only the betatron tune line, whereas in precence of nonlinearities secondary lines appear. The frequency map analysis [2] provided an accurate $\left(\propto 1 / N_{\text {turn }}^{4}\right)$ method to infer all the excited spectral lines of the complex variable $x+i p_{x}$. Experimental applications of this method [3] provided information on excited resonances and nonlinear dynamics.

Constructing the normal form of the one-turn map [4, 5], it is possible to establish the connection between the spectrum of this complex variable and the resonances excited by multipoles, i.e. between the secondary lines and the Resonance Driving Terms (RDT). If several BPMs are available, from the variation along the ring of the RDT it is possible to localize the nonlinearities [6]. This method has been successfully applied at the CERN SPS [7] where the RDT have

[^0]been measured, polarity errors have been detected and linear coupling has been corrected using a method faster than the minimization of $\Delta Q_{\text {min }}$.

So far no explicit relations have been established between the spectral lines, or the RDT, and the corrector magnet strengths. In this paper this link is sketched together with simulations of the GSI heavy ion synchrotron SIS-18. The status of a multi-BPM turn-by-turn acquisition system being currently developed for this purpose is also reported.

## THEORETICAL SKETCHES

The normalized betatron oscillations after $N$ turns under a focusing lattice with skew quadrupoles can be written in terms of Fourier components

$$
\begin{aligned}
& \tilde{x}_{N}-i \tilde{p}_{x N}=a_{0} e^{i 2 \pi N \nu_{x}}+\sum_{j k l m} a_{j k l m} e^{i 2 \pi N\left(\theta_{j k} \nu_{x}+\omega_{l m} \nu_{y}\right)} \\
& a_{0}, a_{j k l m} \in \mathbf{C}, \quad \theta_{j k}, \omega_{l m} \in \mathbf{N}, \quad j+k=1, l+m=1
\end{aligned}
$$

The first term in the r.h.s. is the tune line. The secondary lines contained in the sum are generated by the skew quadrupoles and driven by the corresponding RDT $f_{j k l m}$. For explict expressions of $\theta_{j k}$ and $\omega_{l m}$ see [4]. The amplitude and phase of the excited lines as function of the RDT are listed in Tab. 1. The RDT can be inferred after removing the dependence of the secondary lines on the actions $I_{x, y}$ and betatronic phases $\psi_{x, y, 0}$ using the amplitude and the phase of the tune lines.

> NORMAL QUADRUPOLE TERMS

|  | H-line | V-line | $\left\|a_{0}\right\|$ | $\phi_{0}$ |
| :--- | :--- | :--- | :---: | :--- |
| $y^{2}$ |  | $(0,1)$ | $\sqrt{2 I_{y}}$ | $\psi_{y, 0}$ |
| $x^{2}$ | $(1,0)$ |  | $\sqrt{2 I_{x}}$ | $\psi_{x, 0}$ |

SKEW QUADRUPOLE TERMS $\propto x y$

| $j k l m$ | H-line | V-line | $\left\|a_{j k l m}\right\|$ | $\phi_{j k l m}^{a}+\frac{\pi}{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0110 |  | $(1,0)$ | $2\left\|f_{0110}\right\| \sqrt{2 I_{x}}$ | $\phi_{0110}^{f}+\psi_{x, 0}$ |
| 1001 | $(0,1)$ |  | $2\left\|f_{1001}\right\| \sqrt{2 I_{y}}$ | $\phi_{1001}^{f}+\psi_{y, 0}$ |
| 1010 | $(0,-1)$ | $(-1,0)$ | $2\left\|f_{1010}\right\| \sqrt{2 I_{y}}$ <br> $\phi_{1010}^{f}-\psi_{y, 0}$ |  |
| $\phi_{1010} \mid \sqrt{2 I_{x}}$ | $\phi_{1010}^{f}-\psi_{x, 0}$ |  |  |  |

Table 1: Amplitude and phase of the tune lines (up) and of the secondary spectral lines driven by skew quadrupole RDT (down).

In Fig. 1 an example of measured spectrum obtained after switching on a skew quadrupole is shown. A recursive algorithm has been found to derive the Hamiltonian coefficients from the RDT. We assume to have $W$ skew


Figure 1: Example of horizontal spectrum with a skew quadrupole powered on.
quadrupoles $(M)$ distributed around the ring and the same number of BPMs $(B)$ placed in between (see Fig. 2).


Figure 2: Schematic view of skew quadrupoles and BPMs around the ring.

The Hamiltonian coefficient at the generic location $w$ depends on the difference between the RDT evaluated at the two closest BPMs, via

$$
\begin{align*}
h_{w, j k l m}= & e^{i\left[(j-k) \phi_{w, x}^{M}+(l-m) \phi_{w, y}^{M}\right]} \times \\
& \left(f_{w, j k l m} e^{-i\left[(j-k) \phi_{w, x}^{B}+(l-m) \phi_{w, y}^{B}\right]}-\right. \\
& \left.f_{w-1, j k l m} e^{-i\left[(j-k) \phi_{w-1, x}^{B}+(l-m) \phi_{w-1, y}^{B}\right]}\right), \\
& \text { for } \quad 2 \leq w \leq W \tag{1}
\end{align*}
$$

whereas the first coefficient after the starting point reads

$$
\begin{aligned}
h_{1, j k l m}= & e^{i\left[(j-k) \phi_{1, x}^{M}+(l-m) \phi_{1, y}^{M}\right]} \times \\
& \left(f_{1, j k l m} e^{-i\left[(j-k) \phi_{1, x}^{B}+(l-m) \phi_{1, y}^{B}\right]}-\right. \\
& \left.f_{W, j k l m} e^{i\left[(j-k)\left(Q_{x}-\phi_{W, x}^{B}\right)+(l-m)\left(Q_{y}-\phi_{W, y}^{B}\right)\right]}\right)
\end{aligned}
$$

$Q_{x}$ and $Q_{y}$ are here the betatronic (linear) tunes, whereas $\phi_{w, x}^{B}$ and $\phi_{w, x}^{M}$ are the betatronic phases at the $w$-th BPM and skew quadrupole respectively. Once the Hamiltonian coefficient has been inferred, the magnet (integrated)
strength is computed via

$$
\begin{align*}
& h_{w, j k l m}=-\frac{\left[K_{w, n-1} \Omega(l+m)+i J_{w, n-1} \Omega(l+m+1)\right]}{j!k!\quad l!m!2^{j+k+l+m}} \\
& \times \quad i^{l+m}\left(\beta_{w, x}\right)^{\frac{j+k}{2}}\left(\beta_{w, y}\right)^{\frac{l+m}{2}},  \tag{2}\\
& \Omega(i)=1 \text { if } i \text { is even, } \quad \Omega(i)=0 \text { if } i \text { is odd }
\end{align*}
$$

where $\beta_{w}$ are the beta function at the multipole location. We made use of the multipole expansion
$H_{w}=-\Re\left[\sum_{n \geq 2}\left(K_{w, n-1}+i J_{w, n-1}\right) \frac{\left(x_{w}+i y_{w}\right)^{n}}{n!}\right]$
where $K_{n-1}$ and $J_{n-1}\left[\mathrm{~m}^{-(n-1)}\right]$ are respectively the integrated strengths for the normal and the skew multipoles that we want to measure.

## SIS-18 SIMULATIONS

To check the validity and the applicability of eqs. 1 and 2 to the commissioning of the new eight skew quadrupoles recently installed in the GSI SIS-18 several simulations have been run. The MIMAC [8] platform developed at GSI has been used for both optics calculations ( $\beta$ and $\phi$ at the skew quadrupole and BPM locations) and single particle tracking (to simulate the turn-by-turn data from all the available BPMs). The first goal was to verify the validity of the algorithm, i.e. the capability of reconstructing the skew quadrupole strength from the beam position data. The second was to check the robustness of the method against tune working point usually close to the linear coupling resonance $(1,-1)$ during the multi-turn injection. In Tab. 2 an example of skew quadrupole strength reconstruction is shown. In the last line, the negative polarity has been successfully detected, even if approaching the resonance (1,1) the error increases, because of the non-resonant normal form approach used.

| name | $J_{1}$ INPUT | $J_{1}$ FROM BPMs <br> $(4.20,3.40)$ | $J_{1}$ FROM BPMs <br> $(4.28,3.29)$ |
| :---: | ---: | :---: | :---: |
| S01KM3QS | 5.000 | 4.952 | 4.992 |
| S02KM3QS | 5.000 | 5.014 | 5.080 |
| S04KM3QS | 5.000 | 4.980 | 4.744 |
| S06KM3QS | 5.000 | 5.004 | 5.012 |
| S07KM3QS | 5.000 | 5.019 | 4.982 |
| S08KM3QS | 5.000 | 4.981 | 4.734 |
| S10KM3QS | 5.000 | 5.001 | 5.109 |
| S12KM3QS | -2.000 | -2.006 | -2.011 |

Table 2: Simulated magnet strength reconstruction for two different tune working points. The units of $J_{1}$ are $10^{-3} \mathrm{~m}^{-1}$

## EXPERIMENTAL ASPECTS

From the experimental point of view, the crucial point is the correct turn-by-turn construction of the complex signal $\tilde{x}-i \tilde{p}_{x}$, which means:

1. BPM turn-by-turn synchronization and triggering
2. Correct $p_{x}$ reconstruction from the neighbor BPM
3. Correct transformation to the normalized coordinates

Point 2. requires that the region between two consecutive pick-ups is free from nonlinearities. In this case the momentum can be inferred from the data of the neighbor BPMs [9]. Since the real linear optics not always corresponds to the model, the optical functions needed in point 3. are derived directly from the spectrum of $x$ and $y$ at each BPM [10]. Both points 2. and 3. depend anyway on the proper BPM data acquisition: not synchronized data would produce uncorrelated time series $x_{N}$ and $p_{x_{N}}$ and wrong optical functions.

## Status of the new BPM Data Acquisition System

A new PC data acquisition system has been developed in the last year and is still presently under development. The analog signals ( $\Sigma$ and $\Delta$ ) from a pick-up are digitized by means of a 20 MHz 4 -channel PCI card installed on PC. As trigger the event generator (Q-kicker signal) is used, whereas the sampling (turn-by-turn) clock is provided by the RF master signal (divided by the harmonic number $h=$ 4). The data are stored in 12-bits integer words, providing a resolution $\delta \Sigma / \Sigma=\delta \Delta / \Delta \simeq 5 \times 10^{-4}$ and $\delta x / x \simeq 10^{-3}$. A post-processing software converts these numbers in mm and performs the FFT of the time series (a "von Hann" filter with a peak-search routine [11]).

This approach differs from the one used, for example, at the CERN PS [12] where the sampling is performed in continuous mode with a fast digitizer at high frequency (500 MHz ). The beam position at each turn is then obtained from the full data stream with a peak-search algorithm. The advantages of our PCI card system are the lower hardware costs (a factor ten with respect to the fast digitizer) and smaller storage memory requirements and output files, already suitable for post-processing. The drawback is that the synchronization must be guaranteed during the acquisition since a wrong setting would compromise the entire acquisition, the full data being nonrecoverable. With a continuous fast sampling (each 2 ns ) this risk is automatically avoided.

A first test using one BPM was performed in 2003 when the betatron tunes were measured with an accuracy of $0.3 \%$. In the first half of 2004 a second PCI card was installed and a second BPM connected. To synchronize the two acquisitions both the trigger and the sampling clock of the second card have been delayed by the time-distance between the two BPMs. Having the SIS-18 a strictly 12-fold superperiodicity and being the 12 available BPMs at the end of each period, this distance corresponds to $1 / 12$ of the revolution time ( $4.689 \mu \mathrm{~s}$ ). All tests were performed at the injection energy ( $11.4 \mathrm{MeV} / \mathrm{u}$ ). To check the synchronization of the two cards we compared the phase advances between the two BPMs from the model with the ones obtained from the measured spectra $\psi_{x, y, 0}^{B P M 2}-\psi_{x, y, 0}^{B P M 1}$. In Fig. 3 the result of this test is plotted, showing a poor agreement
made worse by the proximity of the vertical phase advance to $90^{\circ}$, which makes the optical functions reconstruction problematic using the algorithm described in [10]. The use of SVD methods would prevent such a problem.


Figure 3: measured phase advance between the two BPMs with respect to the model.

The reason for this bad synchronization is not yet well understood and requires more dedicated tests. One reason might be the 32 -bit bus of the PCI card which might have prevented the simultaneous storage of all the 4 channels.

## CONCLUSIONS

A new beam-based method to infer the magnet strength of corrector magnets from BPM data has been developed. Numerical simulations confirm its applicability to the new SIS-18 skew quadrupole set at the typical tune working point for multi-turn-injection. A dedicated data acquisition system has been set up at the GSI SIS-18 using two BPMs but is still under development phase, because of synchronization problems.

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